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GANIL P 91 17

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Abstract

Intermittency in relativistic heavy-ion collisions and in particular the projectile dependence, multiplicity dependence and source-size dependence are discussed in the frame of the model of spatio-temporal intermitency. Moreover, the recent theoretical results in intermittency studies of the nuclear multifragmentation are presented.

1 Introduction

Intermittency is a manifestation of scale invariance and randomness in physical systems. It was discussed first in a theory of turbulent flow [1] and , more recently , it was found in the spectra of particles produced in most of the high-energy experiments (for a recent review see [2]) and in the charge distribution of fragments following the break-up of high energy nuclei in the nuclear emulsion [4, 5] . To identify the intermittent pattern of fluctuations in high energy spectra Bialas and Peschanski have proposed to calculate the scaled factorial moments (s.f.m.) [3] which have the advantage of measuring dynamical fluctuations without the spurious influence of statistical fluctuations. The intermittency in this case means that s.f.m. exhibit the power law behaviour vs. the resolution :

$$F_i(\delta) = F_i(\Delta)(\Delta/\delta)^{f_i} \tag{1}$$

, where the strength of intermittency is characterized by the intermitency exponents f_i which are directly related to the anomalous fractal dimensions d_i :

$$d_i = f_i/(i-1) \ . \tag{2}$$

The theoretical understanding of intermittency in high energy experiments is missing as yet because we are unable to relate this phenomenon to the fundamental properties of the theory of strong interactions. Also the appearance of intermittency in the mass/charge distribution of fragments is not understood in terms of the dynamical theory of nuclear fragmentation. The self-similar random cascading models [3] , short range singular correlations [6, 7, 8] , phase transition [9, 10] , and spin-glass phase transitions [11] were discussed in connection with this phenomenon. The finite-size effects of different kind can modify the effect strongly and many of

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them have been discussed recently in connection with the phase transition in finite size percolation and Ising models [12] and in random cascading for systems with a finite multiplicity of particles [13] . Also the dimensional projection can modify the phenomenon as discussed in [7, 8] . Hence this talk neither serves as a systematical discussion of the accomplished theoretical development nor it gives the complete list of all the various developments in this new field. Instead we concentrate on discussing only two points :

- the spatio-temporal intermittency and in particular a role of the size of the pion source and the length of the temporal evolution in the strength of the intermittency in the final spectra of hadrons
- similarity between fluctuations of the fragment-size distribution in the percolation and in the nuclear multifragmentation

Extensive discussion of problems omitted deliberately in this talk can be found in the recent review articles [2] and in the original papers.

2 Spatio-temporal intermittency in ultrarelativistic nuclear collisions

The experiments on the relativistic heavy-ion collisions raised the question about a possible existence of exotic phenomena, such as the creation of the quark-gluon plasma. In the first approximation, the nucleus-nucleus collision may be considered as a superposition of independent nucleon-nucleon collisions, the number of which is determined by the geometry of the reaction. This picture of the ultrarelativistic dynamics leads to accurate predictions for a number of observables. Many geometrical models are successful in describing the relation between hadron-hadron and hadron-nucleus collision, and other models can relate within the same framework both the hadron-nucleus and the nucleus-nucleus collisions [14] . The fundamental assumption of all those models is that the primary excited objects such as nucleons, quarks, tubes, strings or ropes produce particles independently and hence the final particle distribution can be considered as the superposition of distributions from elementary collisions (sources). In those models based on the concept of the creation and subsequent decay of strings, the picture of independent fragmentation of strings seems to be not justified. The density of generated strings, which is $2-3$ strings/fm^2 , denies the reliability of such a scenario and leads to the necessity of taking the string interaction into account.

The multiparticle correlations can be built up at any stage of the collision. They can arise either from random cascading structure of the parton cascade or from the string fragmentation process. But, as we will show below, they are not a product of independent fragmentation of strings (sources). The inclusion of a non-negligible interaction between the strings is crucial in order to reproduce the experimental data.

Moment	Reaction	Data	Data (rescaled multiplicity)	Model
F_2	p-Em [18]	0.019	0.019	0.016
F_2	O-Em [18]	0.016	0.003	0.006
F_3	O-Em [18]	0.042	0.010	...
F_2	S-Em [18]	0.012	0.0015	0.003
F_3	S-Em [18]	0.028	0.006	...

Table 1: Slopes of the scaled factorial moments F_2 and F_3 for different reactions are compared to the rescaled multiplicity data for $p - Em$ reaction. The estimate of the slope from the theoretical model presented below is given for F_2 only (from ref. 18).

of independent fragmentation of strings (sources). The inclusion of a non-negligible interaction between the strings is crucial in order to reproduce the experimental data.

It was claimed that the presence of nonstatistical fluctuations in the spectra of produced hadrons could be a signal of the quark-gluon plasma formation [15, 3]. The effect of nonstatistical fluctuations can be extracted and the comparison between different processes is possible using the method of the scaled factorial moments [3] . The experimental data confirms the existence of such fluctuations and the dependence of s.f.m. on the resolution in rapidity was fitted using the power law (intermittent) relation : $F_i \sim (\delta y)^{-\nu_i}$, where ν_i is the intermittency slope for the s.f.m. of rank i . In the case of N independent fluctuating sources, the superimposed distributions would follow a similar law with the slopes ν_i which are N times smaller than for one source. The discussion of the independent collision model was performed in ref. [16] and the authors concluded that one cannot interpret the observed intermittency slopes as an effect of superimposed nucleon-nucleon collisions, because it gives values $4 - 20$ times too low. One should notice that even if the mechanism responsible for the fluctuations in nucleus-nucleus collisions is different than in hadron-hadron collisions, still there is a lack of consistency in the observed parameters for different projectiles (p, O, S) [8]. In table 1 we show the experimental data for F_2 and F_3 for three projectiles [18] and the predictions for the projectile nucleus A from the p$-$Em data rescaled in accordance with the change of the mean multiplicity : $F_i^A = F_i^p < N_p > / < N_A >$. As was already discussed in the ref. [8] , the observed decrease of the intermittency slopes is not as large as the corresponding increase of the multiplicity.

The theoretical understanding of intermittency is not clear at the moment because , in spite of large body of data exhibiting the power law behaviour of s.f.m. , we do not know what physical effects are at the origin of this phenomenon and what is its relation to the fundamental theory of strong interactions. Bialas and Hwa proposed that the intermittency patterns of fluctuations in the nuclear collisions could be a consequence of a second order phase-transition from quark-gluon plasma into hadrons [9]. In this case the intermittency slopes ν_i , like the correlations fragmentation but instead from the stage of dynamical evolution of the interacting string network with possible phase-transition similar to percolation [19].

In refs. [8, 17] , the model of spatio-temporal intermittency was proposed in which the two-particle density in space-time was taken to be a scale-invariant function , like in the critical point for a second order phase-transition. The correlation function for the proper time τ_2 of the freeze-out can be written in the covariant form :

$$\begin{aligned} d_2(\vec{r}_1,\vec{r}_2) &= C_2\,(\tau_{12})^{-\nu_2} \quad \Big|_{t_i=\sqrt{\tau_2^2+\vec{r}_i^{\,2}}} \quad for \quad \tau_{12} < \tau_{corr} \\ &= C_2\,(\tau_{corr})^{-\nu_2} \qquad\qquad\qquad for \quad \tau_{12} \geq \tau_{corr} \end{aligned} \tag{3}$$

, where $\tau_{12} \equiv \sqrt{(\vec{r}_2-\vec{r}_1)^2-(t_2-t_1)^2}$ and τ_{corr} is the range of the correlations. The finite range of the correlations in this model is exclusively due to causality constraints. Assuming that the phase-transition takes place at a certain proper time τ_1 and that the correlations cease to build at some other value of the proper time τ_2 , one obtains $\tau_{corr} = (\tau_2^2-\tau_1^2)/\tau_1$ for the range of the fluctuations. The intermittency exponent ν_2 describes the strength of the singular correlations and is given by the nature of the mechanism by which the spatio-temporal intermittency is build up. For the $SU(2)$ gauge group and the Ising model phase-transition $\nu_2 = 1.0$ [20, 10] .

For the dynamics of the reaction we assume the Bjorken's scaling in the longitudinal direction. Moreover, the ideal inside-outside correspondence will be smeared out, to simulate realistically the dynamics of interactions at 200 GeV/nucl. The other non-ideal effect taken into account is the presence of the long lived resonances. Here we assume that 50% of the observed pions come from the decay of such resonances. The decay is assumed to be isotropic in the rest frame of the resonance, in which case the rapidity distribution is $1/cosh^2 y$. Such a distribution is well approximated by a gaussian distribution with the half-width equal 0.75 . The one-particle density is assumed to be constant inside a tube of radius R_{tu} and zero outside. The radius of the tube is given by the geometry of the reaction, i.e. by the impact parameter and the projectile radius. The interaction region for central collisions is determined by the radius of the smaller (projectile) nucleus, $R_{tu} = r_0 A^{1/3}$. Integrating the two-particle reduced density over transverse radii and azimuthal angles, one obtains the one-dimensional reduced density :

$$\begin{aligned} d_2^{(1-dim)}(y_1,y_2) = \;(\pi^{-2} R_{tu}^{-4})\int_0^{R_{tu}} R_1 dR_1 \int_0^{R_{tu}} R_2 dR_2 \int_{-\pi}^{\pi} d\phi_1 \int_{-\pi}^{\pi} d\phi_2 \\ d_2(R_1,\phi_1,y_1;R_2,\phi_2,y_2)\,. \end{aligned} \tag{4}$$

This represents the two-particle reduced density of the space-time rapidity distribution of the sources. Following our assumptions this corresponds only approximately to the kinematical rapidity distribution. In order to obtain the distribution in the kinematical rapidity one should fold the space-time rapidity distribution with the phase-space distribution, which we take in the following form :

$$F(y,y^{(st)}) = \frac{exp(-(y-y^{(st)})^2/2\sigma^2)}{\sigma\sqrt{2\pi}}\ , \tag{5}$$

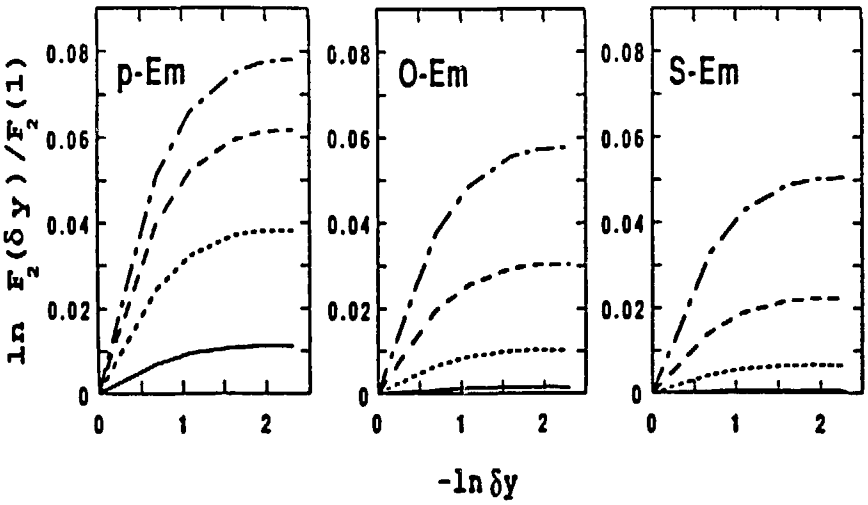

Figure 1: The dependence of the second factorial moment F_2 on the size of the rapidity bin δy for four different freeze out times $\tau_2 = 4fm$ (the solid line), $6fm$ (the dotted line), $10fm$ (the dashed line) and $20fm$ (the dash-dotted line). All curves are normalized at $\delta y = 1$.

with $\sigma = 0.5$. The kinematical rapidity distribution of the sources of the final pions is then :

$$d_2^{(kin)}(y_1, y_2) = \int dy_1^{(st)} \int dy_2^{(st)}\, d_2^{(1-dim)}(y_1^{(st)}, y_2^{(st)}) F(y_1, y_1^{(st)}) F(y_2, y_2^{(st)}) \,. \tag{6}$$

As we already mentioned, only about $q = 50\%$ of the total number of pions are produced directly and the rest comes from the resonances decays. Taking this into account, the two-particle rapidity distribution for pions can be written as follows :

$$\begin{gathered} d_2^{(\pi)}(y_1, y_2) = \int dy_1' \int dy_2'\, d_2^{(kin)}(y_1', y_2') \\ (q\, \delta(y_1 - y_1') + (1-q)\rho^{(iso)}(y_1 - y_1'))(q\, \delta(y_2 - y_2') + (1-q)\rho^{(iso)}(y_2 - y_2')) \,. \end{gathered} \tag{7}$$

The decay distribution of a resonance $\rho^{(iso)}$ in (8) is approximated by $\rho^{(iso)}(y) = exp(-y^2/1.125)/0.75\sqrt{2\pi}$. From the two-particle reduced density one obtains directly the second s.f.m. :

$$F_2(\delta y) = \frac{1}{(\delta y)^2} \int_0^{\delta y} dy_1 \int_0^{\delta y} dy_2\, d_2^{(\pi)}(y_1, y_2) \,. \tag{8}$$

The results for $F_2(\delta y)$ in the range $\delta y = 0.1 - 1$ are plotted in fig. 1 . As one can see the results depend strongly on the assumed freeze-out time τ_2. The resulting slopes ν_2 of the s.f.m. are of the same order of magnitude as observed experimentally. However, the detailed comparison is not meaningful because the calculated slopes ν_2 depend sensitively not only on the projectile and the impact parameter but also on the unobservable parameters such as the correlation range τ_{corr} , percentage of the direct pions q and the width σ of the rapidity spread around the Bjorken's solution. So the dependence on the projectile and impact parameter as calculated in this model can be treated only as a qualitative relation. In table 1 one can see the comparison of the experimental data with results of calculations obtained for the freeze-out time $\tau_2 = 6\ fm$. Eventhough the numerical values of the intermittency slopes ν_2 are systematically underestimated, they depend on the projectile less strongly than the inverse of the mean multiplicity. In table 1 we make a comparison between central nucleus-nucleus collisions in the model (constant interaction region radius) and a sample of central and medium impact parameter collisions in the experiment. This could be another effect explaining the stronger attenuation of the calculated slopes. It would be interesting to compare the predictions of the model with the data for a sample of central collisions with different projectiles.

This tendency of the results of the model of spatio-temporal intermittency can certainly be made closer to the experimental data, if we would allow for a projectile dependence on both the spatial and the temporal extensions of the interaction region. This means that taking larger freeze-out time τ_2 for heavier projectiles, one can obtain even weaker attenuation of the slopes with the mass of the interacting nucleus. The recent results of the NA35 collaboration [22] on the time of the pion emission in ultrarelativistic collisons gives a very high value of the freeze-out time of the order of $20 fm/c$. The life-time of the pion source in the mid-rapidity was find to be much longer in the asymmetric collisions, $20 fm/c$ for O-Au versus $6 fm/c$ for S-S collisons.

This strong difference in emission time can explain in the framework of our model the increase of the slopes of s.f.m. with the increase of the mass the target, as seen by the EMU01 collaboration [21] . The interaction time in the asymmetric collisions can take a value of $20 fm/c$ and as one can see in fig.1 , it gives a substantial increase of the intermittency signal. On the other hand the KLM data [18] make no distinction in the target mass. One should note that if the freeze-out time τ_2 depends also on the projectile mass , it could give a stronger intermittency signal for larger projectile mass, contrary to the increase of the multiplicity and thus exhibit a behaviour which is incompatible with the independent source models. Such an increase of the s.f.m. with the projectile mass seems to be present in the recent NA35 data [23].

The above presented model allows to compare different reactions and explains the dependence of the intermittency parameters on the target and projectile mass. One can take the relations between the "slopes" of the s.f.m. in our model (see the table 1) as the lower bound. Nevertheless, these relations are in much better agreement with the relations followed by the experimental data on F_2 , than the independent superposition models and serve as a schematic explanation of the observed relations between intermittency exponents for different projectiles and targets. The detailed comparison needs however hints on the values of parameters R_{1u} and τ_2 for different collisions.

The above discussed effects can be due also to some other phenomena , different from the phase-transition from quark-gluon plasma into hadrons. In that case, eventhough we have no longer scale-invariant correlations with the range exclusively due to the causality constraints, still the correlation range should be of the order of the radius of the interaction region. Consequently, the difference between various projectiles or impact parameters means only the different radius of a single correlated source. On the other hand, if the correlation range is small then the collision would be better described as a superposition of independent sources with radius equal to the correlation range and would give smaller values of the intermittency exponents for heavy projectiles than observed.

The intermittency in rapidity distributions comes in this model from the correlations of hadron sources in the space-time. These correlations were first studied in the context of a higher order phase-transition [8], but can be generalized to any kind of dynamically induced correlations with the correlation range of the order of the size of the system. The scale-invariant form of the correlations is most probably washed out due to :

- projection from three-dimensional hyperspace of constant proper time to one-dimensional rapidity distributions [7, 8],
- smearing of the distribution due to non-ideal Bjorken's scaling,
- contribution from the isotropic decay of the metastable resonances,
- damping of the fluctuations during the hydrodynamical evolution [17] .

So even if the interacting system has scale-invariant correlations, there is no reason to expect such a behaviour in rapidity distributions of the final hadrons.

The effect of the dimensional projection [7, 8] is crucial to explain the observed dependence of the s.f.m. on the projectile and on the impact parameter. The range of the correlation, which is of the order of the size of the interacting region, is a basic assumption to relate the size of the system to the magnitude of the "slopes" of the s.f.m. The model of spatio-temporal intermittency gives qualitatively similar relations between s.f.m. as observed experimentally.

In accordance with the above considerations , this can be an important observation concerning the stucture of the correlations in collision dynamics. The detailed nature of the dynamical fluctuations remains however unknown and , because of many effects reducing the correlations, it cannot serve as a direct signal of a higher order phase-transition. Alternative mechanisms involving string interaction and rescattering could build up correlations giving similar intermittent behaviour [24]. It should be emphasized again that at present we do not know how to derive the intermittency from the underlying theory of strong interactions and , consequently, we do not understand why the effect is there. Anyway the considerations which were presented above, show the importance of the correlated evolution of the hadron sources. So that any model such as higher order phase-transition, string interaction or other mechanism, must include correctly the interactions and correlations between hadron sources.

3 Intermittency analysis of the fragment-size correlations

In the nuclear collisions at lower energies ($E/A \leq 1GeV$) the particle production is strongly suppressed and nuclear breakup into fragments dominates. The detailed mechanism of this fragmentation and disintegration is not yet fully understood. Several authors, drawing the analogy between between the distribution of cluster sizes at the percolation threshold [25] or the droplet sizes at the critical point [26] and the distribution of nuclear fragments in mass and charge [27, 28] suggested the possibility of a liquid-gas phase transition in finite systems. A great step forward towards a more quantitative examination of this hypothesis was done by Campi [27] who proposed an event by event analysis of the nuclear multifragmentation using the method of conditional moments and shown that nuclei break up as a finite percolating network. Later the percolation model was employed for the calculation of energy spectra, linear momentum transfer, mass yield distributions, the scaling features of the heaviest fragment in the fragmentation event (for a recent review see [29]) .

There are strong arguments that the physical mechanism underlying phase transitions is the coalescence of droplets which is a geometrical mechanism that should be largely independent of the details of the nucleon - nucleon interaction. That is why the schematic percolation model , which contains the two most essential elements of the fragmentation process , i.e. the locality of the breaking process involved in the cluster formation and the the short range character of the nucleon-nucleon interaction , may be successful in describing the nuclear fragmentation process.

The analysis using the conditional moments is troubled in small system by the strong statistical fluctuations. These fluctuations cannot be separated out and modify the results of the Campi analysis by changing both the position of the maximum of the conditional moments S_j ($j = 2, 3, ...$) plotted versus the reduced fragment multiplicity and its width , differently for the moments of different rank. Moreover, the exponent τ of a scale-invariant power-law behaviour $n_s^{inc} \sim s^{-\tau}$ can be somewhat modified by those fluctuations. But even in large systems , when statistical fluctuations in the fragment-size distribution are not essential and the discontinuity of S_j is unambiguously found at the same reduced multiplicity for moments S_j of each rank j , the position of this discontinuity depends solely on the two critical exponents τ and σ of the scaling expression [25] :

$$n_s(\tau, \sigma) \sim s^{-\tau} f(\epsilon s^{\sigma}), \tag{9}$$

and not on the geometry of the fragmentation process. Hence , in our opinion , this analysis provides a necessary but not sufficient condition for the identification of the mechanism of scaling in different fragmentation processes. Indeed , the Smoluchowski equation for cluster formation which represents the mean field situation , yields the scale-invariant fragment-size distribution for any $\tau \geq 1.5$ if the breakup kernel has the multiplicative form $(ss')^{\omega}$ $(\omega > 0)$. Hence, the unambiguous

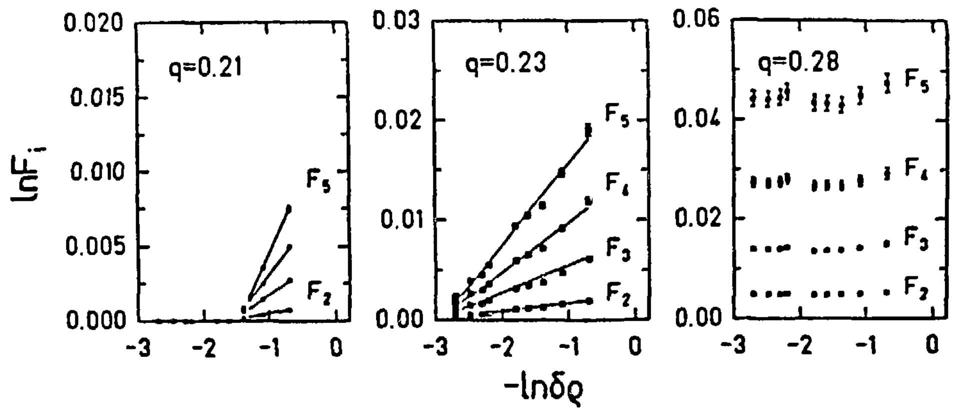

Figure 2: The dependence of the factorial moments on the bin size in the bond percolation model with 6^3 sites, using various values of the bond parameter q (from ref. 4).

identification of the phase-transition in finite systems by analyzing the self-similar fragment-size yields is still lacking and one must be prepared for searching simultaneously for several signatures of the critical behaviour in order to distinguish among different possible explanations of the power-laws. One of the additional features of the critical phenomenon could be the presence of the non-statistical self-similar fluctuations in the fragment size-distribution. In ref. [4] this intermittency analysis has been done for the bond percolation model at around the point $q_\infty = 0.25$ of a second-order phase transition in the infinite ($A \longrightarrow \infty$) system (see fig. 2). The self-similarity of n_s and the intermittent pattern of fluctuations is seen only in the narrow range of bond activation parameters q between 0.21 and 0.27 with the intermittent pattern in the whole range of fragment sizes for $q \simeq 0.24$.

The percolation model is not analytically solvable in general case and hence the detailed study of the correlations between fragments of different sizes is not possible. However , as disscussed in [30] , by studying the properties of the second scaled factorial cummulant $f_{jk} = (<n_j n_k> - <n_j><n_k>)/<n_j><n_k>$ one can identify three main contributions to the correlations :

- the upper mass bound $<n_j n_k> = 0$ for $j + k > A$, which is numerically not important,
- the repulsive correlations arising from the fact that the second fragment is chosen in a smaller system $(A - j)$,
- the attractive correlations arising from the configurations where the two fragments have one or more common empty links in their perimeters.

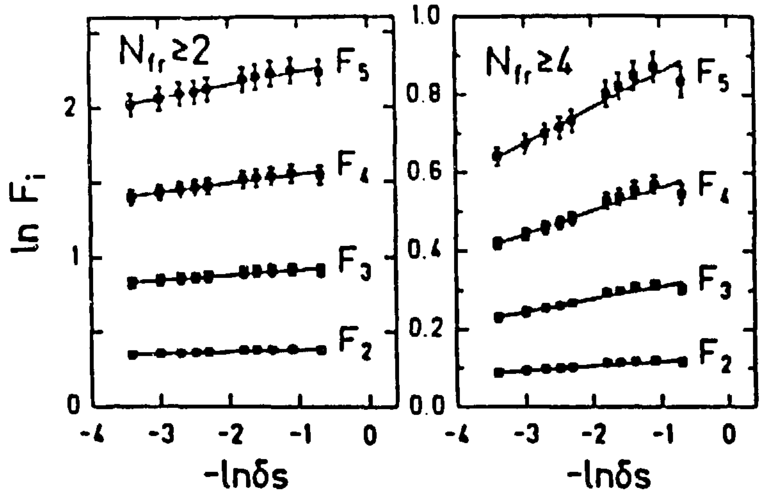

Figure 3: The dependence of the factorial moments on the bin size as calculated for the events of Au-emulsion interactions at $E = 1\mathrm{GeV/nucleon}$. The events have been selected according to the total number of heavy fragments N_{fr} of size $s_{fr} \geq 3$. Solid lines represent linear fits to the data (from ref. 5).

For simplicity let us consider the correlations between one large fragment of mass s and one fragment of mass $s' \equiv 1$ ($< n_s n_1 >$). The mean number of clusters of mass n_s is [25] :

$$< n_s > = A \sum_t g_{st} q^s (1-q)^t \ , \tag{10}$$

where g_{st} is the number of clusters with size s and perimeter t per lattice site. Taking into account only the configurations where the two clusters have at most one common link in the perimeter one obtains :

$$f_{s1} \sim \frac{1}{A}\left(\frac{t}{1-q} - s - t\right) \tag{11}$$

The perimeter t at the percolating point takes the form [31] :

$$t = s(1-q)/q + const\ s^{\sigma} \ , \tag{12}$$

where the second term is not present below p_c. From (11) and (12) one obtains :

$$f_{s1} \sim \frac{const\ q}{A(1-q)} s^{\sigma} \ . \tag{13}$$

The appearance of those attractive correlations should be linked with the apperance of the excess perimeter $\sim const\ s^{\sigma}$ at the percolating point. Above it the number of fragments of "finite" size decreases and consequently the importance of the correlation effects between those fragments diminish. This is not only true for the fragments of size s and $s' \equiv 1$ but for any two fragments of "finite" size. The decrease of the total number of fragments in favour of the increasing mass of the infinite cluster could be responsible for the disappearance of the intermittency signal above the percolating point. The above disscusion shows qualitatively the source of the positive correlations in the fragment size distribution at the percolating point. This mechanism leads to the rise of the s.f.m at the percolating point in the higher dimensional lattices (fig.2). Obviously, the magnitude and in particular the slopes of this dependence cannot be extracted from this simple analysis.

In a recent publications [4, 5] we have shown an evidence for the intermittent behaviour, and hence for the self-similarity of fluctuations in the charge distribution of fragments in the breakup of Au-nuclei of energy $E \sim 1GeV/A$ in a nuclear emulsion. Contrary to the analysis in the percolation model, the experimental data cannot be analyzed as a function of the local connectivity parameter such as the bond activation parameter and one has to look for other ways of selecting critical events. In ref. [4] it was proposed to select events according to the number of heavy fragments . The experimental data [32] consists of about 400 events in which the charge of all fragments has been measured for each event separately. The dependence of the averaged moments F_i on the bin size δs , where for s we take the charge of the fragments ($1 \leq s \leq 79$) , is shown in fig. 3. Both for $N_{fr} \geq 2$ and $N_{fr} \geq 4$ the linear growth of factorial moments clearly manifests the intermittent pattern of fluctuations in the charge distributions.

The experimental data was successfully simulated using the bond percolation model on a cubic lattice containing about the same number of sites as nucleons in the fragmentating Au -nucleus and with the randomly chosen value of the bond activation parameter q in each event [4, 5] . These random - q events have been then filtered by the condition on N_{fr} analogously as in the experimental analysis. Both the intermittency slopes as well as the values of the s.f.m. have been reproduced well. Interestingly, the cascading model of nuclear fragmentation [33] which reproduces reasonably well the features of the conditional moments as seen in the Au-data , fails to reproduce the fluctuation properties of the data as seen in the s.f.m analysis. On the contrary, the microcanonical model by Gross et al. [35] of the fragmentation of hot equilibrated nucleus, when introducing an additional filter on the efficiency in detecting light particles seems to successfully reproduce the data. Should we conclude from these examples that the nuclear fragmentation at $1GeV/A$ is a prompt process rather than a cascading process and that we see the signal of the critical behaviour in the sample of data selected according to the N_{fr} ? We believe that the answer to this question requires further studies because the type of correlations associated to the critical behaviour in the percolation model (second-order phase transition) and in the fragmentation model of Gross et al. [35] (first-order phase transition) are different. In particular, one has to analyze carefully the "background" fluctuations in finite systems due to large fluctuations between different groups of events.

Recently, Mekjian proposed a simple model [34] , which provides an analytical framework for addressing many of those problems. In this statistical model one

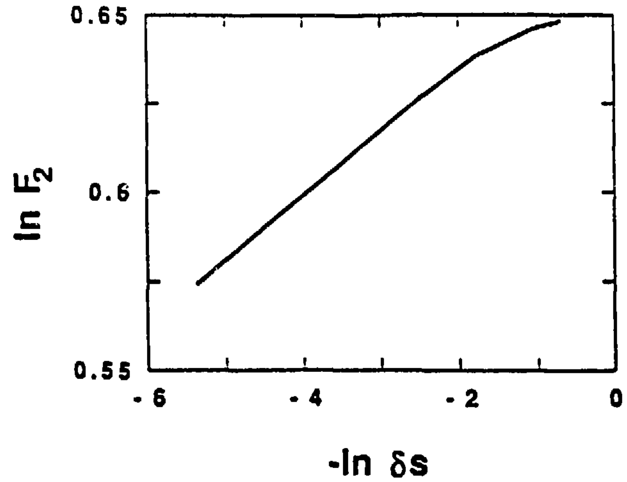

Figure 4: The scaled factorial moment F_i $(i = 2,3)$ in the Mekjian's model with $A = 216$ constituents are calculated for the ensemble composed in the equal parts of events with $x = 0.5, 1.0$ and are plotted as a function of the size of the bin. The solid line denotes F_2 whereas F_3 is depicted with the dashed line.

considers the partitions of A particles into groups with n_j clusters of j elements $(j = 1,2,...,A)$. Various statistical models differ one from another by the choice of the weight of each partition $(n_1, n_2, \ldots, n_A)$, where $A \equiv \sum j\ n_j$. The weight function which leads to an exactly soluble model is [34] :

$$W(\{n_j\}, x; A) = \frac{A!}{n_1!1^{n_1}n_2!2^{n_2}\ldots n_A!A^{n_A}}x^m \equiv M_2(\{n_j\}; A)x^m \ , \qquad (14)$$

where $m = \sum n_j$ is the fragment multiplicity and x is a free parameter of the model. When $x = 1$, the frequency distribution of the fragmentation scheme as measured by its mass distribution, is given by a scale-invariant power law $< n_j >= 1/j$ [34]. The generating function for $W_A(\{x_j\})$ can be written as :

$$G(u, \{x_j\}) = \sum_{A=0}^{\infty} W_A(\{x_j\}) \frac{u^A}{A!} = exp\left[ux_1 + u^2\frac{x_2}{2} + u^3\frac{x_3}{3} + \ldots\right] . \qquad (15)$$

The distribution of clusters $< n_j >$ and the factorial moments can be obtained by derivating the generating function $G(u, x_j$ with respect to u and x_j [30]. For $x = 1$ the only difference with the uncorrelated Poissonian distribution of fragments is due to the repulsive correlations when the mass of the fragments exceeds the total mass of the system. S.f.m. are negative in large bins and tend to zero for small bins. Also for all other values of the tuning parameters x , s.f.m. are negative and extremely small. Mixing in one ensemble events of very different nature, associated for example with different excitation energy or impact parameter and therefore, characterized by different distribution n_s , is equivalent to introducing new, in general attractive correlations and the distribution of fragments in this case may differ from Poissonian distribution strongly. An example of this kind is shown in fig. 4 which presents the results of calculations for the statistical ensemble composed in equal parts of events with $x = 1$ and $x = 0.5$. Eventhough for each x separately the correlations are very small and repulsive, the mixing of very different events leads to large *positive* values of s.f.m. in the whole range of bin-sizes and to approximately linear growth of s.f.m. vs. the resolution. The positive values of moments are due to large fragment-size fluctuations between groups of events distributed according to $n_s(x)$ and the mixed ensemble characterized by n_s^{inc} . One should stress that the similar effect could be present in the random percolation model with the events selected according to N_{fr} and in the canonical fragmentation models. It remains to be seen how much of the fluctuations in the random percolation model [4, 5] , as measured by s.f.m. , result from "background" correlations due to large fluctuations between different groups of events and how much of the effect is due to the genuine attractive correlations which are present in the percolation events at around the critical point.

Instead of presenting conclusions, let us close this paper by a saying that eventhough we seem to see the evidence for the scale-invariant fluctuations in the spectra of produced particles and the charge distributon of fragments following the nuclear multifragmentation, we still do not know how to relate the phenomenon to the more fundamental properties of the theory of strong interactions. Consequently, at the moment we encounter difficulty in separating various finite-size correlation effects and various non-ideal effects contributing at certain scales to the presence of strong non-statistical fluctuations , from the scale-invariant correlations due to the finite system phase transition. To clarify the situation we need much more of experimental and theoretical work. This is particularly true in the domain of low energy nuclear collisions where the high quality exclusive data are virtually non-existant.

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