Conf-9209108--1

BNL--46788

DE92 012519

A GENERALIZED MODEL FOR COINCIDENCE COUNTING*

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1992

§1. Introduction

The aim of this paper is to provide a description of the multiplicative processes associated with coincidence counting techniques, for example in the NDA (non-destructive analysis) of plutonium bearing materials. The model elucidates both the physical processes and the underlying mathematical formalism in a relatively simple but comprehensive way. In particular, it includes the effect of absorption by impurities or poisons, as well as that of neutron leakage on a parallel basis to the treatment of induced fission itself. The work thus parallels and generalizes the methods of Böhnel, of Hage and Cifarelli, and more recently of Yanjushkin. These references are cited in Yanjushkin 1991.

This paper introduces the concept of a dual probability generating function to account for both the basic physical multiplication phenomena, as well as the (instrumental) detection phenomena. The underlying approach extends the idea of a simple probability generating function, due to De Moivre. The basic mathematical background may be found, for example, in Feller 1966.

§2. The Physical Model

It is assumed that each neutron in the system, regardless of the nature of its origin in space or time, has the same but independent probabilities of "interaction", viz.,

p_d, of being detected by the counter,
p_f, of inducing a fission,
p_l, of escaping (i.e. leaking out of) the system, and
p_c, of absorption other than that inducing fission.

If these interactions embrace all the possible fates of the neutrons, then

* This work was performed under the auspices of the U. S. Department of Energy by Brookhaven National Laboratory under Contract No. DE-AC02-76CH00016. Opinions expressed in this paper are solely those of the authors and do not necessarily reflect those of the Brookhaven National Laboratory or of the U. S. Department of Energy.

$$p_d + p_f + p_l + p_c = 1 . \qquad (2.1)$$

It is also assumed that the neutron emission probabilities for spontaneous and induced emission are respectively given by S_i and $\mathscr{I}_i$, where i denotes the number of emitted neutrons. Thus,

$$\sum S_i = 1, \text{ and} \qquad (2.2)$$

$$\sum \mathscr{I}_i = 1. \qquad (2.3)$$

The paper derives expressions for measurable quantities, e.g., factorial moments of various orders of the detected neutrons in the physical model described, including indefinite number of generations starting from an initiating spontaneous fission or (α,n) reaction.

§3. Mathematical Model

The physical problem described above will now be addressed mathematically using a two-variable PGF. The "variables" will be g for the generated neutrons, and d for the detected neutrons.

Starting from a ^{240}Pu spontaneous fission event one has the initial generating function

$$\begin{aligned} P_0(g) &= S_0 g^0 + S_1 g^1 + S_2 g^2 + \ldots \\ &= \sum_{i=0}^{\infty} S_i g^i \qquad (3.1) \\ &\equiv \mathcal{S}(g) . \end{aligned}$$

Each g in the above expression represents a neutron, g^i represents i neutrons generated in the fission. $\mathcal{S}(g)$ represents the spontaneous fission distribution defined by equation (2.2). In accord with the physical description given in §2 above, each neutron can now interact symbolically as described by the relation

$$\begin{aligned} g &\Rightarrow p_d d + p_l + p_c + p_f(\mathscr{I}_0 g^0 + \mathscr{I}_1 g^1 + \mathscr{I}_2 g^2 + \ldots) \\ &= p_d d + p_l + p_c + p_f \mathcal{I}(g) \qquad (3.2) \end{aligned}$$

where $\mathcal{I}(g)$ now represents the induced fission distribution described by equation (2.3).

Substituting equation (3.2) into equation (3.1) one obtains the next order PGF

$$P_1(d, g) = P_0(g = p_d d + p_l + p_c + p_f(\mathcal{I}_0 g^0 + \mathcal{I}_1 g^1 + \mathcal{I}_2 g^2 + \ldots)_1)$$

$$= \sum_{i=0}^{\infty} S_i \,(p_d d + p_l + p_c + p_f I(g)_1)$$

$$= \mathcal{S}\,(p_d d + p_l + p_c + p_f I(g)_1)\,. \tag{3.3}$$

The index 1 denotes that the first generation of induced fission is involved.

Continuing on similarly to the next generation

$$P_2(d, g) = P_1\,(p_d d + p_l + p_c + p_f I(g)_1)$$

$$= \mathcal{S}(p_d d + p_l + p_c + p_f I(p_d d + p_l + p_c + p_f I(g)))\,.$$

At this point, one can develop a recursion formula for the dual PGF (providing the process converges), in which the coefficients of the various (say n-th) powers of d and g represent the probability of the corresponding number of neutrons (i.e. n) detected and generated respectively.

Starting from one neutron (and simplifying the notation in an inessential way), one may write for the first generation

$$G_1(d,g) = p_d d + p_l + p_c + p_f I(g)\,. \tag{3.4}$$

For the next generation, one then finds

$$G_2(d, g) = G_1(d, g = p_d d + p_l + p_c + p_f I(g))$$

$$= p_d d + p_l + p_c + p_f I(p_d d + p_l + p_c + p_f I(g))$$

$$= p_d d + p_l + p_c + p_f I(G_1(d,g))\,.$$

Proceeding in this way, one arrives at the general recursion relation

$$G_n(d,g) = p_d d + p_l + p_c + p_f I(G_{n-1}(d,g)) \tag{3.5}$$

which is fundamental for the dual PGF. Iterating on this relation, one obtains the ultimate limiting relation (again assuming convergence)

$$G(d,g) = p_d d + p_l + p_c + p_f \mathcal{I}(G(d,g)) \quad (3.6)$$

(where $G(d,g) \equiv G_\infty(d,g)$).

In the present application one is interested in the PGF for the neutrons detected, and thus in G(d,g) as a function of d alone. (If one is interested in counting the number of neutrons generated, G(d,g) is treated as a function of g.) To get this one simply sets g = 1 in G(d,g) so that the limiting function G(d,1) now satisfies

$$G(d,1) = p_d d + p_l + p_c + p_f \mathcal{I}(G(d,1)) \, . \quad (3.7)$$

The expected number of neutrons detected, <n>, then becomes (see Feller, *loc. cit.*)

$$<n> = G'(1) = \frac{\partial G(d,1)}{\partial d}\bigg|_{d=1} = p_d + p_f \frac{\partial \mathcal{I}(G(d,1))}{\partial d}\bigg|_{d=1}$$

$$= p_d + p_f \frac{\partial G}{\partial d} \frac{\partial \mathcal{I}(G)}{\partial G}$$

$$= p_d + p_f G'(1)\bar{\nu}$$

so that

$$<n> = \frac{p_d}{1 - p_f \bar{\nu}} \quad (3.8)$$

where

$$\bar{\nu} = \nu_1 = \sum_i i \, \mathcal{I}_i .$$

For the case when neutrons are originally generated by spontaneous fissions, one must begin with the spontaneous fission PGF

$$\mathcal{S}(g) = \sum_{i=0}^{\infty} S_i g^i$$

and recognize that in the limit g must be replaced by G(d,1), as was done above. The compound PGF now becomes

$$\mathcal{S}(G(d,1)) \, .$$

The revised value of <*n*> is now

$$\langle n \rangle = \mathcal{S}'(1) = \frac{\partial \mathcal{S}(G)}{\partial G} \frac{\partial G}{\partial d} \Big|_{d=1}$$

$$= \bar{\nu}_s \left(\frac{p_d}{1 - p_f \bar{\nu}} \right)$$

where

$$\bar{\nu}_s = \frac{\partial \mathcal{S}(g)}{\partial g} \Big|_{g=1} = \sum_i i\, S_i \, .$$

Higher order factorial moments can be obtained similarly. They agree with the BHC formalism, provided $p_d = 1 - p_f$ (i.e. in the absence of neutron leakage or capture). In practical applications, this difference is often taken care of by the introduction of an *ad hoc* "efficiency" factor ε, such that $p_d = \varepsilon(1 - p_f)$. In terms of a nominal intrinsic detector efficiency ε_0 determined without the consideration of induced fission or other absorption effects,

$$\varepsilon = \frac{\varepsilon_0 (1 - p_f - p_c)}{(1 - p_f)} \, .$$

§5. Conclusions

A generalized model for coincidence counting has been developed based on the dual probability generating function introduced. The model accounts explicitly and simultaneously the effects of multiplication, absorption by poison and instrument detection and is applicable for a wide class of NDA including Pu in waste.

References

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