IC/92/186
INTERNAL REPORT
(Limited Distribution)

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

BETHE–SALPETER EQUATION
FOR NON–SELF CONJUGATE MESONS IN A POWER–LAW POTENTIAL

Sameer M. Ikhdair *
International Centre for Theortical Physics, Trieste, Italy
and
Ramazan Sever
Department of Physics, Middle East Technical University, 06531 Ankara, Turkey.

ABSTRACT

We develop an approach to the solution of the spinless Bethe–Salpeter equation for the different–mass case. Although the calculations are developed for spin–zero particles in any arbitrary spherically symmetric potential, the non–Coulombic effective power–law potential is used as a kernel to produce the spin–averaged bound states of the non–self–conjugate mesons. The analytical formulae are also applicable to the self–conjugate mesons in the equal–mass case. The flavor–independent case is investigated in this work. The calculations are carried out to the third–order correction of the energy series. Results are consistent with those obtained before.

MIRAMARE – TRIESTE

July 1992

* Permanent address: Department of Physics, Middle East Technical University, 06531 Ankara, Turkey.

I. Introduction

The relativistic Bethe–Salpeter equation [1-3] is appealing because it is explicitly covariant and has at least formal connection with quantum field theory and perturbation theory. A standard approximation is introduced by replacing the interaction kernel by an instantaneous local potential and neglecting spin and the coupling of the "large–large" and "small–small" components of the wave function [3].

The spinless Bethe–Salpeter (SBS) equation retains the relativistic kinematics and is suitable for describing the spin–averaged spectrum of two bound fermions of masses m_1 and m_2 and total energy $M(Q\bar{q})$ [4]. Numerical and analytical techniques are used to solve the SBS equation. However, neither of these techniques is well adapted for efficient variation of the potential. These are based on the Fourier transform of the equation into the momentum space [5] and the expansion of the wave functions in some complete set of basis states [2,6]. Durand and Durand [7] constructed an analytic solution to the spinless S–wave Salpeter equation for a Coulombic potential. Nickisch et al., [3] presented and tested two numerical methods for the solution of the relativistic SBS equation of the relativistic quark–antiquark bound states. Cea and co–workers developed a WKB approximation technique for central confining potential [8]. However, the semiclassical treatment of BS equation renders the method applicable only to the case of regular or logarithmically divergent potential. They have also extended the analysis of Ref.[8] to a larger class, which includes singular potentials [6]. Also they showed that the two WKB methods do not sensibly differ when applied to regular potentials. Jacobs and co–workers [9] developed a unified approach to the solution of the Schrödinger and the spinless Salpeter equations and then tested it for the quarkonia. We applied a new approach in the shifted 1/N expansion technique to the equal–mass Bethe–Salpeter BS equation and then solved it for the some well–known static and power–law quarkonium potentials to predict the spin–averaged energy levels and bound–state self–conjugate meson masses [10].

The aim of this paper is to extend the previous work to a more general case considering two fermions of different masses. It would be interesting to see to what extent the same approach is applicable to quarkonia with light quarks u, d, and s. Also, we show that the analysis of this work does not sensibly differ from the equal–mass case.

The plan of the paper is as follows: In Sec.II we formulate a general solution for the Bethe–Salpeter equation with $m_1 \neq m_2$ for any spherically symmetric potential. In Sec.III we use the non–Coulombic power–law potential to obtain the non–self–conjugate mesons $D(c\bar{d})$, $F(c\bar{s})$, $B(b\bar{d})$, $G(b\bar{s})$, and $H(b\bar{c})$. Finally Sec.IV is devoted for our conclusions.

II. The Method

We consider a general coordinate–space wave equation of the general form [4]

$$[(-\nabla^2 + m_1^2)^{1/2} + (-\nabla^2 + m_2^2)^{1/2} + V(\vec{r}) - M(Q\bar{q})]\,\psi(\vec{r}) = 0\,, \qquad (1)$$

where the kinetic terms involving the operation $(-\nabla^2 + m_{1,2})^{1/2}$ are nonlocal and are defined in terms of their Fourier transform.

The radial part of Eq.(1), in the N–dimensional space, for the two–body system of different–masses expanded formally in the powers of $(v/c)^2$ up to two terms to obtain a Schrödinger equation with relativistic corrections [3,4]

$$\left\{-\frac{\nabla_N^2}{2\mu}+\frac{\nabla_N^4}{8\eta^3}+V(r)\right\} R(r)=E_{n\ell}R(r)\ , \tag{2}$$

where $E_{n\ell}=M(Q\bar{q})-m_Q-m_q$. μ is the reduced mass and η is a useful parameter which are defined by

$$\mu=\frac{m_1m_2}{m_1+m_2}\ , \tag{3}$$

and

$$\eta=\left(\frac{m_1m_2}{m_1m_2-3\mu^2}\right)^{1/2}\mu\ . \tag{4}$$

We write the operator ∇_N^2 in the spherical polar coordinate in the N–dimensional space

$$\nabla_N^2=\frac{\partial^2}{\partial r^2}+\frac{N-1}{r}\frac{\partial}{\partial r}-\frac{L_{N-1}^2}{r^2}\ . \tag{5}$$

Employing the transformation

$$R(r)=r^{-(N-1)/2}\phi(r)\ , \tag{6}$$

and introducing the shifting parameter $\bar{k}=k-a$ with $k=N+2\ell$, we obtain a factorized Schrödinger equation of the form

$$\left(\frac{1}{8\eta^3}\frac{\partial^2}{\partial r^2}-g_1(r)\right)\left(\frac{\partial^2}{\partial r^2}-g_2(r)\right)\phi(r)=0\ , \tag{7}$$

where the functions $g_1(r)$ and $g_2(r)$ come out from the reduction of the fourth–order differential equation. They take the general forms

$$g_1(r)=\frac{1}{4\mu}+\frac{[\bar{k}-(1-a)][\bar{k}-(3-a)]}{32\eta^3r^2}$$
$$\mp\frac{1}{4\mu}\left[1+\frac{E_{n\ell}-V(r)}{(\eta^3/\mu^2)}+\frac{(E_{n\ell}-V(r))^2}{2(\eta^3/\mu^2)^2}+O[(v/c)^3]\right], \tag{8}$$

and

$$g_2(r)=\frac{2\eta^3}{\mu}+\frac{[\bar{k}-(1-a)][\bar{k}-(3-a)]}{4r^2}$$
$$\pm\frac{2\eta^3}{\mu}\left[1+\frac{E_{n\ell}-V(r)}{(\eta^3/\mu^2)}+\frac{(E_{n\ell}-V(r))^2}{(\eta^3/\mu^2)^2}+O[(v/c)^3]\right]. \tag{9}$$

Thus, the second order Schrödinger–like equation to order $(v/c)^2$ becomes

$$\left\{-\frac{1}{2\mu}\frac{d^2}{dr^2}+\frac{[\bar{k}-(1-a)][\bar{k}-(3-a)]}{8\mu r^2}+V(r)-E_{n\ell}\right.$$

$$\left.-\frac{(E_{n\ell}-V(r))^2}{2m'}+\cdots\right\}\hat{\phi}(r)=0, \tag{10}$$

where $m'=(m_1m_2\mu)/(m_1m_2-3\mu^2)$ and

$$\hat{\phi}(r)=\frac{1}{2\eta}\left\{-\frac{1}{2\mu}\frac{d^2}{dr^2}+\frac{[\bar{k}-(1-a)][\bar{k}-(3-a)]}{8\eta r^2}+\frac{\eta^2}{\mu}\right.$$
$$\left.\pm\frac{\eta^2}{\mu}\left[1+\frac{(E_{n\ell}-V(r)}{(\eta^3/\mu^2)}+\frac{(E_{n\ell}-V(r))^2}{2(\eta^3/\mu^2)^2}+\cdots\right]\right\}\phi(r)\ . \tag{11}$$

where $\hat{\phi}(r)$ is called the Bethe–Salpeter reduced wavefunction. The $(E_{n\ell}-V(r))^2$ perturbation term in the last equation is significant only where it is small, i.e., $(E_{n\ell}-V(r))/2m'\ll 1$. This condition is verified by the confining potentials used to describe heavy–quark systems except near the color–Coulomb singularity at the origin, and for $r\to\infty$. However, it is always satisfied on the average as stated by Durand and Durand [4]. We now proceed to solve Eq.(10) for which the negative sign has been chosen for a physical solution. Shifting the origin by

$$x=\bar{k}^{1/2}(r-r_0)/r_0\ , \tag{12}$$

and expanding

$$V(r)=(\bar{k}^2/Q)[V(r_0)+V'(r_0)r_0x/\bar{k}^{1/2}+V''(r_0)r_0^2x^2/2\bar{k}+\cdots]\ , \tag{13}$$

and

$$E_{n\ell}=E_0+E_1/\bar{k}+E_2/\bar{k}^2+E_3/\bar{k}^3+\cdots\ , \tag{14}$$

and substituting these into Eq.(10), one gets

$$\mathcal{E}_{n_r}=\frac{r_0^2}{Q}\,[\,\bar{k}(E_0+\frac{E_0^2}{2m'})+(E_1+\frac{E_0E_1}{m'})+(E_2+\frac{E_0E_2}{m'}+\frac{E_1^2}{2m'})\frac{1}{\bar{k}}$$
$$+(E_3+\frac{E_0E_3}{m'}+\frac{E_1E_2}{m'})\frac{1}{\bar{k}^3}+\cdots]\ . \tag{15}$$

Q is a scale whose magnitude is to be determined later. Thus comparing Eq.(10) with its counterpart Schrödinger–like equation for the one–dimensional anharmonic oscillator problem which has been investigated in detail by Imbo et al., [11-13] we calculate all the relevant quantities δ's and ε's. Their detailed expressions are presented in the Appendix A. The final analytic expression for the $1/\bar{k}$ expansion of the energy eigenvalues appropriate to the BS particle is

$$\mathcal{E}_{n_r}=\bar{k}\left[\frac{1}{8\mu}+\frac{r_0^2V(r_0)}{Q}-\frac{r_0^2V(r_0)^2}{2m'Q}+\frac{r_0^2E_0V(r_0)}{m'Q}\right]$$
$$+\left[(1+2n_r)\frac{\omega}{2}-\frac{(2-a)}{4\mu}\right]+\frac{1}{\bar{k}}\left[\beta^{(1)}+\frac{r_0^2E_2V(r_0)}{m'Q}\right]$$
$$+\frac{1}{\bar{k}^2}\left[\beta^{(2)}+\frac{r_0^2E_3V(r_0)}{m'Q}\right]+O\left[\frac{1}{\bar{k}^3}\right]\ , \tag{16}$$

where n_r is called the radial quantum number. The quantities $\beta^{(1)}$ and $\beta^{(2)}$ appearing in the correction to the leading order of the energy expression are defined and listed in the Appendix A.

Comparing the terms of Eq.(15) with those of Eq.(16) and equating terms of same order in $\bar{k}$ yields

$$(\frac{r_0^2}{Q})\left[E_0 + \frac{E_0^2}{2m'}\right] = \left[\frac{1}{8\mu} + \frac{r_0^2 V(r_0)}{Q} - \frac{r_0^2 V(r_0)^2}{2m'Q} + \frac{r_0^2 E_0 V(r_0)}{m'Q}\right] , \quad (17)$$

$$(\frac{r_0^2}{Q})\left[E_1 + \frac{E_0 E_1}{m'}\right] = \left[(1+2n_r)\frac{\omega}{2} - \frac{(2-a)}{4\mu}\right] , \quad (18)$$

$$(\frac{r_0^2}{Q})\left[E_2 + \frac{E_0 E_2}{m'} + \frac{E_1^2}{2m'}\right] = \left[\beta^{(1)} + \frac{r_0^2 E_2 V(r_0)}{m'Q}\right] , \quad (19)$$

$$(\frac{r_0^2}{Q})\left[E_3 + \frac{E_0 E_3}{m'} + \frac{E_1 E_2}{m'}\right] = \left[\beta^{(2)} + \frac{r_0^2 E_3 V(r_0)}{m'Q}\right] . \quad (20)$$

From Eq.(17) we get

$$E_0 = V(r_0) - m' + \sqrt{m'^2 + \frac{Q}{4r_0^2(\mu/m')}} . \quad (21)$$

where r_0 is chosen to minimize E_0 [11]. That is,

$$\frac{dE_0}{dr_0} = 0 \qquad \text{and} \qquad \frac{d^2 E_0}{dr_0^2} > 0 , \quad (22)$$

therefore, r_0 satisfies the equation

$$r_0^3 V'(r_0)\left(m'^2 + \frac{Q}{4r_0^2(\mu/m')}\right)^{1/2} = \frac{Q}{4(\mu/m')} . \quad (23)$$

To solve for the shifting parameter a, the next contribution to the energy eigenvalue is chosen to vanish [12], i.e., $E_1 = 0$, which implies that

$$a = 2 - 2\mu(1+2n_r)\omega , \quad (24)$$

where ω is given by

$$\omega = \frac{1}{2\mu}\left[3 + r_0 V''(r_0)/V'(r_0) - 4r_0^4 V'(r_0)^2/Q(m'/\mu)\right]^{1/2} . \quad (25)$$

and Q satisfies Eq.(23), which is written as

$$Q = \frac{2\mu}{m'}[r_0^2 V'(r_0)]^2(1+\beta) , \quad (26)$$

with

$$\beta = \sqrt{1 + [2m'/r_0 V'(r_0)]^2} . \quad (27)$$

Equations (24) and (26) along with Eqs.(25) and (27), with $Q = \bar{k}^2$, yield

$$1 + 2\ell + 2\mu(2n_r+1)\omega = r_0^2 V'(r_0)\left(\frac{2\mu}{m'} + \frac{2\beta\mu}{m'}\right)^{1/2} , \quad (28)$$

which is an explicit equation in r_0. Once r_0 is determined, Eq.(21) gives E_0, Eq.(19) gives E_2 and Eq.(20) gives E_3. Finally Eq.(14) gives

$$E_{n\ell} = E_0 + \frac{\beta^{(1)}}{r_0^2(1 + \frac{E_0 - V(r_0)}{m'})} + \frac{\beta^{(2)}}{r_0^2(1 + \frac{E_0 - V(r_0)}{m'})\bar{k}} + O\left[\frac{1}{\bar{k}^2}\right] . \quad (29)$$

which is an explicit expression for the total energy eigenvalues of the system.

III. Application to the Non-Self Conjugate Mesons

The power−law potential can generate the relativistic bound states of the self−conjugate $Q\bar{Q}$ mesons in close conformity with the spin−averaged data (SAD). We obtain the relativistic spin−averaged mass spectra for the atom−like mesons $D(c\bar{d}), F(c\bar{s}), B(b\bar{d}), G(b\bar{s})$ and $H(b\bar{c})$.

The $\psi(4s)$ and $\Upsilon(4s)$ are predominately decay into mesons containing charmed and beauty flavors. The mesons $c\bar{q}$ and $b\bar{q}$ are in the $^1S_0$ states. The excited meson states D^* and B^* are observed at or above the $\psi(4s)$ and $\upsilon(4s)$ states. These are considered the triplet $^3S_1$ $c\bar{q}$ and $b\bar{q}$ states and have therefore spin 1.

The potentials which do not include spin−spin forces ignore the hyperfine splittings. The energy levels obtained by solving the Bethe−Salpeter equation for the $S-, P-$ and $D-$states can then be considered as the center of gravity (cog) or the spin−averaged data (SAD) of the singlet and triplet states. These states can be weighted by their respective spin statistical factor.

The Bethe−Salpeter mass spectra of the above mentioned non−self−conjugate mesons are calculated in the following manner ($\hbar = c = 1$)

$$M_{n\ell}(Q\bar{q}) = m_Q + m_q + E_{n\ell} , \quad (30)$$

where $E_{n\ell}$ is the binding energy of the quarks.

We investigate a static non−Coulombic power−law potential of the form

$$V(r) = Ar^{\nu} + V_0 , \quad (31)$$

where ν is close to 0.1 and $A > 0$. We inspired to use the simultaneous parameters fit to obtain the spectra of the unlike−flavored quark−antiquark configurations presented herein, in other words flavor−independent case. We fix the quark masses as [14]

$$[m_u, m_d, m_s] = [0.005, 0.01, 0.325] \text{ GeV} ,$$

and

$$[m_c, m_b] = [1.6179, 5.0114] \text{ GeV} .$$

Therefore, our results are reported in Table 1.

IV. Conclusions

We have presented a unified study of some higher spin−averaged mass levels of $Q\bar{q}$ systems in the Bethe−Salpeter equation with any spherically symmetric potential. These calculations are tested using a simple non−QCD−based power−law potential to produce the non−self−meson spectra. Flavor independence is strictly maintained by taking the same set of potential parameters and the quark masses. We point out that with the power−law potential which describes the Bethe−Salpeter mass spectra of the self−conjugate $Q\bar{Q}$ or $q\bar{q}$ mesons reasonably well in the last study, an excellent relativistic description of the non−self−conjugate $Q\bar{q}$ mesons can be obtained using current quark masses for light quarks.

Slight departures from the experimental data is observed. We remark here that fitting the potential parameters quite properly would have enabled us to obtain quantitative agreement with the experimental mass spectra although a very qualitative agreement has been noticed. Finally we stress that the accuracy as well as the convergence increases as the principal and orbital quantum numbers increases. This is quite apparent for one glance on the leading and the corrected terms to the energy.

Acknowledgements

One of the authors (S.M.I.) would like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for the kind hospitality at the International Centre for Theoretical Physics, Trieste, where this work was finalized.

Appendix

We list here the analytic expressions of $\beta^{(1)}$, $\beta^{(2)}$, ε_i's and δ_j's in case of the spinless Bethe−Salpeter equation.

$$\beta^{(1)} = \frac{(1-a)(3-a)}{8\mu} + [(1+2n_r)\bar{\varepsilon}_2 + 3(1+2n_r+2n_r^2)\bar{\varepsilon}_4] - \omega^{-1}[\bar{\varepsilon}_1^2 + 6(1+2n_r)\bar{\varepsilon}_1\bar{\varepsilon}_3 + (11+30n_r+30n_r^2)\bar{\varepsilon}_3^2] , \tag{A1}$$

$$\begin{aligned}\beta^{(2)} = {} & (1+2n_r)\bar{\delta}_2 + 3(1+2n_r+2n_r^2)\bar{\delta}_4 + 5(3+8n_r+6n_r^2+4n_r^3)\bar{\delta}_6 \\ & -\omega^{-1}[(1+2n_r)\bar{\varepsilon}_2^2 + 12(1+2n_r+2n_r^2)\bar{\varepsilon}_2\bar{\varepsilon}_4 + 2\bar{\varepsilon}_1\bar{\delta}_1 \\ & +2(21+59n_r+51n_r^2+34n_r^3)\bar{\varepsilon}_4^2 + 6(1+2_r)\bar{\varepsilon}_1\bar{\delta}_3 \\ & +36(1+2n_r)\bar{\varepsilon}_1\bar{\varepsilon}_2\bar{\varepsilon}_3 + 8(11+30n_r+30n_r^2)\bar{\varepsilon}_2\bar{\varepsilon}_3^2 \\ & +24(1+2n_r)\bar{\varepsilon}_1^2\bar{\varepsilon}_4 + 8(31+78n_r+78n_r^2)\bar{\varepsilon}_1\bar{\varepsilon}_3\bar{\varepsilon}_4 \\ & +12(57+189n_r+225n_r^2+150n_r^3)\bar{\varepsilon}_3^2\bar{\varepsilon}_4] \\ & -\omega^{-3}[8\bar{\varepsilon}_1^3\bar{\varepsilon}_3 + 108(1+2n_r)\bar{\varepsilon}_1^2\bar{\varepsilon}_3^2 + 48(11+30n_r+30n_r^2)\bar{\varepsilon}_1\bar{\varepsilon}_3^3 \\ & +30(31+109n_r+141n_r^2+94n_r^3)\bar{\varepsilon}_3^4] ,\end{aligned} \tag{A2}$$

where

$$\bar{\varepsilon}_i = \frac{\varepsilon_i}{(2\mu\omega)^{i/2}} , \quad i = 1,2,3,4. \qquad \bar{\delta}_j = \frac{\delta_j}{(2\mu\omega)^{j/2}} , \quad j = 1,2,...6. \tag{A3}$$

$$\varepsilon_1 = \frac{(2-a)}{2\mu} , \qquad \varepsilon_2 = -\frac{3}{4\mu}(2-a) , \tag{A4}$$

$$\varepsilon_3 = -\frac{1}{2\mu} + \frac{r_0^5}{6Q}\left[V'''(r_0) - \frac{V(r_0)V'''(r_0)}{m'} - \frac{3V'(r_0)V''(r_0)}{m'} + \frac{V'''(r_0)E_0}{m'}\right] , \tag{A5}$$

$$\begin{aligned}\varepsilon_4 = {} & \frac{5}{8\mu} + \frac{r_0^6}{24Q}\left[V''''(r_0) - \frac{V(r_0)V''''(r_0)}{m'} - \frac{4V'(r_0)V'''(r_0)}{m'}\right. \\ & \left. - \frac{3V''(r_0)V''(r_0)}{m'} + \frac{V''''(r_0)E-0}{m'}\right] ,\end{aligned} \tag{A6}$$

$$\delta_1 = -\frac{(1-a)(3-a)}{4\mu} + \frac{r_0^3 E_2 V'(r_0)}{m'Q} , \quad \delta_2 = \frac{3(1-a)(3-a)}{8\mu} + \frac{r_0^4 E_2 V''(r_0)}{2m'Q} , \tag{A7}$$

$$\delta_3 = \frac{(2-a)}{\mu} , \qquad \delta_4 = -\frac{5(2-a)}{4\mu} , \tag{A8}$$

$$\begin{aligned}\delta_5 = {} & -\frac{3}{4\mu} + \frac{r_0^7}{120Q}\left[V'''''(r_0) - \frac{V(r_0)V'''''(r_0)}{m'} - \frac{5V'(r_0)V''''(r_0)}{m'}\right. \\ & \left. - \frac{10V''(r_0)V'''(r_0)}{m'} + \frac{V'''''(r_0)E_0}{m'}\right] ,\end{aligned} \tag{A9}$$

$$\begin{aligned}\delta_6 = {} & \frac{7}{8\mu} + \frac{r_0^8}{720Q}\left[V''''''(r_0) - \frac{V(r_0)V''''''(r_0)}{m'} - \frac{6V'(r_0)V'''''(r_0)}{m'}\right. \\ & \left. - \frac{15V''(r_0)V''''(r_0)}{m'} - \frac{10V'''(r_0)V'''(r_0)}{m'} + \frac{V''''''(r_0)E_0}{m'}\right] .\end{aligned} \tag{A10}$$

References

[1] E.E.Salpeter and H.A.Bethe, Phys. Rev. **84**, 1232 (1951); J.Schwinger, Proc. Natt. Acad. Sci. U.S.A. **37**, 452 (1951); **37**, 455 (1951).

[2] D.P.Stanley and D.Robson, Phys. Rev. **D21**, 3180 (1980); L.Hostler and W.Repko, Ann. Phys. (N.Y.) **130**, 329 (1980); S.Godfrey and N.Isgur, Phys. Rev. **D32**, 189 (1985).

[3] L.J.Nickisch et al., Phys. Rev. **D30**, 660 (1984).

[4] B.Durand et al., Phys. Rev. **D25**, 2312 (1982); ibid. **D30**, 1904 (1984).

[5] A.B.Henriques et al., Phys. Lett. **64B**, 85 (1976); P.Ditsas et al., Nucl. Phys. **B146**, 191 (1978).

[6] P.Cea et al., Phys. Rev., **D28**, 2291 (1983).

[7] B.Durand and L.Durand, Phys. Rev., **D28**, 396 (1983).

[8] P.Cea et al., Phys. Rev. **D26**, 1157 (1982).

[9] S.Jacobs et al., Phys. Rev. **D33**, 3338 (1986).

[10] S.M.Ikhdair and R.Sever, (to be published in Z.Phys.C).

[11] T.Imbo and U.Sukhatma, Phys. Rev. **D28**, 418 (1983).

[12] T.Imbo et al., Phys. Rev. **D29**, 1669 (1984).

[13] T.Imbo et al., Phys. Lett. **105A**, 183 (1984).

[14] S.N. Jena, and T.Tripati, Phys. Rev. **D28**, 1780 (1983).

TABLES

TABLE I.

State	$c\bar{d}$	$c\bar{s}$	$b\bar{d}$	$b\bar{s}$	$b\bar{c}$
$[A,\nu,V_0]^\dagger = [1.709^{1.1}, 0.1, -2.028]$GeV					
$1S$	1.9047	2.1223	5.2969	5.4981	6.6738
$2S$	2.0714	2.2908	5.4635	5.6641	6.8377
$3S$	2.1672	2.3890	5.5594	5.7609	6.9329
$4S$	2.2364	2.4596	5.6285	5.8304	7.0011
$5S$	2.2910	2.5150	5.6830	5.8851	7.0547
$1P$	2.0619	2.2542	5.4541	5.6297	6.7943
$2P$	2.1602	2.3641	5.5523	5.7375	6.9033
$1D$	2.1547	2.3397	5.5468	5.7149	6.8731
$[A,\nu,V_0] = [1.709^{1.1}, 0.1, -1.960]$GeV					
$1S$	1.9748	2.1901	5.3670	5.5658	6.7418
$2S$	2.1389	2.3583	5.5310	5.7314	6.9055
$3S$	2.2346	2.4564	5.6267	5.8282	7.0007
$4S$	2.3037	2.5270	5.6958	5.8977	7.0689
$5S$	2.3583	2.5824	5.7504	5.9524	7.1224
$1P$	2.1301	2.3222	5.5222	5.6977	6.8623
$2P$	2.2280	2.4319	5.6201	5.8052	6.9712
$1D$	2.2227	2.4077	5.6148	5.7829	6.9411

†Same parameter fits as Ref.[14]

- Table 1. Spin−averaged Bethe−Salpeter mass spectra $M_{n\ell}(Q\bar{q})$ of $c\bar{d}$, $c\bar{s}$, $b\bar{d}$, $b\bar{s}$, and $b\bar{c}$ systems.