IC/92/164

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

FLAVOUR DEMOCRACY CALL FOR THE FOURTH GENERATION

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INTERNATIONAL ATOMIC ENERGY AGENCY

UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

MIRAMARE-TRIESTE

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and
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ABSTRACT

It is argued with the help of an illustrative mode, that the inter species hierarchy among the fermion masses and the quark mixing angles can be accommodated naturally in the standard model with (approximate) flavour democracy provided there are four families of sequential quark–leptons with all members of the fourth family having roughly equal masses. The special problem of light neutrino masses (if any) and possible solutions are also discussed.

MIRAMARE – TRIESTE
July 1992

The neutrino counting at LEP has established with high statistical significance that the number of light neutrinos ($m_\nu << m_Z/2$) is three [1]. This has led to the widespread belief that at least within the framework of the Glashow–Salam–Weinberg minimal standard model (MSM) with sequential quark–lepton families, the number of such families has also been determined to be three[2, 3]. This belief hinges on the notion that a heavy sequential Dirac neutrino ($m_\nu >> m_Z/2$) belonging to the fourth generation necessarily implies an unnaturally large hierarchy among the Yukawa couplings of different neutrinos with the Higgs boson . Though the naturalness argument has an intuitive appeal, it is basically a philosophical outlook and,therefore, is not beyond debate. However the purpose of this note is to point out, without questioning the validity of the philosophy of naturalness, that any unnatural hierarchy is not inevitable for the existence of such a heavy neutrino .On the contrary the completely opposite scenario [4, 5, 6] referred to in the literature as flavour democracy , which requires that the Yukawa couplings of all fermions of a particular type with the Higgs boson before the diagonalisation of the mass matrix are equal, can be implemented in a four family model with a heavy neutrino without requiring any large hierarchy among the Yukawa couplings of the fermions of *different types.* This is no longer possible in a three family model in view of the rather stringent lower bound on the top quark mass and the relatively low masses of the other fermions (see below).

The elements of the n x n mas matrix M^f of the fermions of the type f (up, down, charged lepton or neutrino) in an n–generation model with approximate flavour democracy can be written as

$$M^f \ = \ Y^f(M^0 \ + \ \lambda M_f') \tag{1}$$

where M^0 is the fully democratic matrix with all elements equal to unity, Y^f can be interpretated within the framework of the MSM as the common Yukawa coupling of the fermions of this type (in units of the vacuuam expectation value of the Higgs boson). The parameter λ is introduced purely for book keeping purposes and we take it to be 0.1. The matrix M' with elements $O(1)$ parametrises small departures from perfect democracy. In the limit when M' vanishes this mass matrix can be motivated by imposing a permutation symmetry [4] or by an underlying BCS like dynamics [6]. Any theoretical idea which pinpoints the origin of flavour democracy and the mechanisms for small departures from it will, of course, be an important step forward. In this phenomenological work focussed mainly on the *naturalness* of the hierarchies in the fermion masses and mixing angles within the frame work of the MSM, we shall not speculate about the above points. It is well-known that n − 1 eigenvalues of M^0 are equal to zero while one is equal to n. This illustrates that a large hierarchy among the masses of a given species f does not necessarily require a corresponding hierarchy among the elements of the mass matrix. The problem of a three family model is apparent once the inter- species mass hierarchy is taken into account. This model predicts:

$$m_t : m_b : m_\tau : m_\nu \; = \; Y^u : Y^d : Y^l : Y^\nu \tag{2}$$

where m_ν stands for the mass of the heaviest neutrino. Using the known values of m_b and m_τ and the bounds $m_t \geq 91$ GeV [7] and $m_\tau \leq .035$ GeV [8], one requires $Y^u : Y^d \geq 18$, $Y^u : Y^l \geq 50$ and $Y^u : Y^\nu \geq 2570$! While the first ratio is barely consistent with the philosophy of naturalness the second one is certainly not so. The third one, implying the largest hierarchy, is a reflection of the well-known neutrino mass problem . In contrast if nature indeed prefers to have four families of quark–leptons with $m_T \simeq m_B \simeq m_E \simeq m_N$ (the subscripts refer to fermions belonging to the fourth family), she may do it without hurting anybody's cravings for naturalness.

The non–zero masses of the lighter fermions within a given type f are generated by M' and at the first sight it seems that there are too many paramers to fit the observed fermion spectrum and the quark mixing angles. This, however, is not the case. The requirement that the departure from the democratic structure is small i.e., the elements of M' are $O(1)$ makes this model quite restrictive. In the following we shall illustrate with an extremely simple model that the known phenomenology of the quarks and leptons can be understood without destroying approximate democracy or without resorting to fine tuning. Since the mass hierarchy in the charged lepton sector is not too different from the down quark sector a similar model can be applied there. As long as there is no positive evidence for non-zero masses for the lighter neutrinos a fully democratic mass matrix, rather than the omission of the right–handed neutrinos by hand, seems to be more natural for this sector. However, in order to accommodate a small non–zero mass of any one of them consistent with the present upperbounds [8] some fine

tuning is required (see below) though no large ratio of the Y^f's needs to be introduced. Should such a situation arise the simple interpretation of eq.(1) as the roughly equal Yukawa couplings of a single Higgs boson may appear to be unattractive and physics beyond the MSM may be called for. We will comment on it at the end of the paper.

In order to construct an illustrative simple model we obtain the approximate eigenvalues and eigenvectors of M^f by applying perturbation theory. Since the degenerate eigenvectors of M^0 can not be determined uniquely without specifying additional symmetries we have used instead the matrix elements $(M^f)_{\alpha\beta}$, where α , β = 1,2,3 label the degenerate eigenvectors of M^0 while 4 refers to the eigenvector corresponding to the heavy state, as the phenomenological parameters for the subsequent analysis.It should ,however be emphasised that the above ambiguity is an artifact of using perturbation theory and no physical observable is affected by it. For simplicity we also take M' to be real symmetric. It is well-known that CP violation in a four generation model with three observable phases is less restrictive than in a three generation model with a single observable phase[9]. It is, therefore, not unreasonable to assume that the introdution of three small phases will adequately describe CP violation without drastically altering the pattern of quark masses and mixing angles obtained here.

The fact that $m_u, m_c << m_t$ gives a strong hint regarding the input values of these matrix elements. It is natural to assume that u and c remain massless and degenerate upto first order in perturbation theory while the t quark picks up a mass.There are two attractive ways of achieving this without adjusting the detailed numerology of the matrix elements : i) Assume that all $(M')_{\alpha\beta}$ except (M'_{33}) are zero in the subspace of the degenerate eigevectors or ii) $(M')_{\alpha\beta}$ also has a democratic structure in this subspace. The second alternative can be reduced to case i) by diagonalising the perturbation matrix and can be analysed similarly. Using assumption i) we have ten remaining free parameters $(M'_q)_{33}$, $(M'_q)_{44}$ and $(M'_q)_{i4}$ (i=1,2,3; q=u or d) which are to be determined phenomenologically.

Applying the standard tools of degenerate perturbation theory it is now straightforward to see that one quark remains massless to all orders in perturbation theory,which can be identified with the lightest quark of a given type (up or down). The 2 x 2 block of the CKM matrix involving the first two families are generated in the zeroth order. All other elements are $O(\lambda)$ and are smaller. The relevant formulae for the remaining masses are :

$$m_q^{(4)} = Y^q(4 + \lambda(M_q')_{44}) \quad (3)$$

$$m_q^{(3)} = \lambda Y^q(M_q')_{33} \quad (4)$$

$$m_q^{(2)} = \frac{\lambda^2 Y^q}{4}((M_q')_{14}^2 + (M_q')_{24}^2) \quad (5)$$

where q = u or d and the superscripts 2,3,4 respectively denotes c, t, T (s, b, B) quarks in the up (down) sector. The elements of the CKM matrix , whose measured values are used to fix the remaining free parameters of the model,are given by(upto $O(\lambda)$)

$$V_{ud} = N_u N_d(1 + x_u x_d) \quad (6)$$

$$V_{us} = N_u N_d(x_d - x_u) \quad (7)$$

$$V_{ub} = \eta_{ub}\frac{\lambda}{4N_d}V_{us}\frac{(M_d')_{34}(M_d')_{24}}{(M_d')_{33}} \quad (8)$$

$$V_{cb} = \eta_{ub}\eta_{cb}N_u N_d\frac{|V_{ub}|}{V_{us}}(1 + x_u x_d - \eta_{ub}\frac{|m_c|}{m_t}\frac{(M_u')_{34}}{(M_u')_{24}}\frac{|V_{ub}|}{V_{us}}) \quad (9)$$

where $N_{u,d} = (1 + x_{u,d}^2)^{-1/2}$, $x_{u,d} = ((M_{u,d}')_{14}/(M_{u,d}')_{24})$ and η_{ub}, $\eta_{cb} = \pm 1$ refer to the signs of V_{ub} and V_{cb}. $(M_u')_{14}$ and $(M_d')_{24}$ also involves sign ambiguities. We choose them to be positive. It is to be noted that eq.(6) holds to all orders in perturbation theory while the remaining three receives higher order corrections. We have computed upto second order corrections to the above formulae. The expressions are rather cumbersome and will be presented in a longer paper [10].The numerical values of these corrections are however given below to demonstrate that perturbation theory makes sense.Finally we have rechecked everything by using numerical diagonalisation without appealing to perturbation theory (see below). In the absence of any experimental guidelines regarding m_T and m_B we have to assume some reasonable values for Y^u and Y^d. We take $Y^u = 150$, $Y^d = 100$. The parameters $(M_u')_{44}$ and $(M_u')_{44}$ do not affect the masses of the quarks belonging to the first three

families or their mixing angles to the lowest order in λ . We have chosen $(M_u')_{44} \simeq -(M_d')_{44} \simeq$ - (8.0–8.5). The last choice is made so that the mass splitting of the fourth generation quarks does not make a large contribution to the ρ parameter [11, 12]. We emphasize that this choice is not necessary if one takes $Y^u \simeq Y^d$. Our choices of Y^u and Y^d gives somewhat better hierarchy between the up and the down sectors but is not crucially important. Using eqs. (3)–(8) we obtain (for $m_c \simeq 1.5\, m_t \simeq 125$, $m_s \simeq .150$, $m_b \simeq 5$ (all in GeV) , $V_{ud} \simeq 0.9747$, $V_{us} \simeq 0.223$, $V_{ub} \simeq 0.004$ and $V_{cb} \simeq 0.04$) : $x_u = 0.2$, $x_d = 0.45$, $(M_u')_{14} \simeq 0.392$, $(M_u')_{34} \simeq 3.76$, $(M_u')_{33} \simeq 8.33$, $(M_d')_{14} \simeq 0.312$, $(M_d')_{34} \simeq -0.462$, $(M_d')_{33} \simeq 0.5$. It is gratifying to note that no large hierarchy in the matrix elements is required.The remaining elements of the CKM matrix are predictions of this model and the full matrix is given by

$$V = \begin{pmatrix} 0.9747(0.0) & 0.2235(-0.00008) & 0.004(-0.0009) & 0.004(-0.0009) \\ -0.2235(0.0003) & 0.9747(-0.0012) & 0.04(0.001) & -0.031(-0.017) \\ 0.005(0.002) & -0.039(-0.004) & 1.00(-0.006) & -0.105(-0.036) \\ -0.011(-0.002) & 0.03(0.01) & 0.105(0.037) & 1.00(-0.006) \end{pmatrix} \quad (10)$$

where the numbers in the parentheses give the second order corrections. While the detailed numerology of the predictions of this matrix is somewhat dependent on the values of the input parameters, a key prediction independent of such details is that the 2 x 2 block involving the first two families should be almost identical. The present experimental values $|V_{cd}| = 0.204 \pm 0.017$ and $|V_{cs}| = 1.00 \pm 0.20$ [8] leave ample room for deviations from this prediction. The discovery of the fermions belonging to the fourth family by direct searches in conjuction with accurate measurements of V_{cd} and V_{cs} at a tau–charm factory may provide an an interesting test of this model.

Using the phenomenologically determined $(M_{u,d}')_{\alpha\beta}$'s and a specific choice for the degenerate eigenvectors of M^0 it is an easy numerical excercise to determine the elements of $M_{u,d}'$. As has already been mentioned the physical observales are of course independent of this choice. As example we present below two such matrices . One can readily diagonalise them numerically to verify that they indeed reproduce all the known phenomenology of the quarks and a close agreement with the CKM matrix given above.

$$M^u = \begin{pmatrix} 1.0775 & 0.9435 & 1.0497 & 0.5287 \\ 0.9435 & 0.8095 & 0.9158 & 0.3947 \\ 1.0497 & 0.9158 & 1.0220 & 0.5010 \\ 0.5287 & 0.3947 & 0.5010 & 1.0910 \end{pmatrix} \tag{11}$$

$$M^d = \begin{pmatrix} 1.2505 & 1.1960 & 1.2280 & 1.2348 \\ 1.1960 & 1.1415 & 1.1735 & 1.1803 \\ 1.2280 & 1.1735 & 1.2055 & 1.2124 \\ 1.2348 & 1.1803 & 1.2124 & 1.2858 \end{pmatrix} \tag{12}$$

As expected the departure from democracy is rather small in the down sector. In the up sector the departure from democracy outside the 3 x 3 block is quite significant. Whether this is an artifact of our input parameters has to be checked by a detailed numerical computation[10]. We also note that the above precise values of the elements of the mass matrices are not required to get the hierarchy in the masses qualitatively. These are required to reproduce the rather precisely determined V_{ud} and V_{us}.

As has already been mentioned , nonvanishing neutrino masses of a few MeV or smaller may revive the naturalness problem in this model. For example, a tau-neutrino mass close to its present upper limit would imply $\lambda Y^\nu (M'_\nu)_{33} \leq 0.035$ GeV or $(M'_\nu)_{33} \leq 0.003$ (assuming $Y^\nu \simeq 100.0$). This would reintroduce large hierarchies among the matrix elements. In the context of the neutrinos it may, therefore, turn out be more appealing to use $(M'_\nu)_{33} = 0.0$ as an input. Such a model would lead to two massless neutrinos to all orders in perturbation theory and a light neurino with mass

$$m_\nu^{(3)} = \frac{\lambda^2 Y^\nu}{4}((M'_\nu)_{14} + (M'_\nu)_{24} + (M'_\nu)_{34}) \tag{13}$$

A tau neutrino with mass $\simeq 0.035$ GeV would then imply (assuming approximate equality of all matrix elements) $(M'_\nu)_{i4} \simeq 0.2$, which does not seem to be unnatural.

It is well-known that if all the neutrino masses turn out to be non-zero and very small, they can be accommodated in the three family model only

at the expense of introducing large hierarchies in the inter-species Yukawa couplings. In a four family scenario no such large hierarchy needs to be introduced but fine-tuning of the values of these couplings seems to be necessary which makes one feel rather uneasy. Perhaps new physics with an inbuilt see-saw mechanism [13] (as in ref 3) together with family democracy will provide an elegant model. Nevertheless, since no evidence of the Majoraana nature of the neutrinos which is so crucial for the see-saw mechanism has been found , it may be worhwhile to speculate about alternative scenarios within the context of the present model . If the quarks and leptons are composites of more fundamental objects such a scenario can be motivated.In order to make the discussion as much model dependent as is possible we simply summarise the essential features . We assume that the Yukawa couplings of the (composite) fermions with the Higgs boson are fully democratic. The compositeness of the fermions manifests itself through non- renormalisable interactions of the form $(g_{ij}^2/\Lambda^2)\bar{\psi}_{iL}\psi_{jR}\bar{f}_{iL}f_{jR}$ where the ψ's and f's stand for usual quark-leptons and preons respectively[14], g_{ij}'s are coupling constants and Λ is the compositeness scale. The formation of preonic condensates through some new interactions then generates additionnal mass terms for the fermions which lead to departures from exact democracy.The parameter λY^f in eq. (1) can then be identified with $g_{ij}^2 \Lambda_f^3/\Lambda^2$, whwre Λ_f^3 denotes the strengthes of the preonic condensates. If the compositeness scale of the neutrinos is much larger than the other fermions then the departures from the democratic structure in the neutrino sector will naturally be much smaller than those in the other sectors. Assuming $\lambda Y^f \simeq 10$, $g_{ij} \simeq 1$ and assigning a typical value of 300 GeV^3 characteristic of the electroweak scale to the preon condensates , one needs $\Lambda \simeq 2$ TeV in order to produce the above mass matrices in the quark sector. Such a low compositeness scale in a one scale model ,though allowed by the present experimental data , may lead to unacceptably large flavour changing neutral currents [15] unless special mechanisms are introduced to suppress them. On the other hand in two scale models [14] the above effective interactions may be generated by the exchanges of a composite scalar particle of mass in the TeV region characteristic of the lower scale. In such models flavour changing neutral currents are also naturally suppressed.

In summary, the excitement that the neutrino counting experiments have also determined the number of sequential quark lepton families seems to be rather premature. In this note we have illustrated with the help of an extremely simple model that the hierarchy among the fermion masses at least in the quark and charged lepton sectors can be understood naturally in models with (approximate) flavour democracy provided there are four sequential families of quark–leptons with all members of the fourth family having roughly equal masses. This is not possible in a three family model. In the neutrino sector exact flavour democracy predicts three mass less neutrinos without requiring the absence of right-handed neutrinos. Departures from flavour democracy can naturally explain one massive neutrino belonging to the lighter families provided its mass is in the MeV region. If confronted with non-vanishing but extremely small neutrino masses for the first three families the model may still accommodate them without introducing any large ratios , although some fine tuning in the values of the Yukawa couplings may be needed. The possibility of understanding very light sequential Dirac neutrinos in the democratic scenario with composite quark–leptons has been qualitatively discussed.

ACKNOWLEDGMENTS

The author would like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste, where part of this work was done. He would also like to thank Dr. A. Joshipura for valuable discussions. This research was partly supported by a grant from the Department of Science and Technology, India.

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