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NEUTRINO OSCILLATIONS AND A NEW FARADAY EFFECT

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ABSTRACT

By analogy with the classical Faraday effect for the electromagnetic waves, a Faraday effect for massive neutrinos is found to be a somewhat generic description of neutrino oscillations when the neutrinos traverse a dense medium with or without a magnetic field. We further plot the Faraday angle for the solar neutrino problem as an illustration of the fact that the Faraday effect may yield a conceptually convenient parametrization of various neutrino oscillation scenarios.

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The well-known classical Faraday effect [1] occurs when a linearly polarized elctromagnetic wave passes through a fixed distance in a magnetic field resulting in the rotation of its plane of polarization, called the Faraday angle. The Faraday angle is found to be proportional to the bireferingence of the medium, i.e., the difference of the refractive indices corresponding to the left- and right- circularly polarized components of the wave. The effect is thus a measure of the properties of the medium (matter and external magnetic field) through which the wave passes.

In the present note we attempt to study an analogous effect for a massive neutrino de Broglie wave assumed to be consisting of a superposition of a Dirac type electron-neutrino (ν_e) and a corresponding muon-neutrino (ν_μ) for a two flavour case. We assume that the two neutrinos in general can have both left- and right- helicities. Our result can also be generalised to the Majorana neutrino case in a straightforward way.

Lim and Marciano [2] and others [3,4] have combined the various oscillations scenarios such as the matter enhanced flavour oscillations (the MSW effect [5], the helicity oscillations (the OVV effect [6]) and the spin-flavour oscillations (SFP) [2,3], by a 4×4 Hamiltonian evolution matrix operating on a 4 component chiral neutrino base (ν_e^L , ν_μ^L , ν_e^R , ν_μ^R). They further showed that these scenarios could be described effectively by 2×2 submatrices as given in the following evolution equation:

$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \begin{pmatrix}\frac{\Delta}{2E_\nu}\sin^2\theta + \sum_i \mathbf{a}^i_{\nu_e} & H_{12}\\ H_{21} & \frac{\Delta}{2E_\nu}\cos^2\theta + \sum_i \mathbf{a}^i_{\nu_\mu}\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}, \tag{1}$$

where θ is the vacuum mixing angle, $\Delta = m_2^2 - m_1^2$ is the difference of mass-squared eigenvalues and $\mathbf{a}^i_{\nu_{e/\mu}}$ is the interaction potential exerted by the ith matter particle on the traversing electron-neutrino/muon-neutrino. The off- diagonal elements $H_{12} = H_{21}$ for different oscillation scenarios take different values, viz. for the matter-enhanced resonant flavour oscillations (MSW effect):

$$H_{12} = \frac{\Delta}{2E_\nu}\sin 2\theta\,; \tag{2}$$

for the spin-flavour scenario:

$$H_{12} = \mu_{21}B\,; \tag{3}$$

whereas for the helicity oscillations (OVV effect):

$$H_{12} = \mu_{11}B \quad \text{and} \quad \Delta = 0\,, \tag{4}$$

where μ and B are the magnetic moment and the transverse magnetic field respectively. From the evolution matrix in eq.(1), the resonance condition can easily be obtained as:

$$\sum_i(\mathbf{a}^i_{\nu_e} - \mathbf{a}^i_{\nu_\mu}) = \frac{\Delta}{2E_\nu}\cos 2\theta. \tag{5}$$

This condition holds for the above mentioned oscillations with appropriate values of the parameters θ and Δ . These set of values for instance can be found when the matter potentials are parametrized as in the Solar Neutrino Problem.

The usual diagonalization of the evolution matrix appearing in Eq.(1) in the presence of matter leads to the eigenvalues

$$\mathbf{E}^m_{1,2} = \frac{1}{2}\left[\sum_i(\mathbf{a}^i_{\nu_e} - \mathbf{a}^i_{\nu_\mu}) + \frac{\Delta}{2E_\nu} \pm \sqrt{(\sum_i(\mathbf{a}^i_{\nu_e} - \mathbf{a}^i_{\nu_\mu}) - \frac{\Delta}{2E_\nu}\cos 2\theta)^2 + (2H_{12})^2}\right]. \quad (6)$$

The mixing angle θ_m, between the two oscillating states in the presence of matter is defined by

$$tan 2\theta_m = \frac{2H_{12}}{\sqrt{(\mathbf{E}^m_1 - \mathbf{E}^m_2)^2 - (2H_{12})^2}}, \quad (7)$$

where, using Eq.(6):

$$\mathbf{E}^m_1 - \mathbf{E}^m_2 = \sqrt{(\sum_i(\mathbf{a}^i_{\nu_e} - \mathbf{a}^i_{\nu_\mu}) - \frac{\Delta}{2E_\nu}\cos 2\theta)^2 + (2H_{12})^2}, \quad (8)$$

is the energy eigenvalue splitting. Clearly at resonance, defined by Eq.(5), θ_m is maximum and the splitting at the resonance which is given by the discriminant Eq.(8) drops to its minimum value:

$$(\mathbf{E}^m_1 - \mathbf{E}^m_2)_{res.} = 2H_{12}. \quad (9)$$

Next, introduce the index of refraction n_i for the neutrino mass eigenstates as a dispersion relation

$$k_i \equiv E_\nu n_i = E_\nu - \mathbf{E}^m_i, i=1,2, \quad (10)$$

where k_i are the corresponding wave numbers. The Faraday angle for the neutrino de Broglie wave, related to the birefringence, is then defined as:

$$\phi_\nu = \ell(k_2 - k_1),$$

which using Eq.(10) becomes

$$\phi_\nu = \ell(\mathbf{E}^m_1 - \mathbf{E}^m_2). \quad (11)$$

Here ℓ is the distance traversed by the neutrino wave. Next, the specific Faraday angle which may be defined as Faraday angle averaged over ℓ, becomes

$$\phi^{(s)}_\nu = \phi_\nu/\ell = \mathbf{E}^m_1 - \mathbf{E}^m_2. \quad (12)$$

In the following, we see some qualitative features of the results obtained in Eqs.(8-12): The angle ϕ_ν evolves as a function of net matter density $\sum_i(\mathbf{a}^i_{\nu_e} - \mathbf{a}^i_{\nu_\mu})$ and distance ℓ traversed by the neutrinos in the medium. If the neutrinos start propagating from some highly dense region, where:

$$\sum_i(\mathbf{a}^i_{\nu_e} - \mathbf{a}^i_{\nu_\mu}) >> \frac{\Delta}{2E_\nu}cos 2\theta,\ 2H_{12}$$

then, using Eqs. (8) and (11-12):

$$\phi_\nu \sim \ell\sum_i(\mathbf{a}^i_{\nu_e} - \mathbf{a}^i_{\nu_\mu}) \quad (13)$$

and

$$\phi^{(s)}_\nu \sim \sum_i(\mathbf{a}^i_{\nu_e} - \mathbf{a}^i_{\nu_\mu}). \quad (14)$$

On the other hand, when the neutrinos propagate out in the less dense region, where

$$2H_{12} >> \sum_i(\mathbf{a}^i_{\nu_e} - \mathbf{a}^i_{\nu_\mu}), \frac{\Delta}{2E_\nu}cos 2\theta,$$

then

$$\phi_\nu \sim \ell(2H_{12}) \quad (15)$$

and

$$\phi^{(s)}_\nu \sim 2H_{12}. \quad (16)$$

If $2H_{12}$ is fixed as in the case of spin-flavour ocillaltions, then the specific rotation angle $\phi^{(s)}_\nu$ turns out to be fixed for every given resonance position, while the rotation angle ϕ_ν is just proportional to the distance ℓ traversed up to the resonance position.

In order to understand the significance of the Faraday angle in a quantitative way, we study its evolution in the solar environment, corresponding to these different possible scenarios of the oscillations. The net interaction potential experienced by the neutrino wave is given by:

$$\sum_i(\mathbf{a}^i_{\nu_{eL}} - \mathbf{a}^i_{\nu_{\mu L}}) = \sqrt{2}G_F N_e \quad (17)$$

in the case of the MSW effect;

$$\sum_i(\mathbf{a}^i_{\nu_{eL}} - \mathbf{a}^i_{\nu_{\mu R}})\delta = \frac{G_F}{\sqrt{2}}2N_e - N_n \quad (18)$$

in the case of the spin-flavour oscillations; and

$$\sum_i(\mathbf{a}^i_{\nu_{eL}} - \mathbf{a}^i_{\nu_{eR}}) = \frac{G_F}{\sqrt{2}}2N_e - N_n \quad (19)$$

in the case of the OVV effect. The electron density N_e and the nucleon density N_n are well approximated [2,7]:

$$N_e \approx 6N_n \approx 2.4 \times 10^{26} exp(-\ell/0.09R_s)/c.c. \quad (20)$$

Using Eqs.(8), (11) and (12) – (20), one can then easily calculate ϕ_ν and ϕ^s_ν for the solar environment. Some results are plotted in Figs.1–3, which illustrate the features mentioned above in more detail. Figs. 1(a–c) give the Faraday angle ϕ_ν vs. distance traversed (in units of solar radius R_S, using values of μB appropriate to the sun) for a typical choice of Δ-values corresponding to small (vacuum) angle θ (fixed) MSW fit of the Gallex experiment [8]. Note that the minima shift with Δ as do the values of ϕ_ν corresponding to large distance ℓ. However, for the same choice of Δ values, the specific Faraday angle ϕ^s_ν behaves in a similar way (Figs.1(d–f)). This trend of ϕ^s_ν is nearly independent of the character of the oscillation. Figs. 2(a–c) give the Faraday angle ϕ_ν versus distance traversed for the SFP, and show that the behaviour of ϕ_ν is nearly the same as that for the MSW (Figs. 1(a–c)). Thus, one may *a* *priori* expect no substantial difference in the SNU rates observed on the basis of these two scenarios as possible solutions of the SNP. Figs. 3(a–b) give the values of the Faraday angle and the specific Faraday angle with distance for the OVV-scenario. This behaviour turns out to be different from the other two scenarios, which may be regarded as a significant result.

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REFERENCES

1. Francis F. Chen *'Introduction to Plasma Physics'*
   Plenum Press, New York and London (1977).
2. C.S. Lim and W.J. Marciano
   Phys. Rev. **D37**, (1988) 1368.
3. E.K. Akhmedov
   Phys. Lett. **B213**, (1988) 64.
4. J. Pulido
   Phys. Rep. **211**, (1992) 167.
5. L. Wolfenstein
   Phys. Rev. **D17**, (1978) 2369; **D20**, (1979) 2364;
   S.P. Mikheyev and A.Yu. Smirnov Nuov. Cim. **9C**, (1986) 17;
   Sov. J. Nucl. Phys. **42**, (1985) 913;
   For a review, see for example: T.K. Kuo and J. Pantaleone
   Rev. Mod. Phys. **61**, (1989) 937.
6. M.B. Voloshin, M.I. Vysotsky and L.B. Okun
   Sov. Phys. JETP **64**, (1986) 446;
   M.B. Voloshin and M.I. Vysotsky
   Sov. J. Nucl. Phys. **44**, (1986) 845.
7. J.N. Bahcall and R.K. Ulrich
   Rev. Mod. Phys. **60**, (1988) 297.
8. P. Anselmann et al. (Gallex Collaboration) *'Implications of the Gallex determination of the solar neutrino flux, June 1, 1992'*
   (submitted to Physics Letters B)

FIGURE CAPTIONS

Figs.1–3 Faraday angle ϕ_ν and specific Faraday angle ϕ_ν^s (defined in the text) for a two-flavour superposition of massive neutrino waves are plotted against the distance ℓ (in units of solar radius) traversed by the wave for the Solar Neutrino Problem (SNP). The typical values of the parameters $\Delta = m_2^2 - m_1^2$ and the small vacuum mixing angle θ have been taken from the Gallex experiment, which may be consistent with the MSW effect.

Figs.1(a–c) Give the plots for the Faraday angle, while

Figs.1(d–f) Give those for the specific Faraday angle.

Figs.2(a–c) Give corresponding values of Faraday angle vs. ℓ for an SFP scenario.

Figs.3(a–b) Here, the Faraday angle and the specific Faraday angle for the OVV effect are plotted, corresponding to the same choice of parameters.

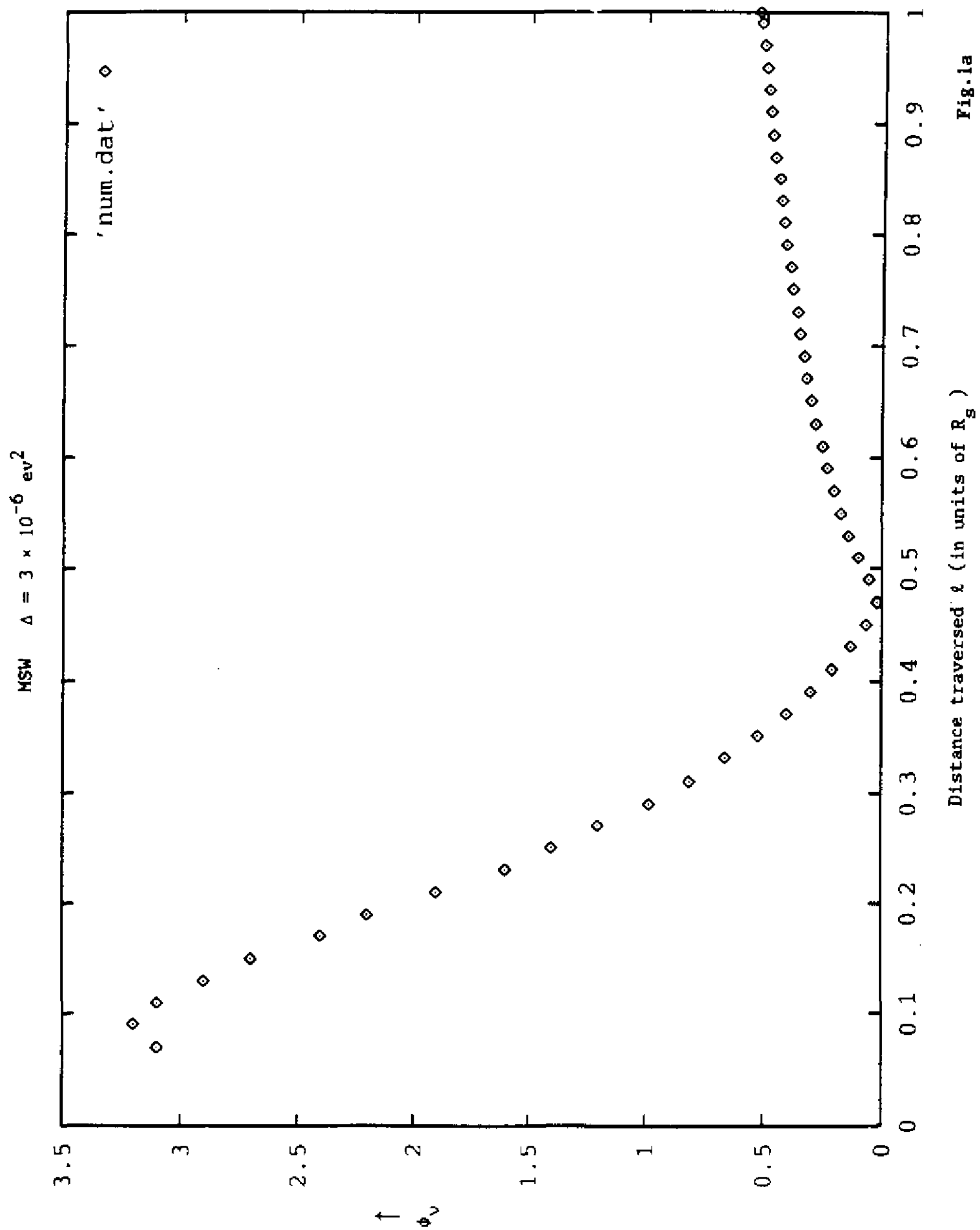
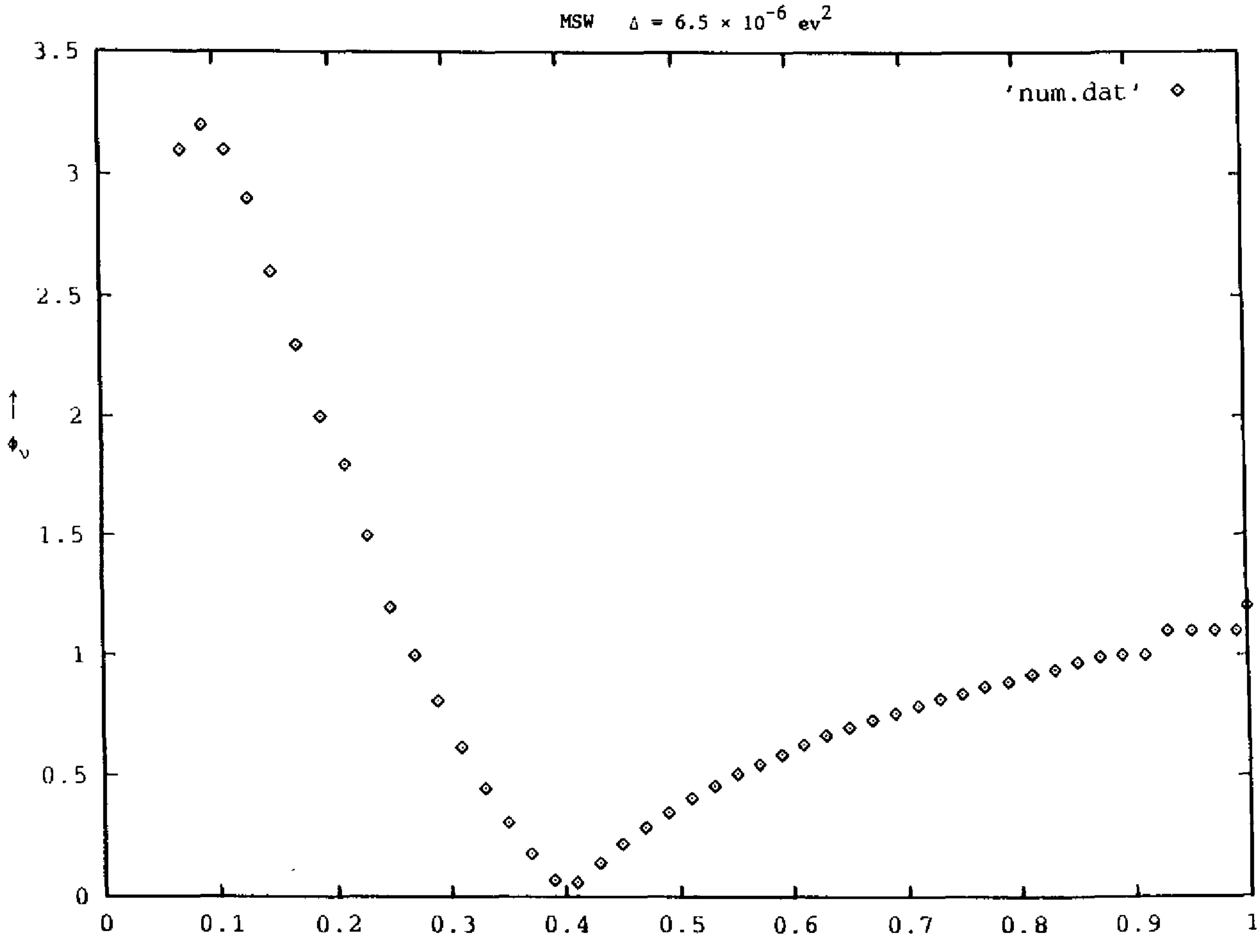
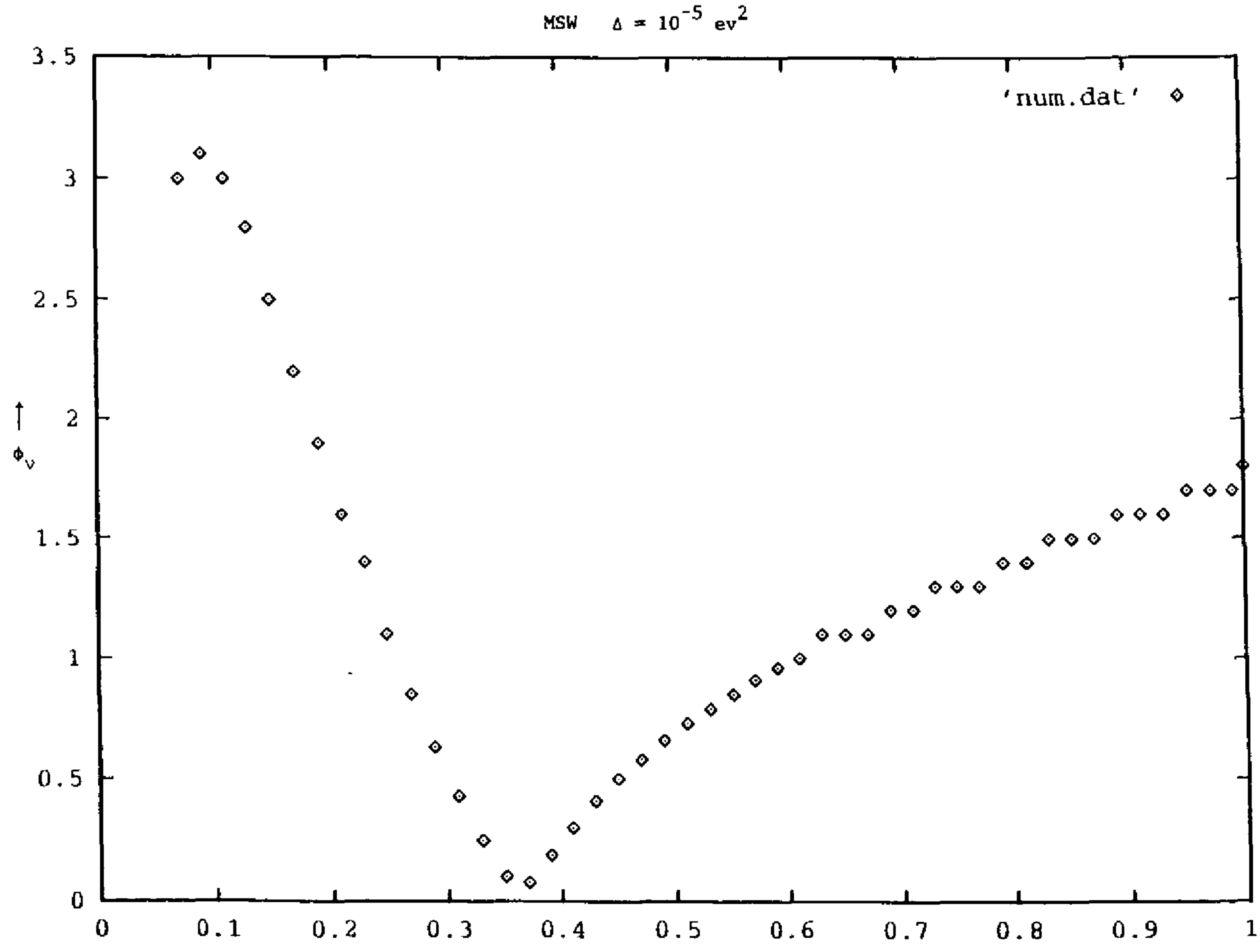


Fig.1a



Distance traversed ℓ (in units of R_s)

Fig.1b



Distance traversed ℓ (in units of R_s)

Fig.1c

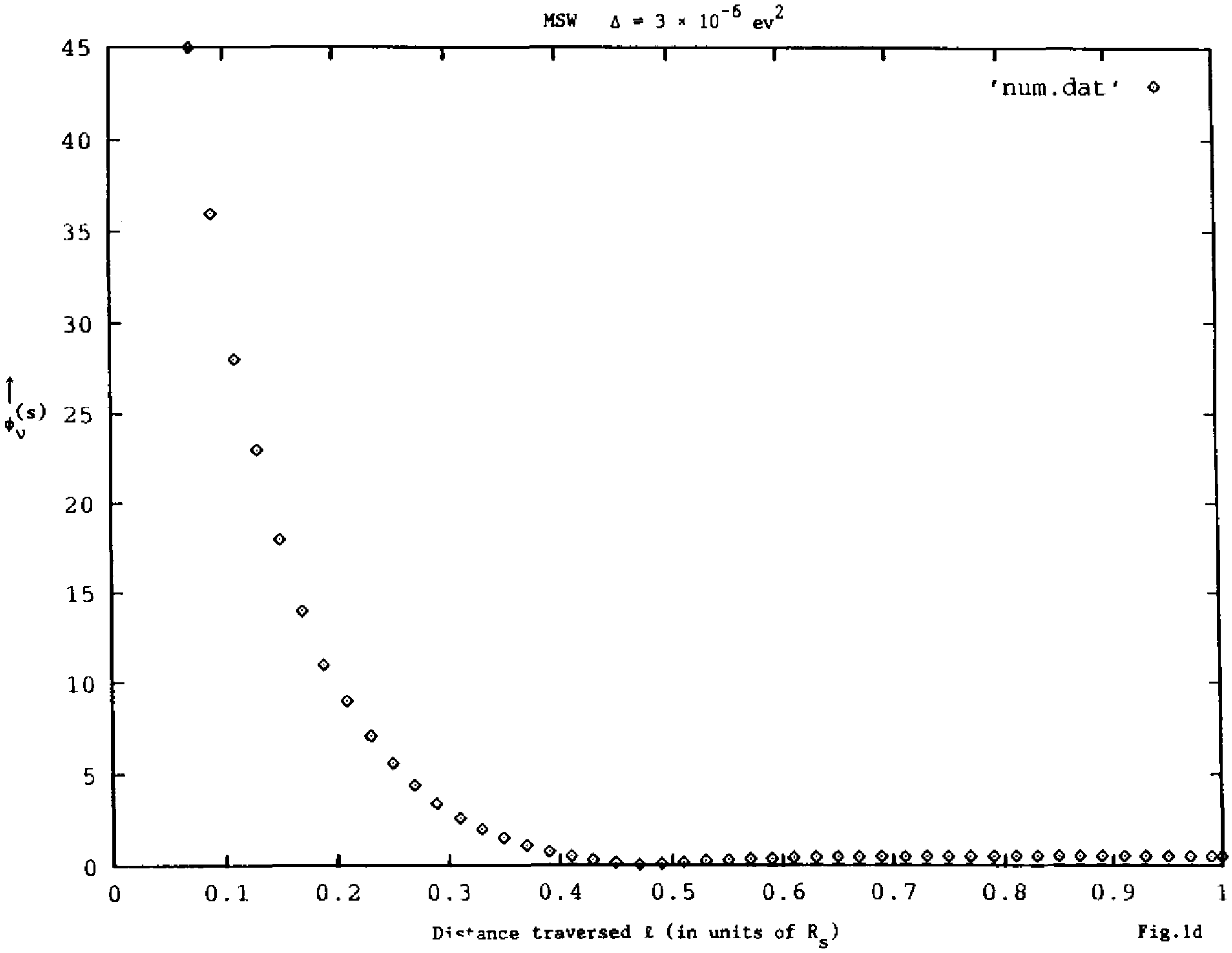

MSW Δ = 3 × 10^-6 ev^2
'num.dat'
45
40
35
30
25
20
15
10
5
0
$\phi_\nu^{(s)}$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Distance traversed ℓ (in units of R_s)

Fig.1d

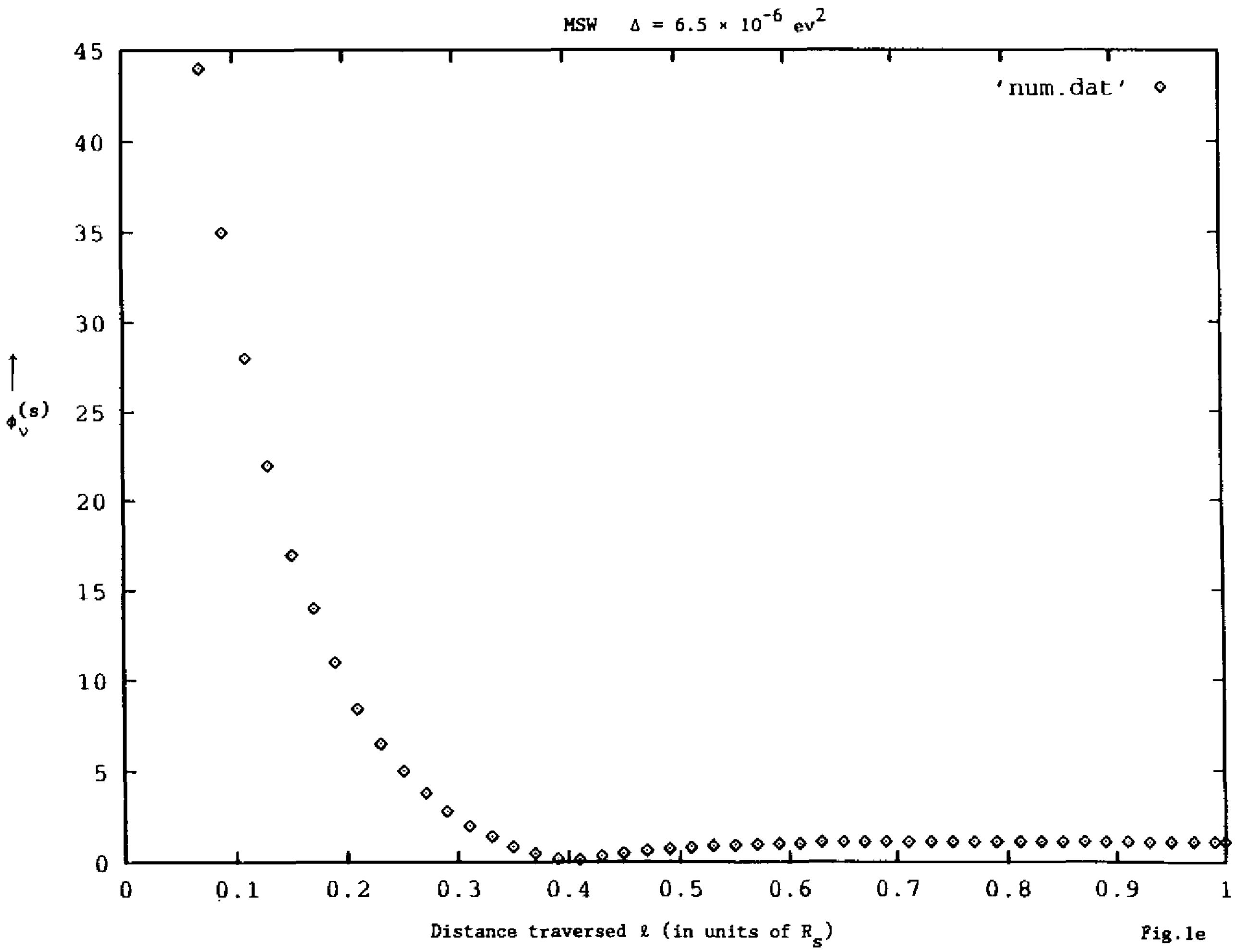
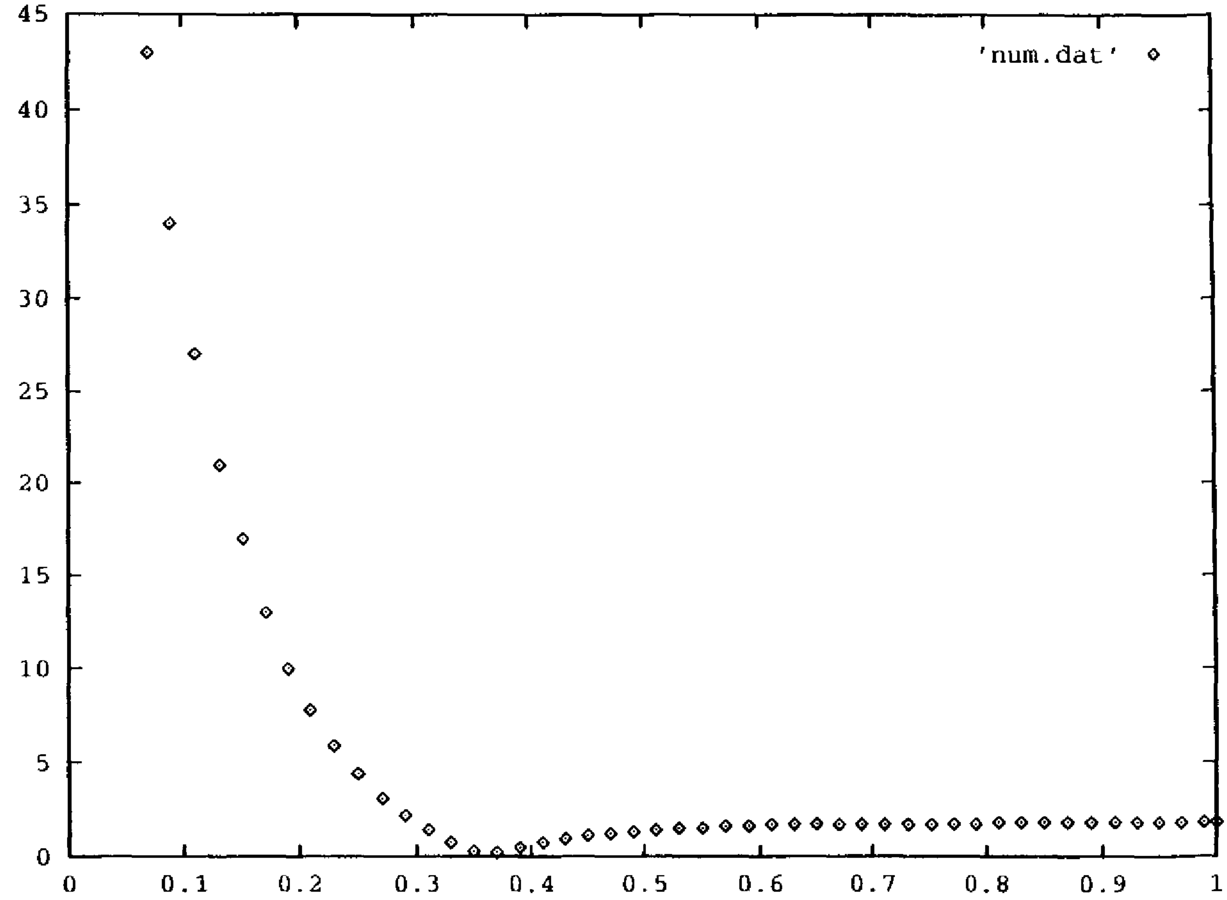

MSW Δ = 6.5 × 10^-6 ev^2
'num.dat'
45
40
35
30
25
20
15
10
5
0
$\phi_\nu^{(s)}$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Distance traversed ℓ (in units of R_s)

Fig.1e

MSW $\Delta = 10^{-5}$ ev^2

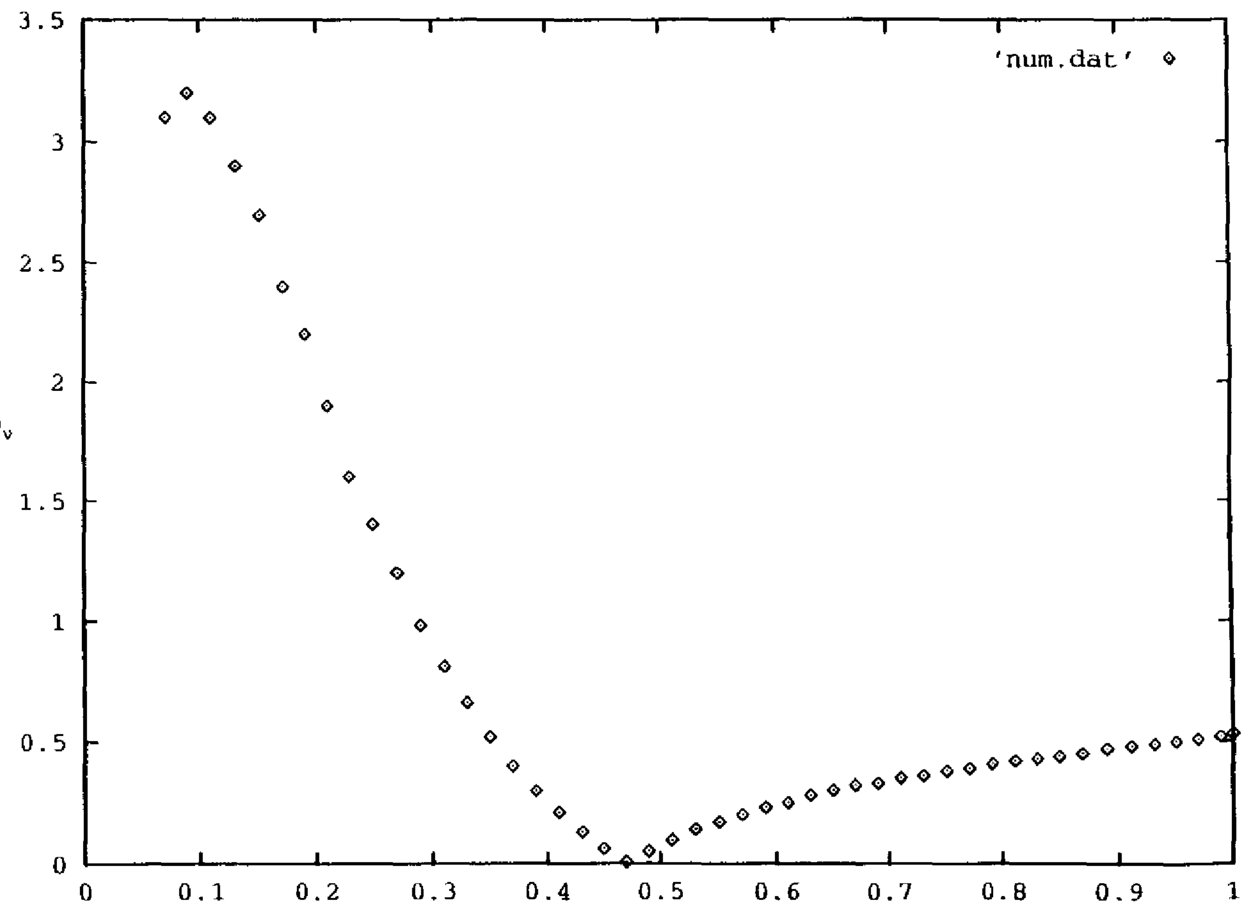


$\phi_\nu^{(s)}$ ↑

Distance traversed ℓ (in units of R_s)

Fig.1f

SFP $\Delta = 3 \times 10^{-6}$ ev^2



ϕ_ν

Distance traversed ℓ (in units of R_s)

Fig.2a

SFP 6.5×10^{-6} ev^2

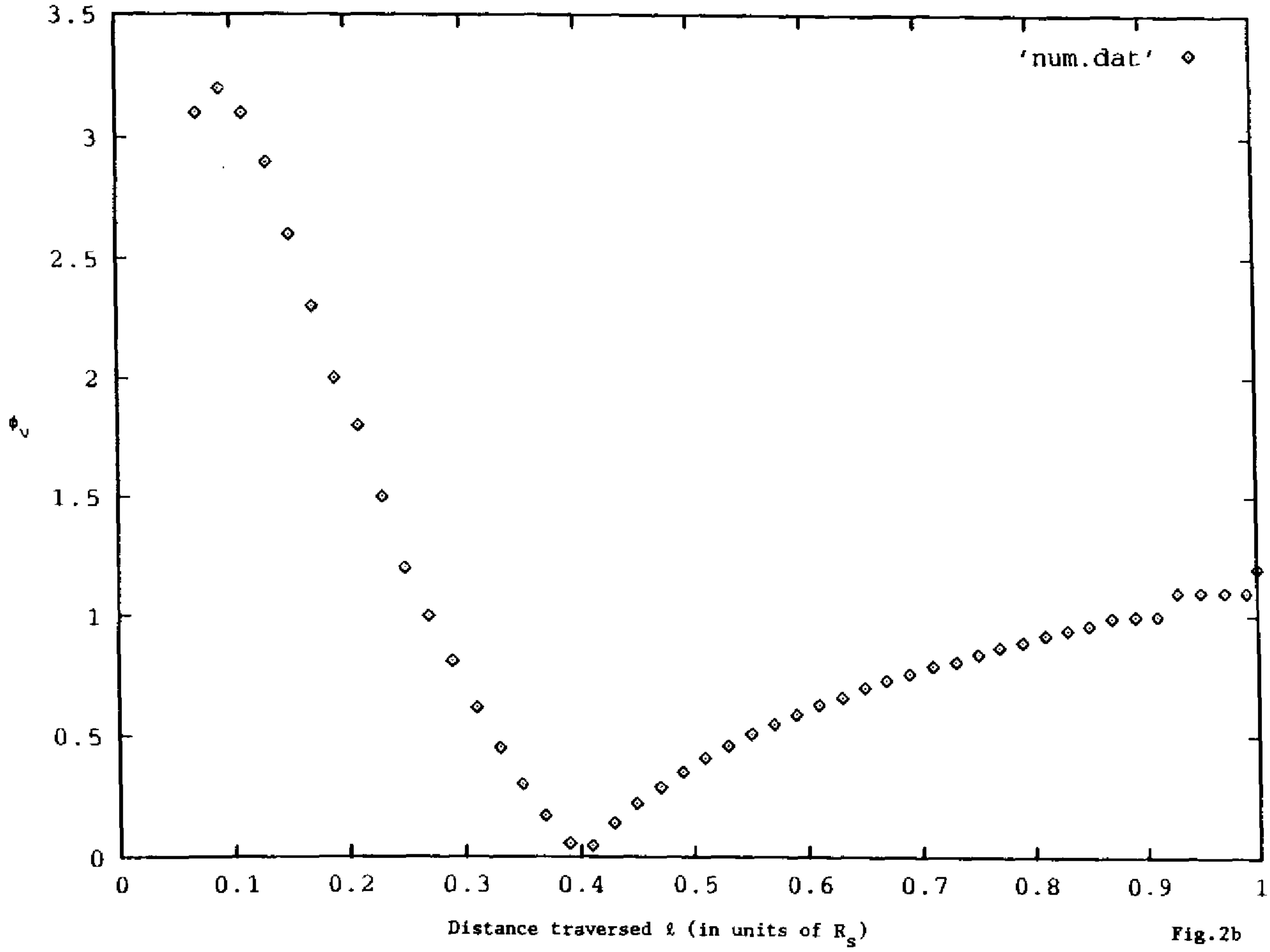


Fig.2b

SFP $\Delta = 10^{-5}$ ev^2

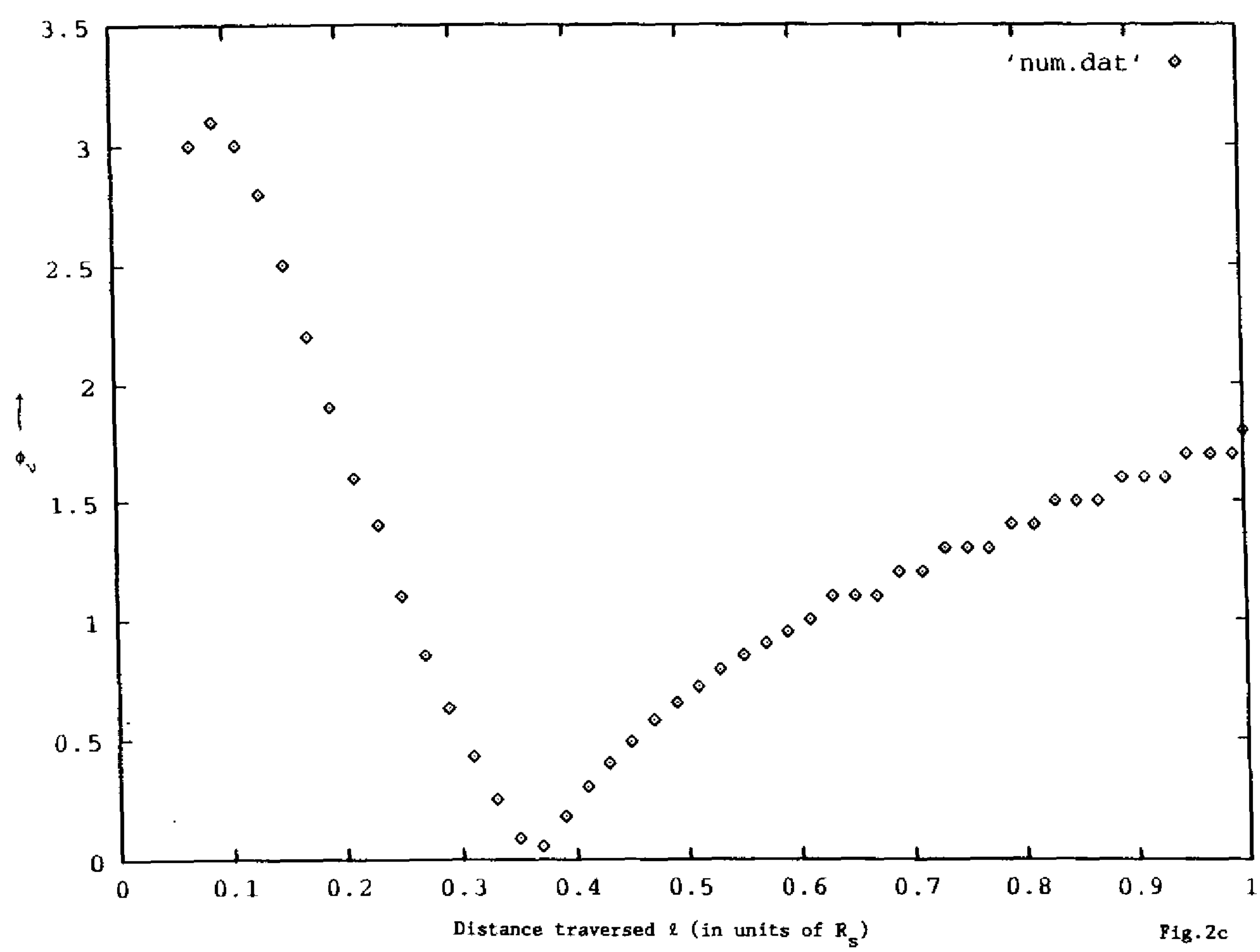


Fig.2c

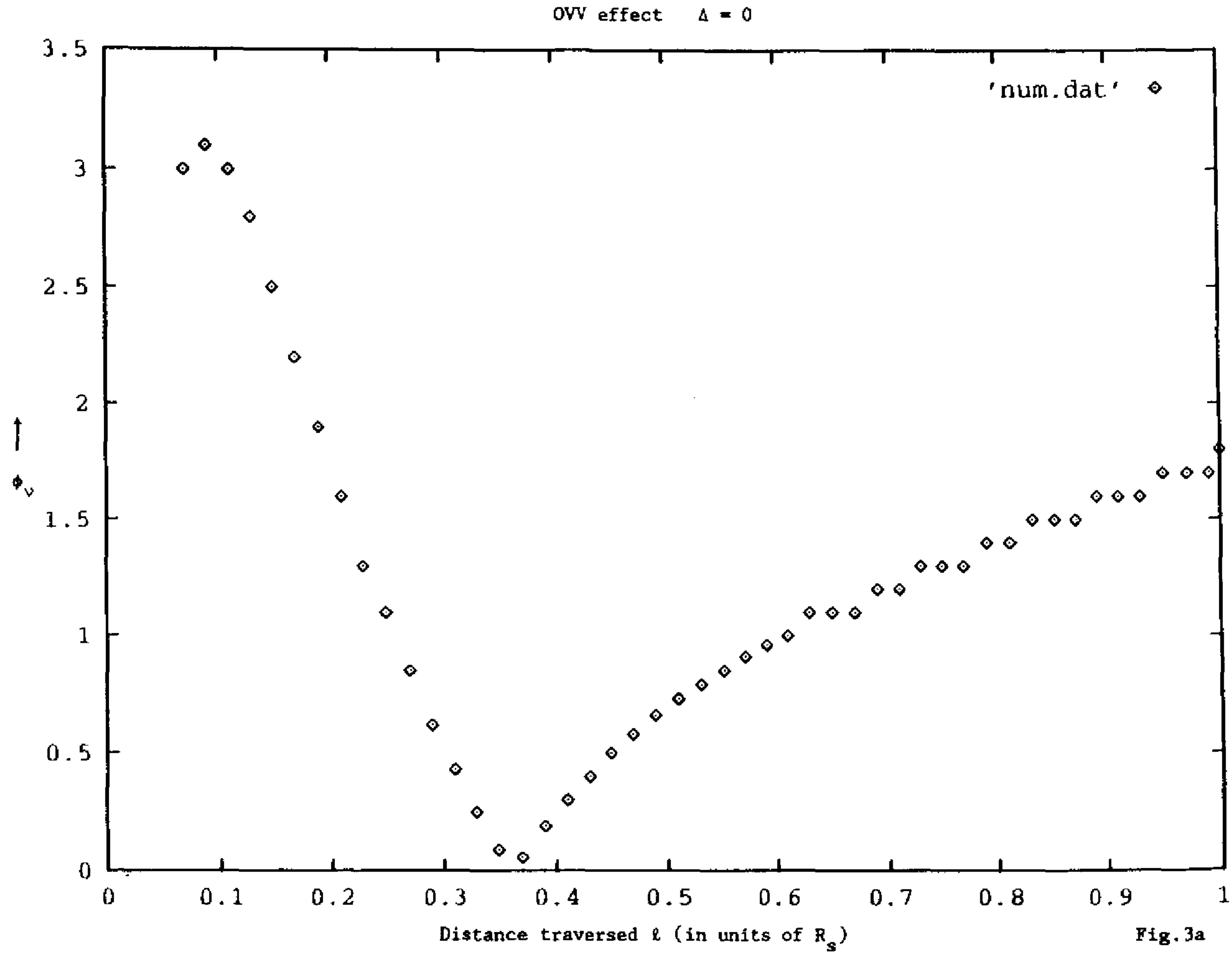

OVV effect Δ = 0
'num.dat'
ϕ_ν
3.5
3
2.5
2
1.5
1
0.5
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Distance traversed ℓ (in units of R_s)

Fig.3a

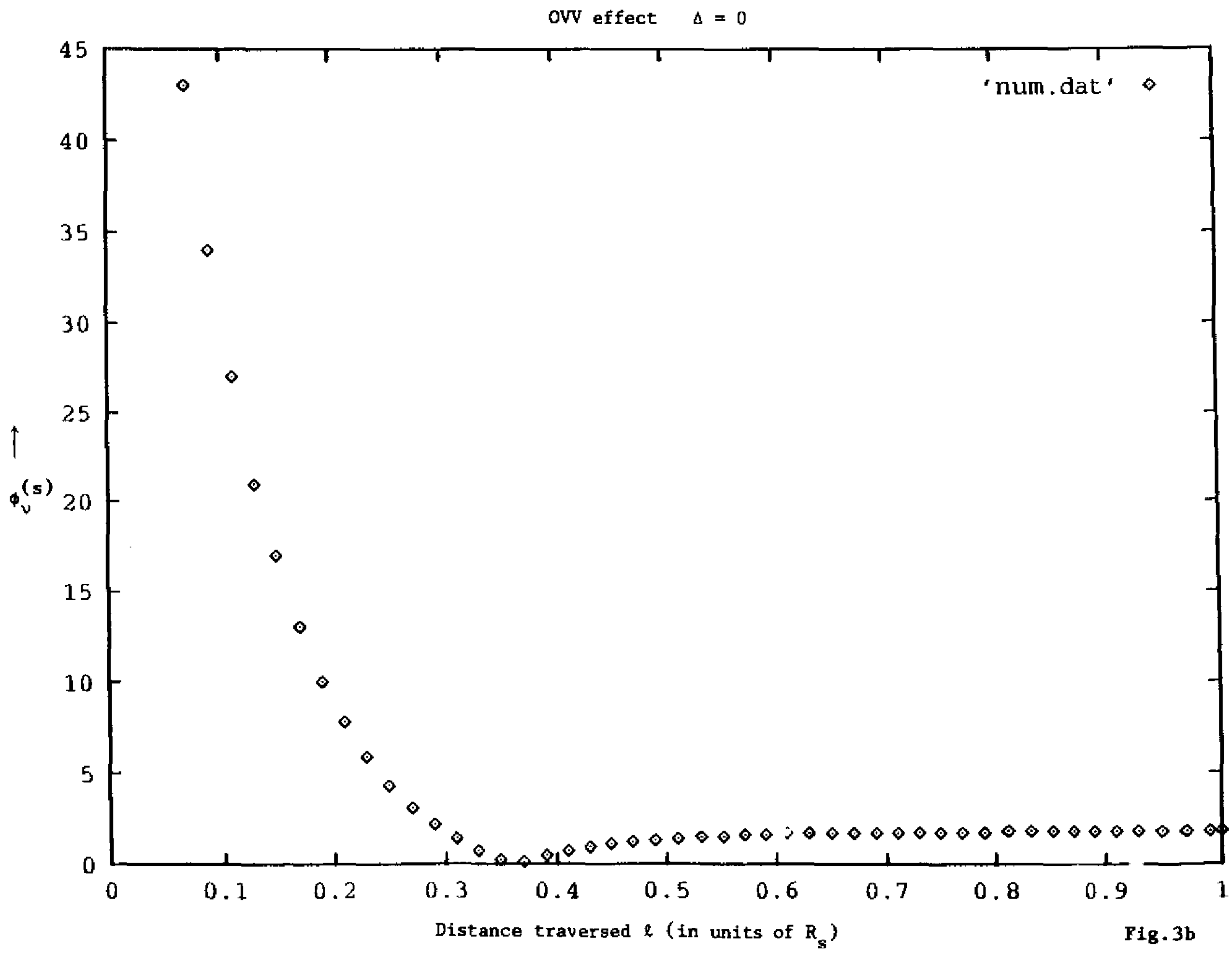

OVV effect Δ = 0
'num.dat'
$\phi_\nu^{(s)}$
45
40
35
30
25
20
15
10
5
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Distance traversed ℓ (in units of R_s)

Fig.3b