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Nonlinear shaping of a two-dimensional ultrashort ionizing pulse

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A theoretical description of ultrashort ionizing wave pulses is presented by means of two different models where the ionization rate increases or decreases, respectively, as a function of the electric-field amplitude. We show that the pulse evolves either into a horseshoe- or a horn-type structure in the time-space domain. In some parameter regions the intensity of the pulse can also increase.

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Ionization of media under the action of a strong electromagnetic field has traditionally attracted interest and research in various branches of physics. Most attention has usually been paid to the problem of breakdown conditions and the properties of the emerging plasma. However, recent development of sources of ultrashort laser and microwave radiation [1,2], has intensified interest in the problem of nonlinear conversion of spectral intensity and the spatial characteristic of the pulse itself. It has been shown that the nonlinear interaction between the electromagnetic pulse and the plasma leads to a number of important effects, such as strong adiabatic frequency up-conversion [3,4], the generation of harmonics [5], pulse compression [6], etc.

Most theoretical models that describe the above-mentioned effects assume that the pulses are one dimensional. However, within the ionized medium, it is likely that refraction phenomena are essential. They are determined by the spatial structure of the electron density, and are therefore related to the ionization-recombination dynamics of the plasma and are a slowly decreasing effect for $|E|>E_m$ [usually E_m corresponds to the electron oscillation energy $eE_m/(4m\omega)\sim 10$ eV].

In the present paper we shall consider the nonlinear dynamics of an ultrashort pulse when the ionization processes play a dominant role. The density growth rate can then be modeled as

$$\frac{\partial N}{\partial t}=\Gamma(N,N_m,|E|)\ , \tag{1}$$

where the function Γ depends on the electron concentration N, the neutral-atom concentration N_m, and the amplitude of the wave electric field $|E|$. Considering electron-impact ionization we choose $\Gamma=f(|E|)NN_m(1-N/N_m)$, where f is a rapidly increasing function of $|E|$ in the interval $0<|E|<E_m$. In reality, this growth rate is a complicated functional of the electron-velocity distribution function and can be computed in the frame of a kinetic plasma description (see, e.g., Ref. [7]). The model dependences used below are in qualitative agreement with results of the kinetic theory and reflect the well-known features of the atom ionization cross section as a function of the incident electron energy.

For field-induced ionization the generally accepted description (see, e.g., Refs. [8] and [9]) is based on the Keldysh theory [10]. In this case $\Gamma=\Gamma(|E|)N_m(1-N/N_m)$, where $\Gamma(|E|)$ is a powerlike (multiphoton ionization) or exponentially growing (tunneling ionization) function of $|E|$ for all wavelength bands (from rf to near uv) that are now occupied by high-power radiation sources. For the uv band or for still higher frequencies, it can, however, occur that Γ decreases for sufficiently large $|E|$. This corresponds to the phenomenon of atom stabilization in superstrong electromagnetic fields [11]. The nonlinear dynamics of such short-wavelength radiation will not be discussed here.

In order to study the dynamics of two-dimensional electromagnetic pulses, we assume that the wave propagation is in the z direction, and that the wave amplitude depends on x and z but not on y. This assumption will facilitate our calculations below. For stationary self-focusing the physical picture would be significantly changed if we also included the y dependence, but for the present problem such a generalization would not lead to any new physical effects. The wave electric field (with frequency ω and wave number k) is thus written in the form $[E(x,z,t)\exp(-i\omega t+ikz)+\text{c.c.}]\hat{y}/2$ where the smoothly varying ($|\partial_x|$ and $|\partial_z|\ll k$) envelope satisfies the well-known equation

$$2ik\frac{\partial E}{\partial z}+\frac{\partial^2 E}{\partial x^2}+\frac{(1-\epsilon)}{c^2\epsilon}\frac{\partial^2 E}{\partial \tau^2}-\frac{\omega_p^2}{c^2}\frac{(N-N_0)}{N_0}E=0\ . \tag{2}$$

In Eq. (2), N_0 is unperturbed electron density, $\omega_p=(N_0q^2/\epsilon_0 m)^{1/2}$ is the plasma frequency, $\tau=t-\omega z/kc^2$ is the local time, and $\epsilon=1-\omega_p^2/\omega^2$ is the dielectric permitivity of the unperturbed plasma. We have neglected dissipative effects connected with the ionization and the electron collisions. This approximation is valid when the frequency-shift emerging and growing due to ionization is much larger than the effective collision frequency. At the same time we assume that the frequency shift is much smaller than ω.

A transversely localized pulse causes the electron concentration to grow faster in the central part than in the periphery when $\partial\Gamma/\partial|E|>0$. The electromagnetic fields will thus diverge, and the longitudinal ionization compression is accordingly less efficient. On the other hand, if $\partial\Gamma/\partial|E|<0$, the transverse refraction leads to field convergence. The interaction between the longitudinal and transverse effects will thus determine the compression as well as the resulting structure of the two-dimensional pulse. Below we shall analyze the nonlinear dynamics for these two cases with the use of computer simulation of the three-dimensional (z,x,τ) problem. The numerical simulations of Eq. (2) with different ionization models have been based on the standard Fourier-transform split-step method demanding periodic boundary conditions in x and τ. Both local time and transverse coordinate intervals have been chosen large enough to simulate localized structures and to eliminate artificial reflection from the boundaries. The validity of the final results has been checked by expanding the computation intervals.

When the increase in the electron density is small during the time the pulse passes through a given point, we can qualitatively analyze both the electron impact (when $|E|<E_m$) and the field-induced ionization mechanisms by using a common expression for Γ. In order to simplify this study we choose the form

$$\Gamma=\gamma_0 N_m |E|^2/E_0^2 \ , \tag{3}$$

where γ_0 and E_0 are constants measured in units of frequency and electric field strength, respectively. This representation of Γ allows one to approximate with certain degree of precision the experimental data corresponding to growing dependences of the ionization rate versus field intensity. The splitting into γ_0 and E_0 can be obviously performed in an arbitrary way by keeping their ratio constant, i.e., it is possible to take E_0 as the maximum initial field amplitude of the pulse. Normalizing z by $[c^2 2k\epsilon/(1-\epsilon)][(N_0/N_m)\epsilon\gamma_0\omega^2]^{2/3}$, x by $[c^2\epsilon/(1-\epsilon)]^{1/2}[(N_0/N_m)\epsilon\gamma_0\omega^2]^{1/3}$, τ by $[(N_0/N_m)\epsilon\gamma_0\omega^2]^{1/3}$ and introducing $n=[(N-N_0)/N_m](N_m\epsilon\omega^2/\gamma_0^2 N_0)^{1/3}$ and $A=E/E_0$, we obtain from (1), (2), and (3) the dimensionless equations

$$i\frac{\partial A}{\partial z}+\frac{\partial^2 A}{\partial x^2}+\frac{\partial^2 A}{\partial \tau^2}-nA=0 \ , \tag{4}$$

$$\frac{\partial n}{\partial \tau}=|A|^2 \ . \tag{5}$$

The normalizations used seem to be rather convenient since they demonstrate the scaling regularities of the problem. Varying γ_0 in proportion to E_0 and renormalizing the pulse characteristics: according to $\tau_p \sim \gamma_0^{1/3}$ and $L\sim\gamma_0^{1/3}$ we obtain for each solution of the dimensionless equations (4) and (5) a whole set of evolution scenarios for different pulses in the plasma with given unperturbed parameters.

In the one-dimensional case ($\partial/\partial x=0$), Eqs. (4) and (5) describe the ionizational compression effect [6], where those parts of the pulse which have large amplitudes at the center of its profile overtake the front parts. This results in a combined self-steepening and collapse process which involves the greater part of the wave energy. The characteristic compression distance [6] is $z_* = \tau_0^{1/2}/A_0$, where τ_0 and A_0 represent the initial duration and amplitude of the pulse. Figure 1 shows the compression of a pulse with the initial Gaussian form $A(0,\tau)=3\exp(-\tau^2/18.5)$, corresponding to the compression distance $z_*=0.69$.

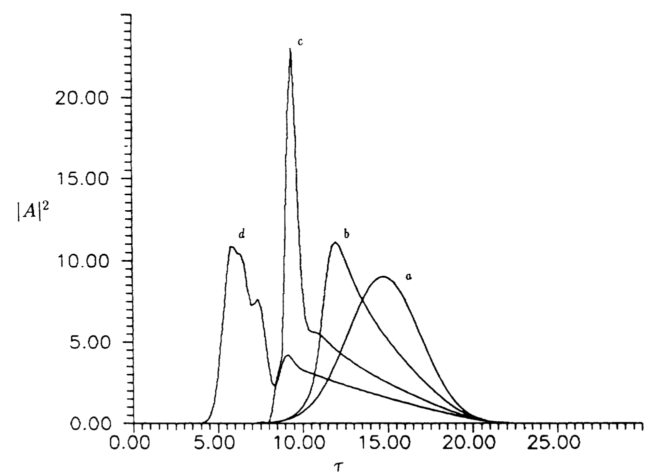

FIG. 1. Evolution of the one-dimensional pulse with the initial distribution $A(0,\tau)=3\exp(-\tau^2/18.5)$; a, b, c, and d correspond to $z=0$, 0.42, 0.69, and 0.84, respectively.

Consider next a slightly nonuniform initial field distribution with a transverse length scale $a_\perp$, and assume that the linear diffraction length $z_d \approx a_\perp^2/4$ is much larger than z_*. It turns out that the frequency increase of the pulse is largest at the center position x_c which also has the largest acceleration. As a result, the pulse is bent in the x-τ plane where it looks like a horseshoe. Figure 2 shows the horseshoe formation of a bell-shaped pulse $A(0,\tau,x)=A_0\exp(-\tau^2/\tau_0^2-x^2/a_\perp^2)$ with $A_0=3$, $\tau_0=18.5^{1/2}$, and $a_\perp=21.3$. The maximum intensity and the compression distance are for this case almost the same as in the one-dimensional example, i.e., the nonlinear refraction is here a minor effect.

Numerical studies show that the maximum compression can be obtained at $z<z^*$ for smaller transverse lengths $a_\perp$, although the compression then is less efficient. In order to compare with the one-dimensional situation, the evolution of pulses with $a_\perp=5.3$ and 2.1 are presented in Figs. 3 and 4, respectively. In Fig. 3 one can see that the nonlinear refraction is significant and that the longitudinal compression is weakened. In Fig. 4, even the electromagnetic field diffraction in the unperturbed plasma prevents the increase of the amplitude. However, also in this case the pulse converts into the form of a horseshoe.

Considering three-dimensional wave packets, it can be shown that the field amplitude in the central part of the pulse decreases somewhat faster as compared to the two-dimensional case. Thus we conclude that, using the model (3), the transverse structure of an ultrashort pulse causes field divergence. The ionization nonlinearities has here a defocusing character.

The sign of the nonlinearity and, accordingly, the direction of the transverse refraction will, however,

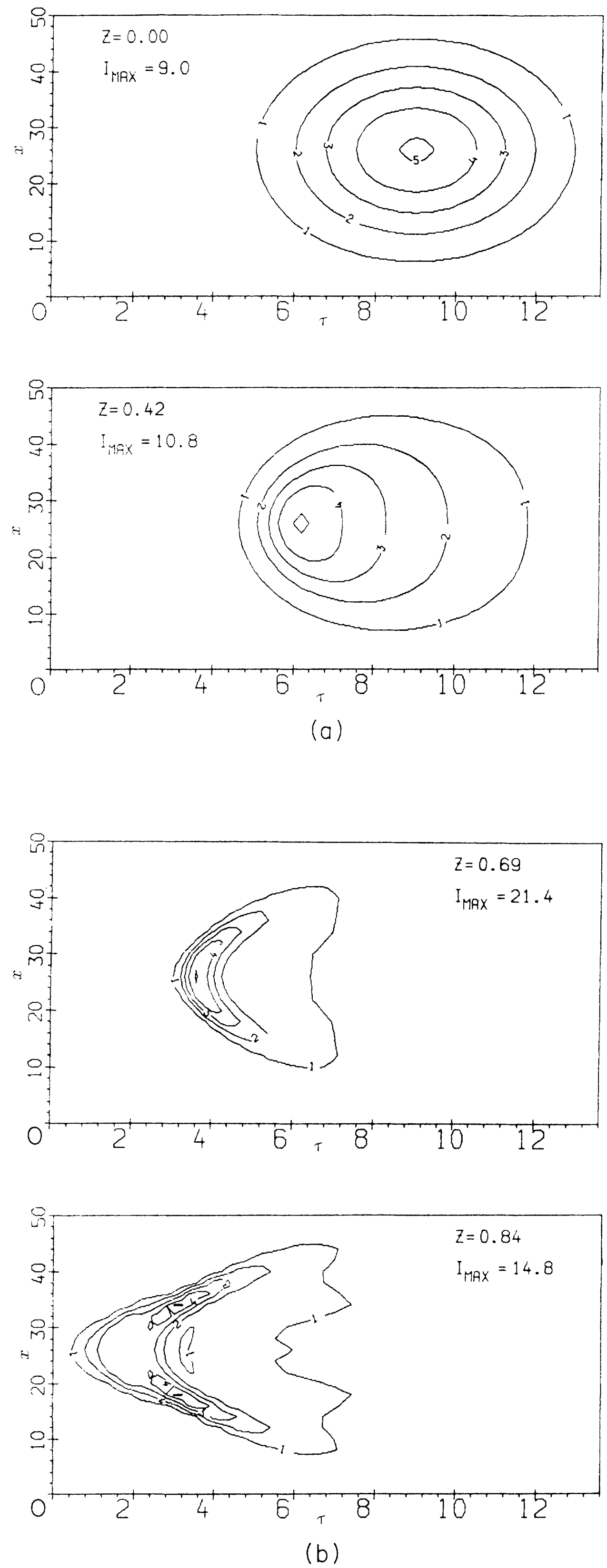

FIG. 2. Equal-intensity contours for two-dimensional evolution of the initial pulse distribution $A(0,\tau,x) = 3\exp[-(\tau/4.3)^2-(x/21.3)^2]$.

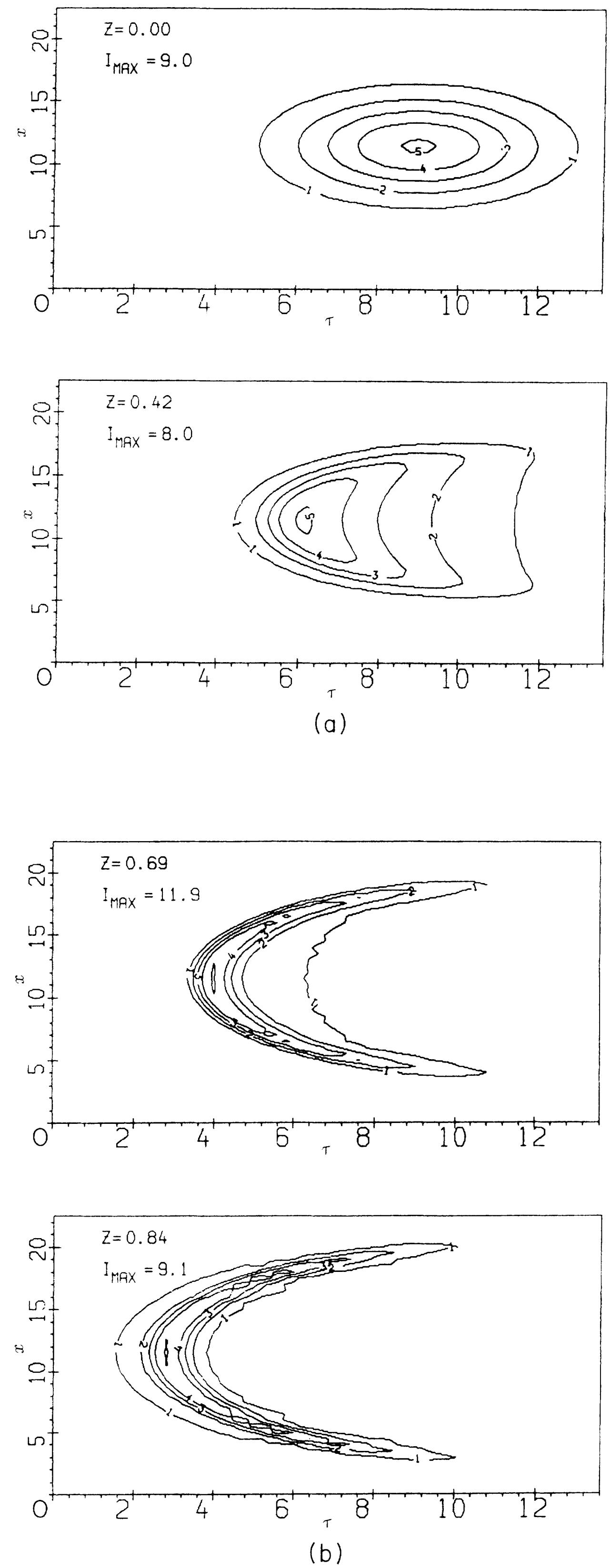

FIG. 3. Equal-intensity contours for two-dimensional evolution of the initial pulse distribution $A(0,\tau,x) = 3\exp[-(\tau/4.3)^2-(x/5.3)^2]$.

change if the ionization cross section decreases with increasing electric-field amplitude. This is a characteristic feature of electron-impact ionization in an electromagnetic field that is so strong that the oscillation energy of the electrons exceeds essentially the ionizaion potential energy. In order to analyze the dynamics of a super-strong, ultrashort electromagnetic pulse, we adopt the model equation

$$\frac{\partial n}{\partial \tau}=\frac{1}{|A|}\exp\left[-\frac{1}{|A|}\right], \tag{6}$$

where we have used our previous normalization for the variables. The maximum ionization rate is accordingly $1/e$. The $1/|A|$ dependence for large amplitudes is typical for the Born approximation. We have thus now obtained a system of equations, namely (4) and (6), which can be used to consider the two-dimensional compression, when the longitudinal self-steepening process is enhanced by the transverse self-focusing. A detailed analysis shows, however, that the longitudinal compression of superstrong field packets is rather inefficient. This can be seen if we write the complex amplitude A in the form $A=\Psi\exp(+i\varphi)$, and rewrite Eqs. (4) and (6) in the form of two new equations for the real amplitude Ψ and the frequency $\Omega=-\partial\varphi/\partial\tau$. Thus

$$\frac{\partial}{\partial z}\Psi^2-2\frac{\partial}{\partial \tau}(\Psi^2\Omega)=0\ , \tag{7}$$

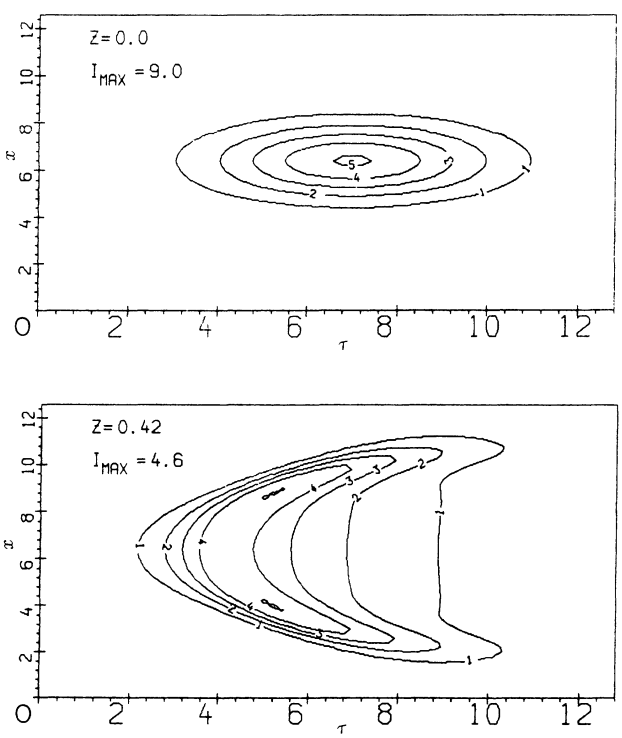

FIG. 4. Equal-intensity contours for two-dimensional evolution of the initial pulse distribution $A(0,\tau,x)=3\exp[-(\tau/4.3)^2-(x/2.1)^2]$.

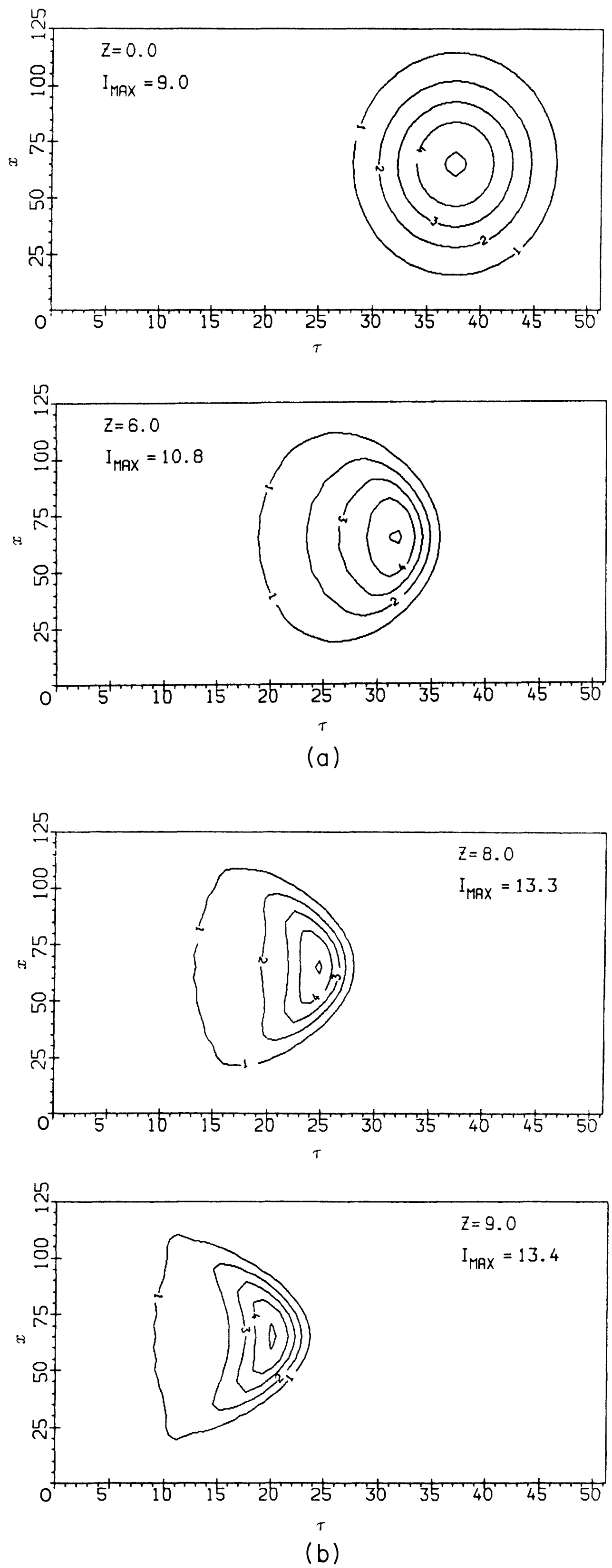

FIG. 5. Equal-intensity contours for the quasi-one-dimensional pulse with the initial Gaussian distribution ($A_0=3$, $\tau_0=10.25$, $a_\perp=53.3$) demonstrating weak longitudinal compression.

$$\frac{\partial\Omega}{\partial z}-\frac{\partial}{\partial\tau}\Omega^2=\frac{\exp(-1/\Psi)}{\Psi}-\frac{\partial}{\partial\tau}\left[\frac{1}{\Psi}\frac{\partial^2\Psi}{\partial\tau^2}\right]. \quad (8)$$

In the parameter range $1 \ll \Psi \ll \tau_0^3$, where the longitudinal dispersion term in Eq. (8) is small, Eqs. (7) and (8) look like an ideal gas system with the "effective pressure" $p=\int_{-\infty}^{\tau} d\tau\Psi$. From gas dynamic theory we know that an initially smooth distribution will evolve to a singularity. In terms of electrodynamics, the frequency modulation will grow faster on the periphery than at the central part of the pulse. This will lead to gathering of all the electromagnetic energy from the back front to the pulse center.

We estimate the minimum duration of the compressed pulses $\tau_{\min} \sim W^{1/7}$, where W is the wave energy involved in the compression. As $W \gg 1$, we note that $\tau_{\min}$ is much larger than the usual ionization time ($\sim W^{-1/2}$). The efficiency of the shortening of the pulse is also limited by an overflow of wave energy from the pulse front. As the frequency modulation, and the corresponding particle velocity, are small at the maximum value of $|A|$, perturbations escaping the compression region will appear in the $-\tau$ direction. This corresponds to a gas dynamical wave evolution where a shock wave cannot be stable if the velocity perturbation is in front of the shock velocity itself [12,13]. The longitudinal pulse shortening is thus reduced.

The evolution (see Fig. 5) of a quasi-one-dimensional wave packet with an initial Gaussian form ($A_0=3$, $\tau_0=10.25$, and $a_\perp=53.3$) shows that the compression is not significant ($|A|^2=I_{\max}=13.4$). The asymptotic form at the pulse again resembles a horseshoe, which this time is bent in the direction opposite to the pulse acceleration.

This scenario changes significantly when the nonlinear refraction is taken into account. As a density hole is formed in the strong field region, the electromagnetic fields will converge against the center. We will thus observe self-focusing of the radiation. The leading front has small field amplitudes and its nonlinear distortion can therefore be neglected. It expands in agreement with the usual diffraction rules. As a result, the bell-shaped pulse will evolve into a horn-type structure, as demonstrated in Fig. 6. As the compression is caused by the transverse dynamics, we can roughly estimate this effect by neglecting the time derivative in Eq. (3), assume a rectangular pulse form along the τ axis, and consider amplitudes $|A| \gg 1$. A similar set of equations was used previously [14] to describe the ionization self-channeling of a quasistationary electromagnetic beam. Expanding the pulse field and the electron density as

$$|A|=A_0\left[\frac{a_0}{a(z,\tau)}\right]^{1/2}\left[1-\frac{x^2}{a^2}+\cdots\right] \quad (9)$$

and

$$n=u(z,t)+v(z,t)x^2+\cdots, \quad (10)$$

we obtain the equations

$$\frac{\partial^2 a}{\partial z^2}=\frac{16}{a^3}-4av \quad (11)$$

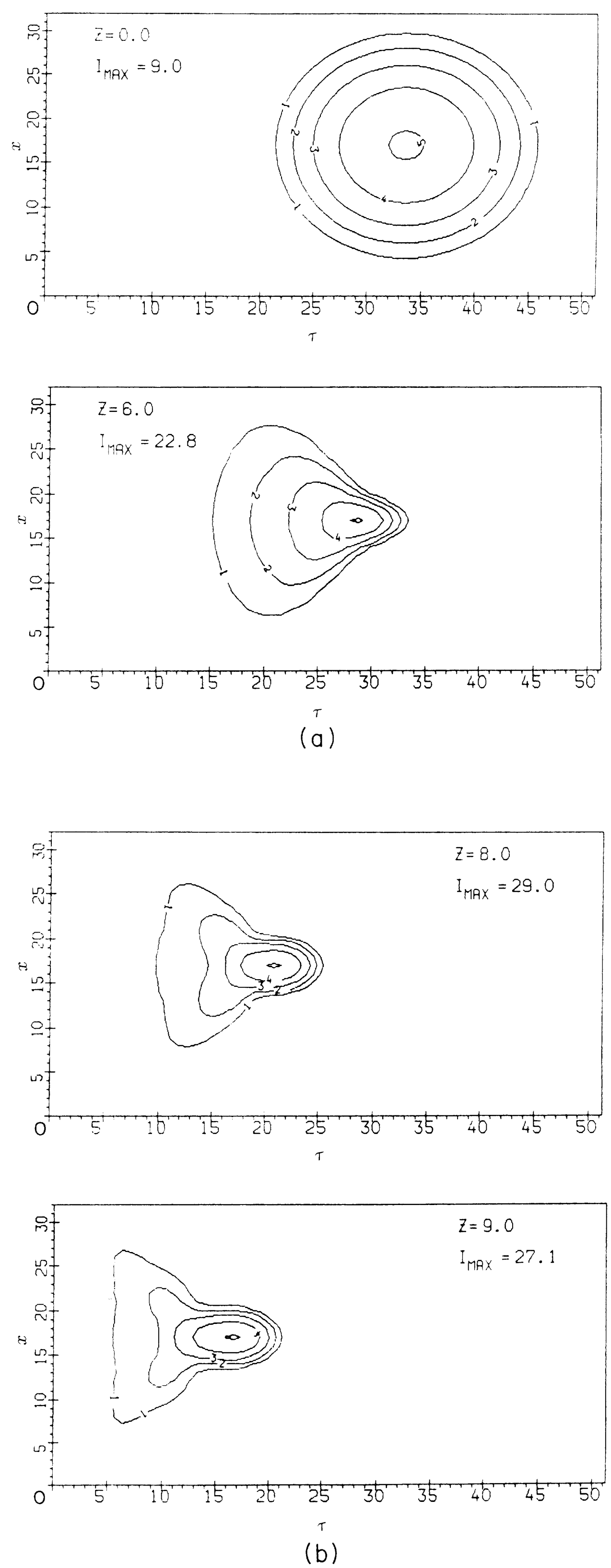

FIG. 6. Equal-intensity contours for the quasi-one-dimensional pulse with the initial Gaussian distribution ($A_0=3$, $\tau_0=10.25$, $a_\perp=10.7$). The effect of transverse compression is well pronounced and results in a horn-type structure of the ionizing pulse.

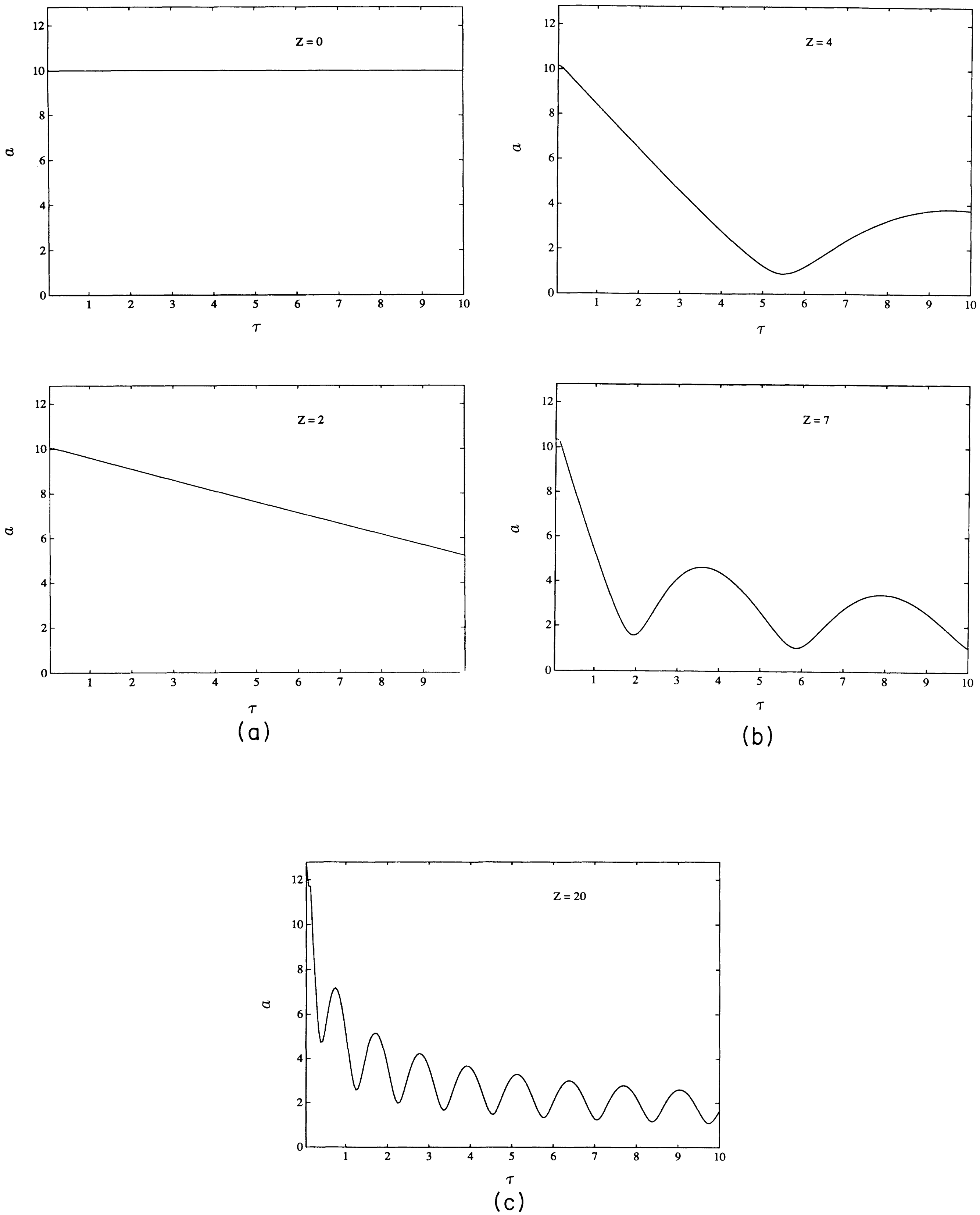

FIG. 7. Snapshots of pulse evolution according to Eqs. (11) and (12) for different distances of propagation through the ionizing medium. The horn-type structure is formed at distances less than the characteristic diffraction distance $z_d = a_0^2/4 = 25$. (For $z > z_d$ the structure will slowly expand.)

and

$$\frac{\partial v}{\partial \tau}=\frac{1}{A_0 a_0^{1/2} a^{3/2}} . \tag{12}$$

If the pulse duration is denoted by τ_p, the transverse length by a_0, and if the initial amplitude satisfies the condition $A_0 < a_0^2 \tau_p/4$, then the nonlinear refraction that is described by the second term on the right-hand side of (8) causes compression. Analysis of Eqs. (11) and (12) shows that for distances $z_* \cong A_0^{1/2} a_0 / 2\tau_p^{1/2}$, a quasistationary horn-type structure is formed; see Fig. 7. It consists of a narrow leading part obeying the diffraction rules and a bulk part tending to form a smooth self-similar background which is modulated by a smaller-scale wave perturbation. The background distribution corresponds to an adiabatic self-similarity in Eqs. (11) and (12), when neglecting the term $\partial^2 a/\partial z^2$ and obtaining $a \cong (32 A_0 a_0^{1/2}/5\tau)^{2/5}$. The modulation of the self-similar background stems from the usual competition between diffraction and nonlinear terms in Eq. (11). However, in comparison with steady-state self-focusing phenomena this effect is more complicated here due to the presence of nonlinear dynamics. For a narrow pulse which satisfies $A_0 > a_0^2 \tau_p/4$, diffraction will always dominate the evolution and lead to a constant expansion of the field distribution in the x direction.

It is not difficult to demonstrate that the horn-type structure appears also for three-dimensional distributions, although the self-similarity conditions then are slightly different from those above, i.e., $a_{3d} \sim \tau^{-1/3}$. In conclusion, we have investigated the nonlinear dynamics of an electromagnetic pulse propagating through and ionizing a medium. Depending on the nonlinear character of the ionization the pulse shape undergoes different evolution. In the case of an increasing ionization rate with the pulse intensity, the nonlinear refraction prevents strong longitudinal pulse compression and gives rise to a "horseshoe" pulse structure. On the other hand, for decreasing ionizaion rate with pulse intensity, the transverse focusing is pronounced and leads to a "horntype" structure of the pulse.

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