

**EFFECT OF SAWTEETH ON ALPHA POWER  
DEPOSITION AND IGNITION IN TOKAMAKS**

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# EFFECT OF SAWTEETH ON ALPHA POWER DEPOSITION AND IGNITION IN TOKAMAKS

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## Abstract

The main features of the alpha particle heating, ignition and thermonuclear burn in a tokamak plasma with sawtooth oscillations are revealed. The sensitivity of results against the various model of sawteeth and characteristics of the safety factor  $q(r)$  is investigated. Analysis of ignition is first applicable to the case of  $\tau_E < \tau_r$ ,  $\tau_E$  being the alpha particle energy loss time,  $\tau_r$  period of sawtooth oscillations.

## 1. Introduction

Sawtooth oscillations are a phenomenon which is inherent in the most of the modern tokamaks. Despite the progress in their stabilization there is no assurance that this kind of the MHD-activity of plasma can be suppressed completely in reactor conditions. Moreover, the experiments where stabilization is carried out by ICRH show that the sawtooth-free period may end with the sawtooth oscillation having large amplitude and large region of localization, which is called "giant" or "monster" sawtooth [1,2].

Realizing the importance of sawteeth, a large attention to experimental and theoretical study of them is paid by researchers since their discovery in 1974 [3]. In addition, when projecting the reactor-tokamaks the special restrictions on plasma parameters are supposed to avoid the sawtooth instability or to reduce the region of its localization. However, till now both the ignition conditions and burning in plasma with sawteeth almost have not been investigated. Only the first steps are made in studying the alpha power deposition profile during sawtooth oscillations [4,5].

The analysis of Refs. [4,5] is based on the model which describes the sawtooth oscillations with the central safety factor,  $q_c$ , equal to unity after the sawtooth collapses. However, experiments on JET [2] and previous experiments on TEXTOR [6] and Tokapole-II [7] indicate that  $q_c$  remains well below unity during sawteeth. This circumstance, as well as a number of the other essential experimented data, agrees with the model of sawteeth suggested in Ref. [8]. That is why it is of interest to study the plasma heating by alphas, basing on the model of Ref. [8]. Such study including calculations of the alpha power deposition profile and its change during sawtooth oscillations is one of the aims of the present work. It is carried out in Sec. 2. The other aim is consideration of the problem of ignition during sawteeth (Sec. 3).

## 2. Alpha power deposition

Following Ref. [8] we proceed from the conservation laws of the ideal MHD for reconnecting layers given by:

$$\psi^-(x_1^-) = \psi^-(x_2^-) = \psi^+(x_1^+) = \psi^+(x_2^+) \quad (1)$$

$$\begin{aligned} \frac{\partial \psi^-(x_1^-)}{\partial x_1^-} dx_1^- &= \frac{\partial \psi^-(x_2^-)}{\partial x_2^-} dx_2^- = \\ &= \frac{\partial \psi^+(x_1^+)}{\partial x_1^+} dx_1^+ = \frac{\partial \psi^+(x_2^+)}{\partial x_2^+} dx_2^+ \end{aligned} \quad (2)$$

$$dV_1^+ = w dV_2^- \quad (3)$$

$$dV_2^+ = (1-w) dV_2^- + dV_1^- \quad (4)$$

Here  $\psi$  is the magnetic flux across the helical  $m = n = 1$  surface,  $dV = 2\pi R z dz$ , subscripts 1 and 2 label values at  $z < z_s$ , and  $z > z_s$ , respectively,  $z_s$  is the radius of the  $q = 1$  surface, minus/plus denote values before/after the sawtooth collapse,  $w = w(\psi)$  determines the fraction of the volume  $dV_2^-$  producing the cold plasma core with  $q < 1$ ,  $x = z^2/(z_s^-)^2$ . We assume that before and after collapse (or after postcursors) the MHD perturbations are absent and therefore

$$\psi(x) = \frac{B_r(z_s^-)^2}{2} \int_0^x dx (\mu - 1) \quad (5)$$

where  $\mu = q^{-1}$ ,  $q$  is a safety factor,  $B_r$  is a toroidal magnetic field. For future calculations we present Eqs. (3), (4) as [8]

$$\frac{dx_1^+}{d\psi} = -w \frac{dx_2^-}{d\psi} \quad (6)$$

$$\frac{dx_2^+}{d\psi} = (1-w) \frac{dx_2^-}{d\psi} - \frac{dx_1^-}{d\psi} \quad (7)$$

In the case of collapses with  $q_0^+ \approx q_0^-$ ,  $q_0 \equiv q(r=0)$  [3 - 5] the value  $w$  may be expressed in terms of observable variables [8] :

$$1 - \mu^-(x_{20}^-) = [\mu^-(x_1^- = 0) - 1] w$$

$$x_{20}^- = 1 + \frac{x_1^+}{w} \quad (8)$$

Eqs. (8) follows from Eq. (6) and condition

$$\left. \frac{d\psi^+}{dx_1^+} \right|_{x_1^+=0} = \left. \frac{d\psi^-}{dx_1^-} \right|_{x_1^-=0} \text{ provided}$$

$$q_0^+ = q_0^- \quad (\text{i.e.})$$

$$w = \begin{cases} \text{const}^+, & x_2^- < x_{20}^- \\ 0 & , x_2^- \geq x_{20}^- \end{cases}$$

Eqs. (1)-(8) enable to find the connection between  $\mu^+(r)$  and  $\mu^-(r)$ , i.e. to find the change of the magnetic configuration due to the collapse provided  $x_1^+$  is known.

To obtain relationships connecting the plasma parameters before and after the collapse we involve into consideration the conservation of the number of particles with given values of  $\xi$ ,  $\lambda$ ,  $\sigma$  (c.f. Ref. [5, 6]) where  $\xi = mv^2/2$  is the particle energy,  $\sigma = \text{sgn } v_{||}$ ,  $\lambda = v_{\perp}^2 B_0 / v^2 B$ ,  $B$  is the magnetic field,  $B_0 = B(r=0)$ ,  $v_{||}$  and  $v_{\perp}$  are the longitudinal and transverse velocity of a particle, respectively:

$$f_1^+ dV_1^+ = w f_2^- dV_2^- \quad (9)$$

$$f_2^+ dV_2^+ = (1-w) f_2^- dV_2^- + f_1^- dV_1^- \quad (10)$$

where  $f = f(r, \xi, \lambda, \sigma)$  is the particle distribution function.

Combining Eq. (9) with Eq.(3) and Eq. (10) with Eq. (4) and taking into account that  $dV \sim dx \sim (\mu-1)^{-1} d\psi$  we get

$$f_1^+ = f_2^- \quad (11)$$

$$f_2^+ = \frac{(1-w)V_2 f_2^- + V_1 f_1^-}{(1-w)V_2 + V_1} \quad (12)$$

where  $V_{1,2} \equiv |M_{1,2}^-|^{-1}$ . As the plasma density and temperature are defined by  $n^\sigma = \int f^\sigma d^3v$ ,  $n^\sigma T^\sigma = \int f^\sigma \frac{m v^2}{3} d^3v$  ( $\sigma$  denotes the particle species), Eqs. (11), (12) are transformed into relationships for  $n^+$  and  $n^-$ ,  $T^+$  and  $T^-$  when replacing  $f \rightarrow n$  and  $f \rightarrow nT$ . Similarly, replacing in Eqs. (11), (12)  $f \rightarrow E_\alpha$ ,  $E_\alpha \equiv \int f_\alpha \frac{m_\alpha v^2}{2} d^3v$  being the  $\alpha$ -particle energy density, we obtain the connection between  $E_\alpha^+$  and  $E_\alpha^-$ .

Thus, we have:

$$M_{1j\sigma}^+ = M_{2j\sigma}^- \quad (13)$$

$$M_{2j\sigma}^+ = \frac{(1-w)V_2 M_{2j\sigma}^- + V_1 M_{1j\sigma}^-}{(1-w)V_2 + V_1} \quad (14)$$

where  $M_{j\sigma} = \int \left(\frac{m_\sigma v^2}{2}\right)^j f^\sigma d^3v$ ,  $j = 0, 1$ ;  $\sigma = e, i, \alpha$ .

Note that Eqs. (11), (12) and Eqs. (13), (14) are valid provided the redistribution during collapse of both plasma and  $\alpha$ -particles is primarily connected with the particle motion along the magnetic field lines. It requires the fulfilment of special conditions. In particular, the banana width of alphas should be small in comparison with the radius of the  $\psi = 1$  surface. The other necessary conditions are considered in Refs. [4, 5].

We assume that during intervals between collapses the alpha energy density is governed by the following equation:

$$\frac{\partial E_\alpha}{\partial t} = -\tau_E^{-1} E_\alpha + E_\alpha S \quad (15)$$

where  $E_\alpha = 3.5$  MeV,  $S$  is the intensity of the alpha source.

Eqs. (13), (14) (for  $j = 1, \sigma = \alpha$ ) and (15) enable to calculate the dynamics of change of the fast alpha energy as well as the plasma heating power due to  $\alpha$ -particles,  $P \equiv \tau_E^{-1} E_\alpha$  provided the evolution of plasma parameters is given. As plasma characteristics are affected by alphas, the problem is, generally speaking, self-consistent. However, if we are interested in a regime of stable burning when plasma parameters averaged over the sawtooth period are constant, we may assume that the functions  $n(r, t), T(r, t)$  are similar to those observed in tokamaks of today. Thus, we take  $n(r) = n_0 (1 - r^2/a^2)^{\gamma_n}$ ,  $T(r) = T_0 (1 - r^2/a^2)^{\gamma_T}$  (where  $a$  is the minor radius of the plasma,  $\gamma_n, \gamma_T, n_0$  and  $T_0$  are parameters) and use Eqs. (13), (14) with  $j = 0$  and to calculate the variation of  $n$  and  $nT$ , respectively. Finally we suppose that the time dependence of both temperature and density of the plasma is linear between collapses.

To find the periodic solution of Eqs. (13)-(15) for  $E_\alpha$  we set an arbitrary initial condition and then repeatedly solve Eqs. (13)-(15) until the process converges (see Ref. [5] for more details).

The main aim of calculations is to investigate the sensitivity of the plasma heating profile to the crash model and parameters of the  $q(r)$ -profile. Experimental data obtained on JET show that shear near the  $q = 1$  magnetic surface is very low ( $\sim 2\%$ ) [2]. On the other hand,  $q_c$  is significantly lower than unity ( $q_c - 1 \sim 0.2$ ) [2]. This means that the profile of  $\mu \equiv q^{-1}$  has the "head and shoulder" shape observed earlier on TEXTOR [6]. To account for this fact we use the following  $\mu$  profile:

$$\mu(r) = \begin{cases} \int \mu_* \frac{r_*^2 - r^2}{r_*^2} + \int \mu_* \frac{r^2}{r_*^2}, & 0 \leq r \leq r_* \\ \int \mu_* \frac{(r_s^-)^2 - r^2}{(r_s^-)^2 - r_*^2} + \frac{r^2 - r_*^2}{r_s^2 - r_*^2}, & r_* \leq r \leq r_s^- \\ \frac{a^2 - r^2}{a^2 - (r_s^-)^2} + \int \mu_a \frac{r^2 - (r_s^-)^2}{a^2 - (r_s^-)^2}, & r_s^- \leq r \leq a \end{cases} \quad (15)$$

where  $\mu_* = 1.005$ ,  $\mu_a = 1/3$ ,  $r_s^-/a = 1/3$ ,  $r_*/r_s^- = 0.7$ . The Eq. (8) was used to calculate  $w$  so that  $q_s^+ = q_s^-$ . If we also take  $\alpha_s^+ = \alpha_*$ , the profile of  $q^+(r)$  calculated from Eqs. (6)-(7) almost coincides with  $q^-(r)$  [8] in accordance with the fact that  $\tau_2$  is much smaller than the skin time for typical sawteeth in modern tokamaks. Note that in this case the parameters characterizing the  $q(r)$  are similar to those of JET experiments [2, 11].

The results of calculations for  $\gamma_n = 0.5$ ,  $n_0 = 1.5 \times 10^{20} \text{ m}^{-3}$ ,  $\gamma_T = 1$ ,  $T_0 = 20 \text{ keV}$  are presented in Figs. 1-4. Fig. 1 shows the radial distribution of the alpha energy deposition  $P_\alpha$  for a flat  $\mu(r)$ -profile ( $\mu_0 = 1.05$ ). One can see that the redistribution of the plasma parameters in this case is close to turning the plasma core inside out. A more realistic case  $\mu_0 = 1.25$  ( $q_0 = 0.8$ ) is presented in Figs. 2-3. It shows that both the radius of the core affected by mixing and the magnitude of sawteeth significantly increase when  $q_0$  decreases. The results for the same  $\mu(r)$  profile but for the case of  $r_s^+ = 0$  are presented in Fig. 4. Note that the case of  $r_s^+ = 0$  ( $q_0^+ = 1$ ) is described by the model in which reconnection results only in joining flux surfaces together (unlike the model [8] in which there is also a current layer where flux surfaces split). This case corresponds to a crash with the rigid shift of the plasma core like in Kadomtsev's model [9]; but it can also correspond to a crash with the quasi-interchange flow (Wesson's model [10]). Comparison of Fig. 2 and Fig. 4 demonstrates that the model [8] leads to stronger plasma



redistribution than model with  $r_s^+ = 0$ . Thus, the alpha-particle energy deposition essentially depends both on the crash model and on the  $q(r)$  profile. Calculations for various values of  $\tau_r$  in the framework of the model [8] lead to the same conclusions as in Kadomtsev's model [9]: when  $\tau_e/\tau_r \ll 1$  then oscillations of the alpha energy deposition occur, whereas the case  $\tau_e/\tau_r \gg 1$  is characterized by relatively weak oscillations having the shape of inverted sawteeth.

### 3. Ignition and burning in a plasma with sawtooth oscillations

It follows from Sec. 2 and from Refs. [4, 5] as well that the alpha power deposition profile can strongly differ from the radial dependence of function  $S(r)$  describing the alpha production. This circumstance and the oscillation of plasma parameters caused by the sawtooth instability complicate the study of physics issues of burning in a plasma, such as ignition, thermal instability etc. The problem is simplified when  $\tau_e \ll \tau_r$ . In this case the difference between the profiles of  $P_\alpha(r)$  and  $S(r)$  is essential only during the short time (about  $\tau_e$ ) immediately after a crash. It makes possible to use the approximation  $P_\alpha(r) \simeq \bar{\epsilon}_\alpha S(r)$  during the time intervals between crashes. The similar reason enables not to care about the proper modeling of the heat pulses produced by crashes outside the reconnection region. Note that the condition  $\tau_e \ll \tau_r$  is expected to be fulfilled in the ITER ignition operation mode.

Taking into account the foregoing and assuming  $\tau_e \ll \tau_r$ , we can write the following equation of the plasma energy balance:

$$3n \frac{\partial T}{\partial t} = \frac{1}{2} \frac{\partial}{\partial r} r \kappa \frac{\partial T}{\partial r} + \epsilon_n S - Q_{br} \quad (16)$$

Here  $T(r, t)$ ,  $n(r)$ ,  $\kappa$  are the plasma temperature, density and heat conductivity, respectively,  $Q_{br}$  is losses due to bremsstrahlung.

The equation (16) is valid for  $\bar{t}_i < t < \bar{t}_{i+1}$ ,  $i = 1, 2$ .  $\bar{t}_i$  being a crash moment, provided the plasma density is sustained unchanged. It means that, first, we consider the ignition scenario with plasma heating which follows obtaining a plasma with required density, second, the radial profile  $n(r)$  at  $r < r_{mix}$  ( $r_{mix}$  is the sawtooth mixing radius) is supposed to be sufficiently flat (in order to neglect the density oscillations caused by sawtooth instability), and third, the particle confinement time should be large in comparison with the plasma energy confinement time.

The equation (16) together with the equations determining the connection between  $T(r, t_i)$  and  $T(r, t_{i+1})$  (Eqs. (1), (2), (5)-(7), (13), (14) plus Eq. (8) in the case of  $q_0^+ = q_0^-$ ) enable to study the ignition and burn in plasma in the presence of sawteeth provided the following parameters are known:  $q_f^-(r)$ ,  $q_0^+$ ,  $r_s^+/a$ ,  $\bar{t}_2$ .

We assume that  $q_f(r)$  has the profile shape determined by Eq. (15) with  $\mu_+ = 1.005$ ,  $\mu_a = 1/3$ ,  $\mu_0^- = 1.5$ . To consider a plasma with usual and "monster" sawteeth the two values of the  $q = 1$  radius before a crash are used in calculations:  $r_s^-/a = 1/3$  and  $r_s^-/a = 1/2$ . In the most calculations we assume  $q_0^+ = q_0^-$ . The sawtooth oscillations are supposed to be periodical with  $\bar{t}_2 = (0.1 \pm 5)$  sec. The large range of the considered values of  $\bar{t}_2$  is connected with both the various experimental conditions and the uncertainty in the scaling for  $\tau_r$  [12].

The calculations were carried out for the following parameters:  $a = 1.6 \text{ m}$ ,  $R = 5.2 \text{ m}$ ,  $\bar{n} = 1.1 \times 10^{20} \text{ m}^{-3}$ ,  $\mathcal{L} = 10^{14} \text{ m}^2 \text{ s}^{-1}$ ,  $W = 50 \text{ MW}$ ,  $\bar{n}$  being the cross-section-averaged plasma density,  $W$  is the power of auxiliary heating. It was assumed that the radial profile of  $\bar{W}(r)$  is parabolic and that

$$n(r) = \begin{cases} n_m (1 - r^2/a^2) + 10^{19}, & r > r_{mix} \\ n_m (1 - r_{mix}^2/a^2) + 10^{19}, & r \leq r_{mix} \end{cases} \quad (17)$$

where  $n_m$  is determined through given value of  $\bar{n}$ .

The main obtained results are presented in Figs. 5+9 where the time evolution of the average plasma temperature ( $\bar{T} = \int n(r)T(r)rdr / \int n(r)rdr$ ) is shown. Figs. 5+7 are relevant to the case of  $r_s/a = 1/3$  (which corresponds to usual sawteeth), and Figs. 8+9 represent results for  $r_s/a = 1/2$  ("monster" sawteeth). It follows from Figs. 5+8 that:

(i) Sawtooth oscillations cause the increase of ignition temperature ( $\bar{T}_{ign}$ ). The largest effect corresponds to large  $r_s/a$  and small  $\bar{\tau}_r/\bar{\tau}_E$ . For instance, when  $r_s/a = 1/2$  and  $\bar{\tau}_r/\bar{\tau}_E \approx 1/6$  ( $\bar{\tau}_E \approx 1.5 \text{ s}$ ) then  $\bar{T}_{ign}/\bar{T}_{ign}^0 = 1.6$  (Fig. 8), when  $r_s/a = 1/3$  and  $\bar{\tau}_r/\bar{\tau}_E = 1/15$  then  $\bar{T}_{ign}/\bar{T}_{ign}^0 = 1.2$  (Fig. 5),  $\bar{T}_{ign}^0$  being the ignition temperature in plasma without sawteeth. It means that using the RF-heating and/or the NBI which increases  $\bar{\tau}_r$  and  $r_s$  can lead either to the increase or to the decrease of  $\bar{T}_{ign}$ , depending on the value of the change in  $\bar{\tau}_r$  and  $r_s$ ;

(ii) The time and energy required for achieving the ignition grow because of sawteeth. These effects are larger also in the case of large region of the sawtooth localization and frequent crashes. Examples:  $\Delta t/\Delta t^0 = 2.2$  for  $r_s/a = 1/2$ ,

$\tau_r/\tau_E = 1/6$  (Fig. 8);  $\Delta t / \Delta t^0 = 1.27$  for  $r_s/a = 1/3$ ,  $\tau_r/\tau_E = 1$  (Fig. 5).

(iii) Sawtooth oscillations produce stabilizing effect on thermal instability. First, they decrease the temperature of the stable burn. Second, they make possible the ignition scenario with slowly varying temperature after ignition, the profile shape of the plasma temperature during the quasi-steady-state phase ( $5 \text{ sec} < t < 15 \text{ sec}$  in Figs. 5, 6, 8) being far from the steady-state ignition profile determined by the steady-state equation of plasma energy balance.

(iv) When  $\tau_r \gtrsim \tau_E$  the average plasma temperature oscillates after achieving the regime of the stable burn ( $t \gtrsim 10 \text{ sec}$  in Fig. 8). This effect exists only in the case of "monster" sawteeth when the temperature in the plasma center is strongly modulated,  $\Delta T(0)/T(0) \sim 30\text{-}50\%$ ; the modulation magnitude  $\Delta T(0)/T(0) \sim 18\%$  which takes place when  $r_s/a \sim 1/3$  is not sufficient to produce the noticeable effect on the average temperature  $\bar{T}$ . Appearance of the time oscillations of  $\bar{T}$  is a consequence of deterioration of the plasma energy confinement after crashes. The reason of such deterioration is the mixing of the plasma at  $r < r_{\text{mix}}$  due to crashes, which leads to the bursts of energy flux from the region  $r < r_s$  to the region  $r > r_s$ . The direct influence of plasma redistribution during a crash on the time dependence of  $\bar{T}$  is negligibly small (that is why  $\bar{T}$  does not oscillate in the most cases presented in Figs. 5-8). This statement becomes clear if one takes into account that: (1) only a very small fraction of the magnetic field energy is transformed into the particle thermal energy (one can show that the change of the plasma energy during a crash may be estimated as  $\delta Q/Q < (\mu_0^- - 1)^2 \varepsilon_s^2 \beta^{-1}$  that results in  $\delta Q/Q \sim 1\%$  for  $\beta \sim \varepsilon_s^2$ ,  $\mu_0^- - 1 \sim 0.1$  where  $\varepsilon_s \equiv r_s^2/R$ ,  $\beta \equiv 8\pi p/B^2$ ,  $Q = 3 \int dV nT$ , the integral is taken over the plasma

volume; (2)  $\bar{T}$  is proportional to  $Q$ ; (3) the number of particles is conserved during a sawtooth crash.

(v) The analysis of the temperature profile during the process of achieving the ignition and the subsequent burn shows that  $T(r)$  even before a crash may be non-monotonic or flat (see Fig.9). Non-monotonic profiles take place in the case of small  $\tau_r/\tau_E$  when the time between crashes is not sufficient to restore the monotonic profile.

Note that in the case of small  $\tau_r$  ( $\tau_r \sim 0.1$  sec) Eq.(16) is, strictly speaking, invalid (especially in the high-temperature region). However, one can expect that the conclusions of the points (i)-(iv) remain true.

It is of interest to know the ignition conditions in the presence of sawtooth oscillations. Obtaining of such conditions is clear to require an analysis including the consideration of radial profiles of plasma parameters. The 1-D analysis of Ref. [13] enabled to obtain the integral criterion of ignition in a plasma without sawteeth. As sawtooth instability results in the relatively small temperature modulation in the region  $r/a \lesssim 1/2$  (at least when "monster" sawteeth are absent) we tried this criterion for plasma with sawtooth oscillations. The numerical calculations showed that it may be used for finding the approximate moment of ignition (i.e. the moment when the auxiliary heating may be stopped).

#### 4. Conclusions

Earlier [4, 5] we have shown that the crucial parameter which determines the behaviour of fast ions in plasmas with the

sawtooth oscillations is the ratio  $\tau_z/\tau_r$ . Depending on this parameter the intensity of plasma heating by alphas may have the shape of normal or inverted sawteeth, the crashes may result in either bursts of plasma heating in region  $r \gtrsim r_s$  or only weak oscillations of  $P_\alpha(r)$ . Here we have found that all these effects are essentially stronger when  $q_0^+ = q_0^-$ , i.e. when crashes weakly affect the central safety factor - situation which seems to be typical for large modern tokamaks [2, 6, 7] and which corresponds to the model of Ref. [8]. In addition, we have found that the safety factor profile  $q(r)$  with the small shear in the vicinity of the  $q=1$  surface and with  $q_0$  well below unity (i.e. the "head-and-shoulder" profile of  $q^{-1}(r)$ ) enhances the effect of sawteeth in comparison with the case of the flat-top  $q^{-1}(r)$  with  $q_0$  close to unity.

Sawtooth oscillations are a phenomenon whose picture depends strongly on both the spatial coordinates and time. Despite this, it turns out convenient to express the ignition temperature and, in general, to characterize the plasma energy during the process of achieving the ignition and the subsequent burn in terms of the average temperature,  $\bar{T}$ , which is shown to be almost not affected (directly) by sawteeth.

An approach based on the use of  $\bar{T}$  enabled us to reveal the main features of ignition and burn in presence of sawteeth. A general conclusion which follows from our analysis is that the sawtooth oscillations have the strong influence on ignition conditions and characteristics of the burn provided the region of their localization is relatively large ( $r_{mix}/a \gtrsim 1/2$ ) and the time between crashes is small ( $\tau_r < \tau_E$ ). However, one should remember that even when sawteeth are localized in a relatively small region, their role in a fusion plasma may be essential due to issues which

are not considered in this work (confinement of alpha particles, effects of the heat pulse propagation, etc.). Finally, we note that our study of ignition is carried out in assumption  $\tau_e \ll \tau_r$ . Despite this assumption is expected to be fulfilled in typical reactor conditions (in particular, in the ignition mode of ITER and, perhaps, in the ITER steady-state mode) our analysis should be extended in the future for other cases. In addition, in the case of  $\tau_e < \tau_r$  the effects of finite ratio  $\tau_e/\tau_r$  should be considered.

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## FIGURE CAPTIONS

Fig. 1. Radial distributions of the energy deposition  $P_x$  before a crash (dashed line) and after a crash (solid line) for

$$\mu_0 = 1.05, \quad \mu_a = 1/3, \quad \mu_* = 1.005, \quad \tau_s^-/a = 1/3, \\ \tau_s^+/\tau_s^- = 0.7, \quad \tau_* = \tau_s^+, \quad \tau_r = 1 \text{ sec.}$$

Fig. 2. The same as in Fig. 1 except  $\mu_0 = 1.25$ .

Fig. 3. Spatial-time evolution of energy deposition  $P_x$  for parameters of Fig. 2.

Fig. 4. The same as in Fig. 2 except  $\tau_s^+ = 0$ .

Fig. 5. Time evolution of  $\bar{T}$ . The auxiliary heating is stopped at the moment  $t_*$  corresponding to a point on a curve noted by a star (\*). Curve 1 -  $t_* = 3.94$  sec, 2 -  $t_* = 5.0$  sec, 3 -  $t_* = 6.0$  sec, 4 -  $t_* = 5.0$  sec, 5 -  $t_* = 3.94$  sec.

Fig. 6. Time evolution of  $\bar{T}$  in the case of  $\tau_s^+ = 0$  (dash-dotted line) and  $\tau_s^+/\tau_s^- = 0.7$  (solid line). The upper and lower curves are the same as curves 2 and 4 in Fig.

Fig. 7. Time evolution of  $\bar{T}$  in the case of  $\tau_s^+ = 0$  (dash-dotted line),  $\tau_s^+/\tau_s^- = 0.7$  (solid line) and  $\tau_r$  exceeding essentially the sawtooth period of Fig. 6.

Fig. 8. Time evolution of  $\bar{T}$  in the case of "monster" sawteeth with  $\tau_r = 5.0$  sec (curve 1),  $\tau_r = 1.0$  sec (curve 2) and  $\tau_r = 0.25$  sec (curve 3).

Fig. 9. Pattern of ignition in presence of "monster" sawteeth auxiliary heating is stopped at  $t = 6.34$  sec,  $\tau_r/\tau_E = \tau_s^-/a = 0.5$ .

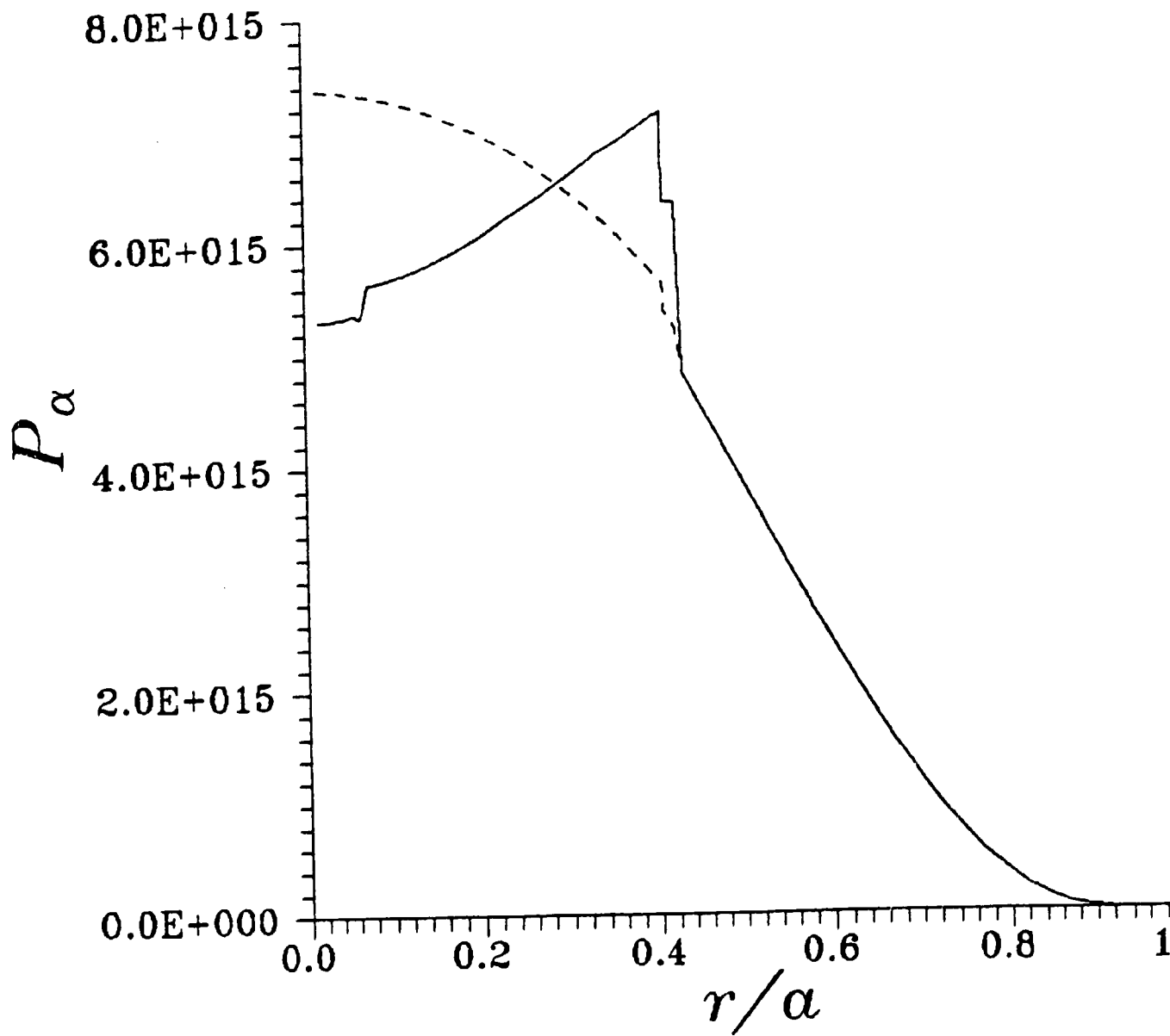


Fig. 1

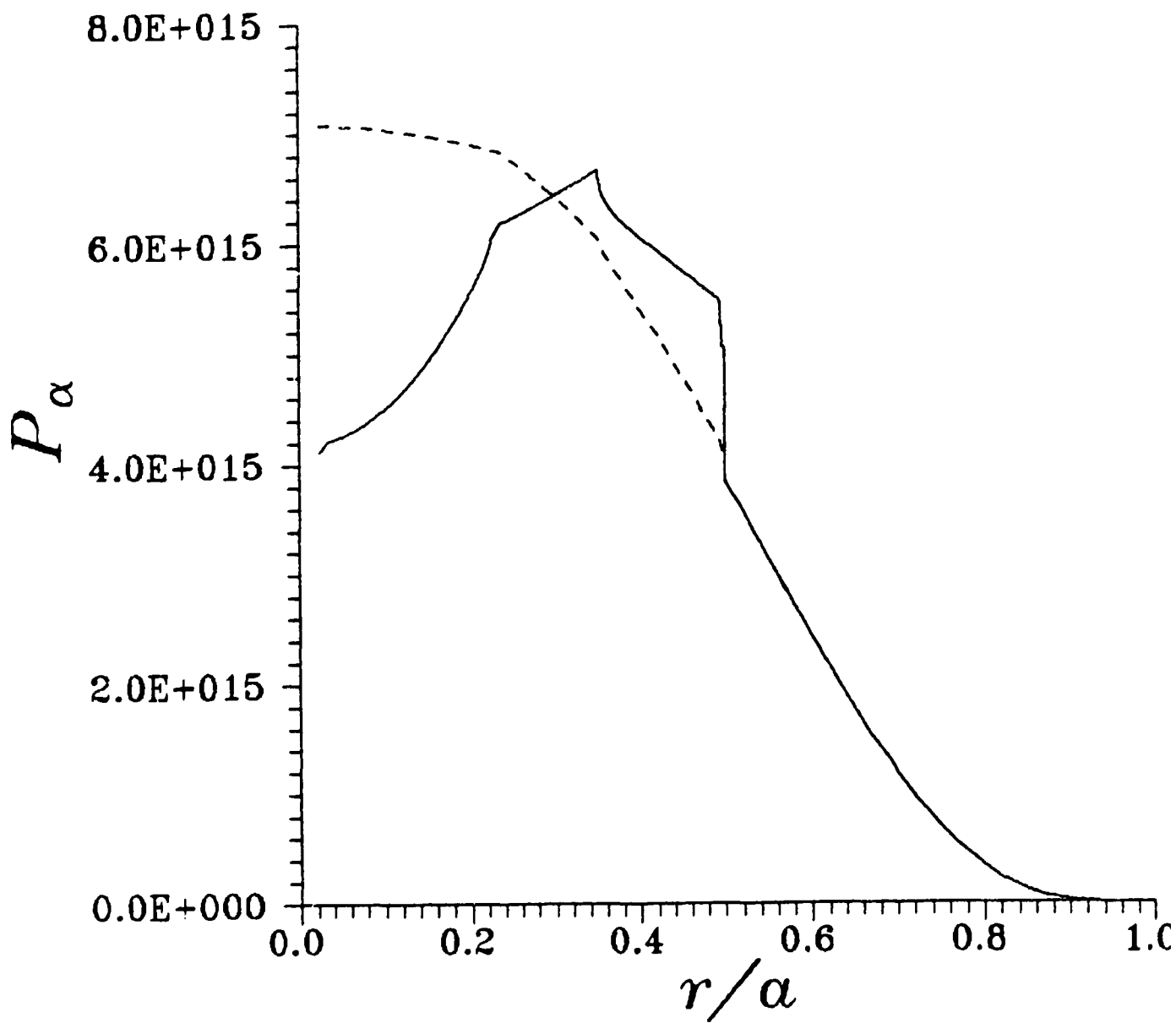


Fig. 2

Palpha(units  $10^{**}14$ )

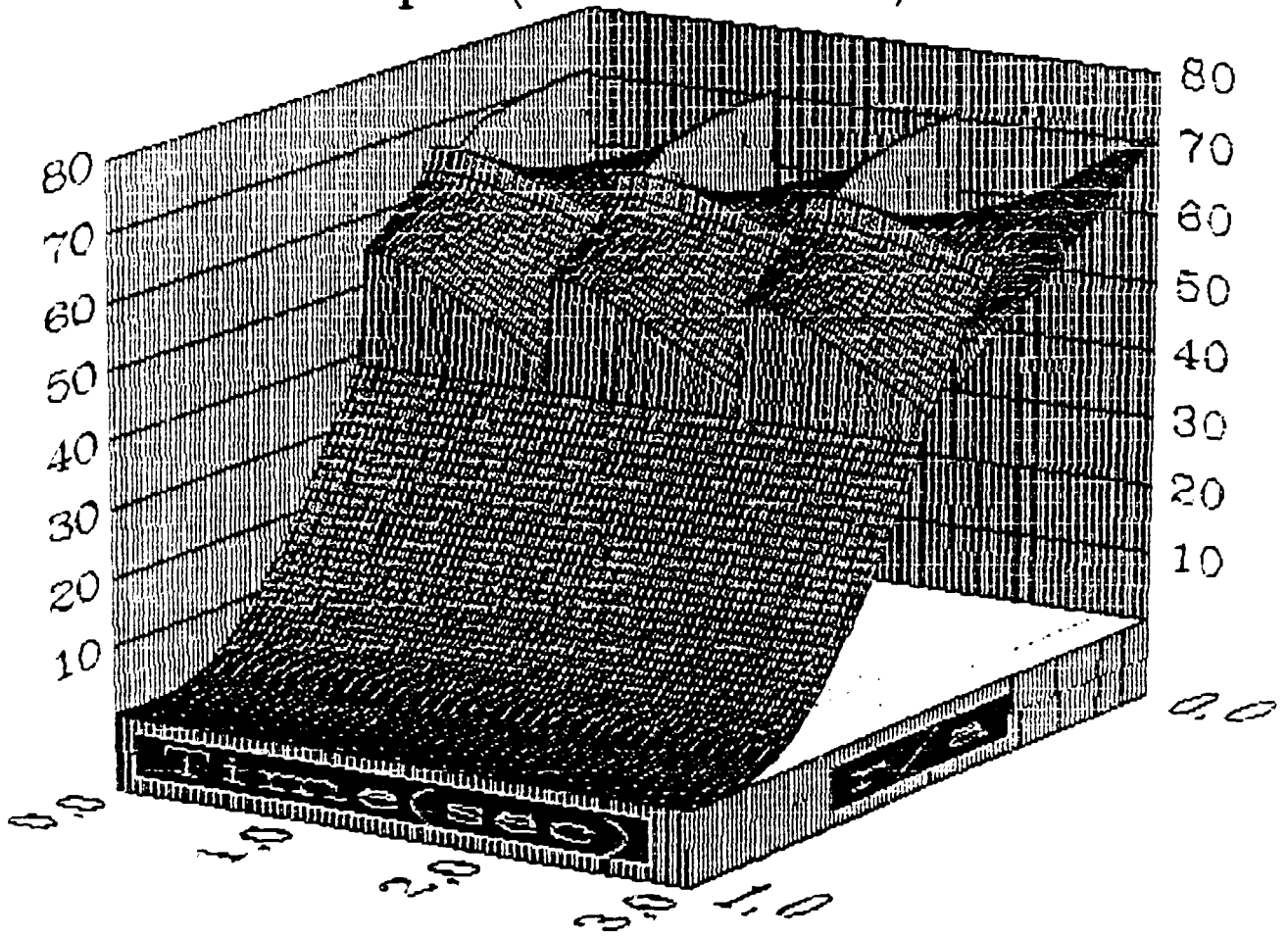


Fig. 3

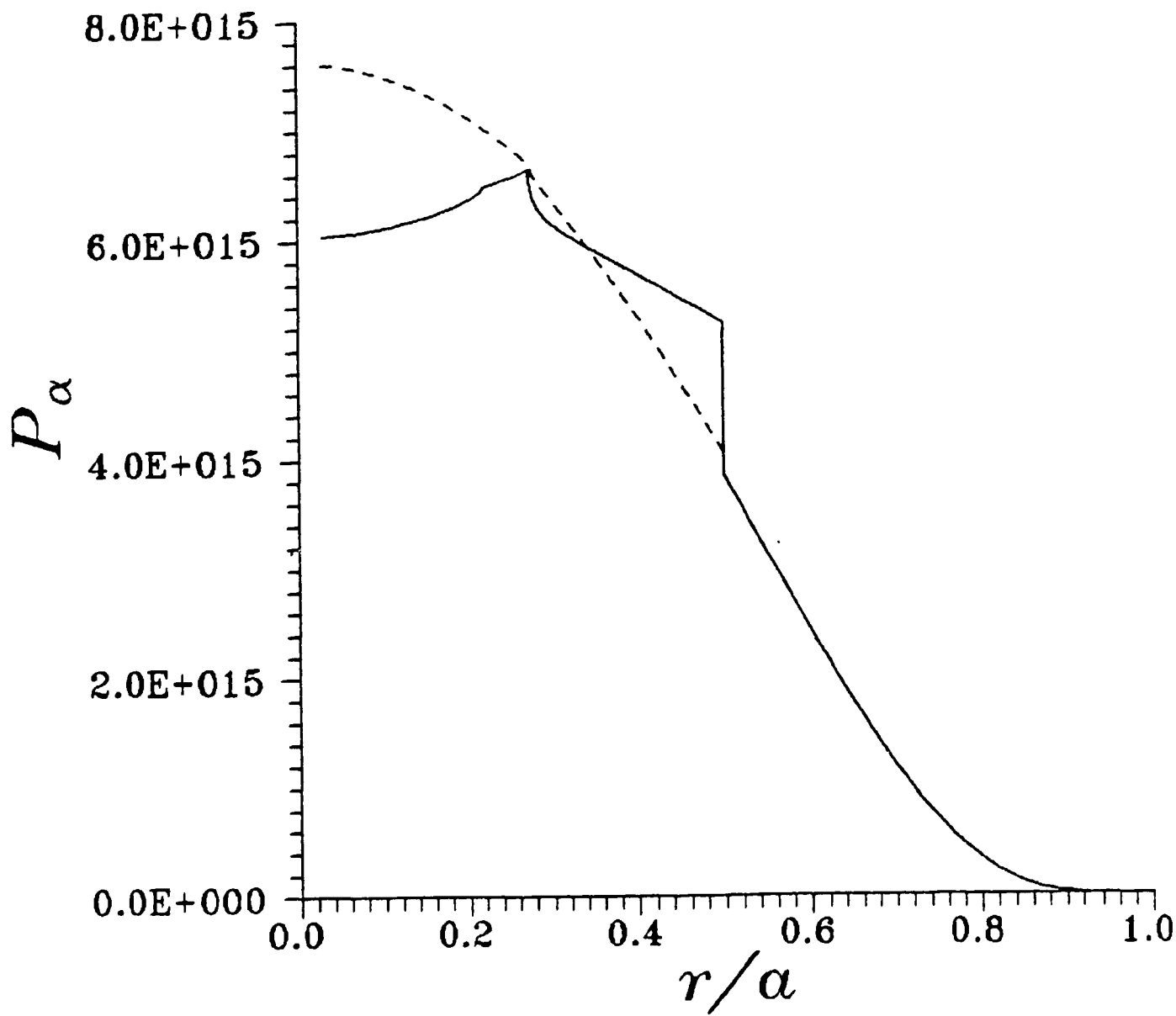


Fig. 4

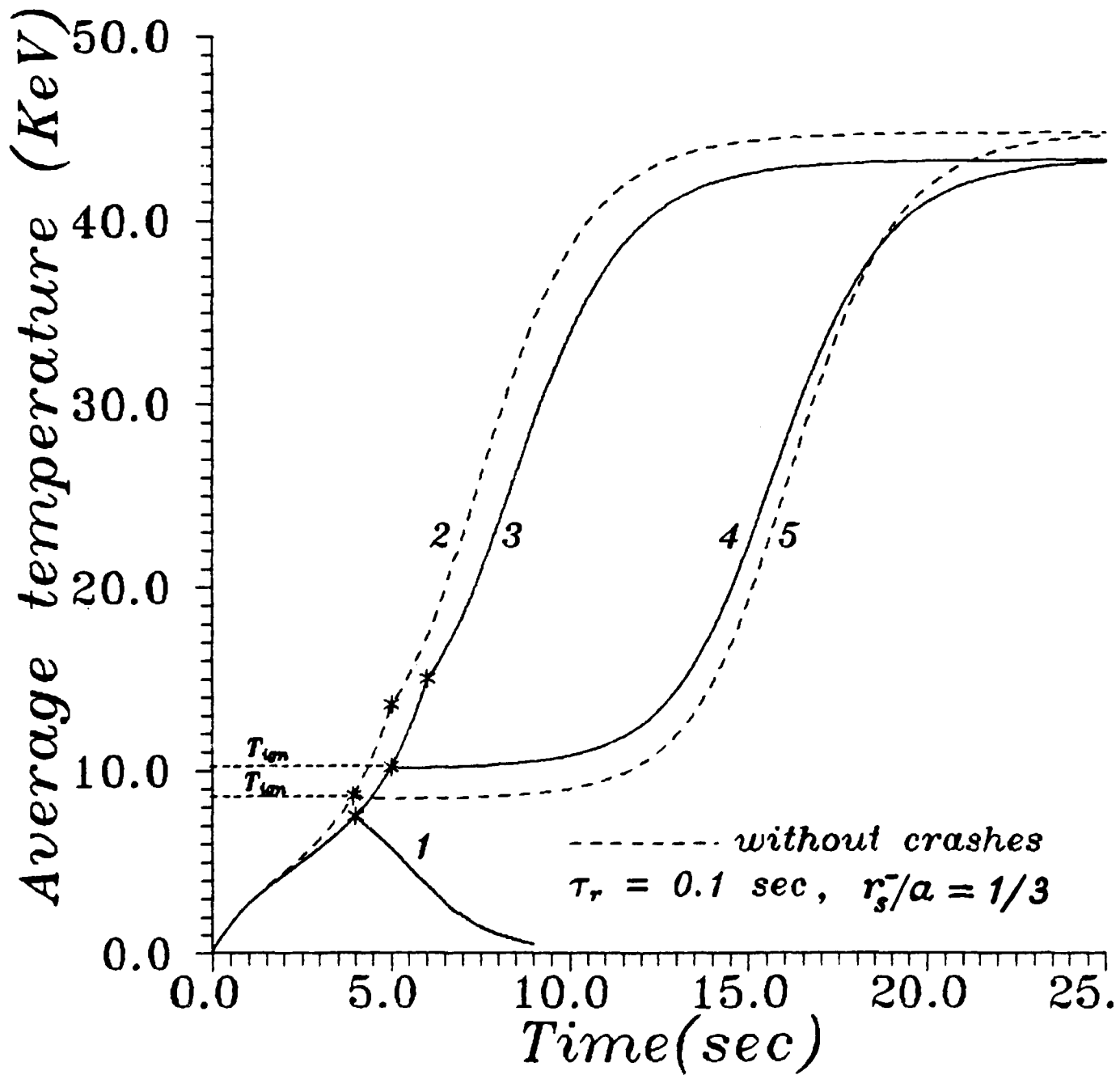


Fig. 5

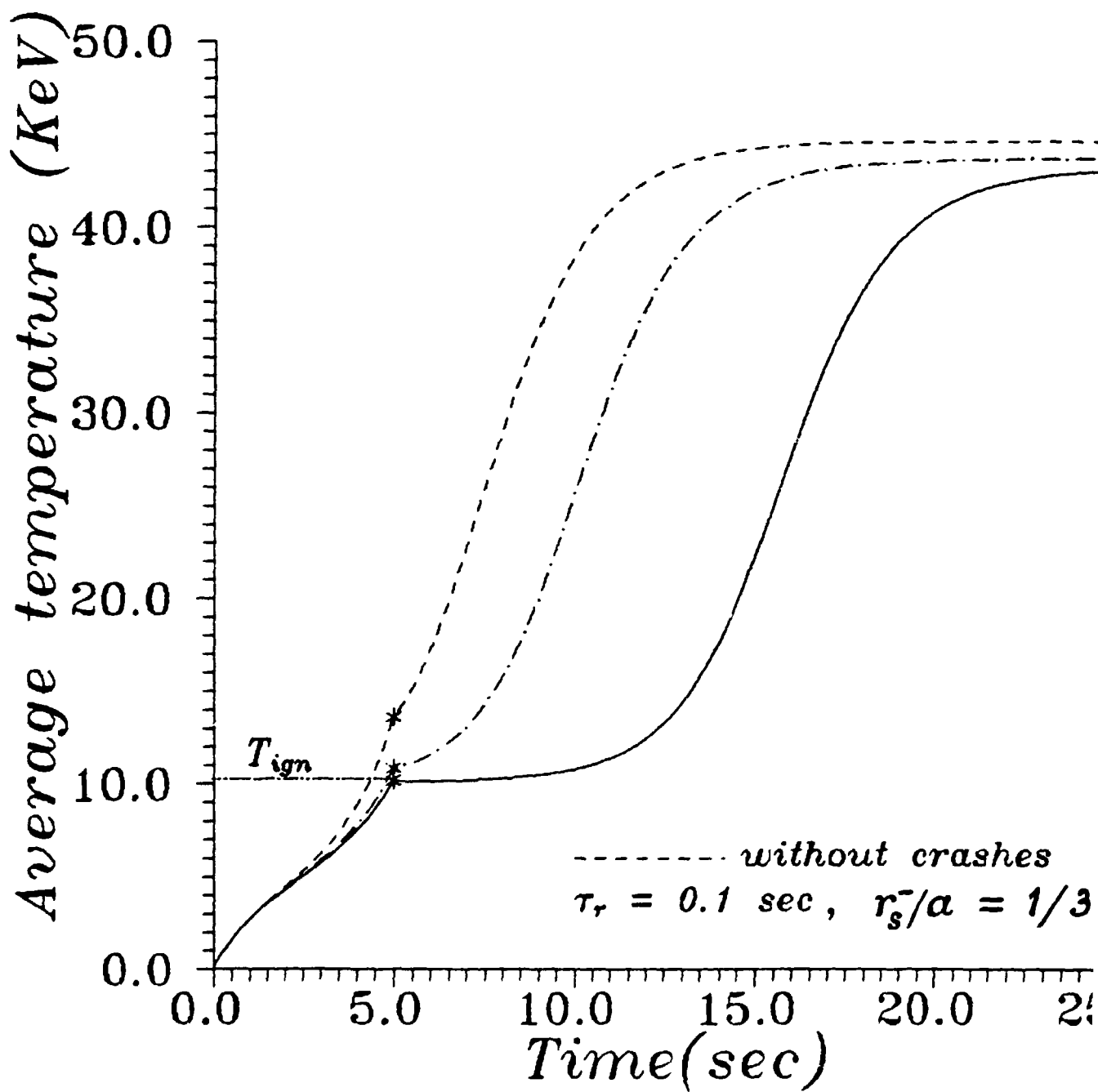


Fig. 6



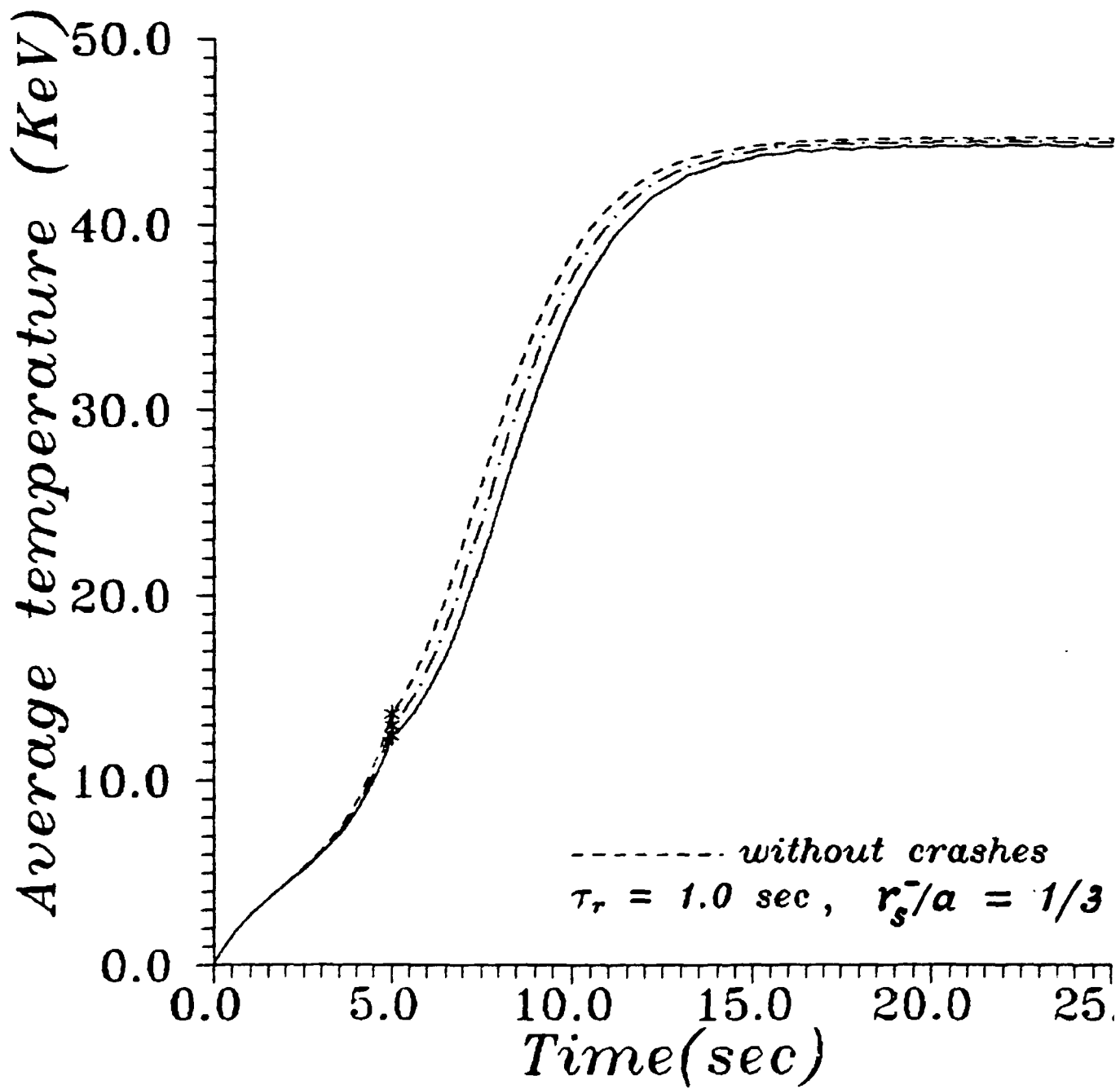


Fig. 7

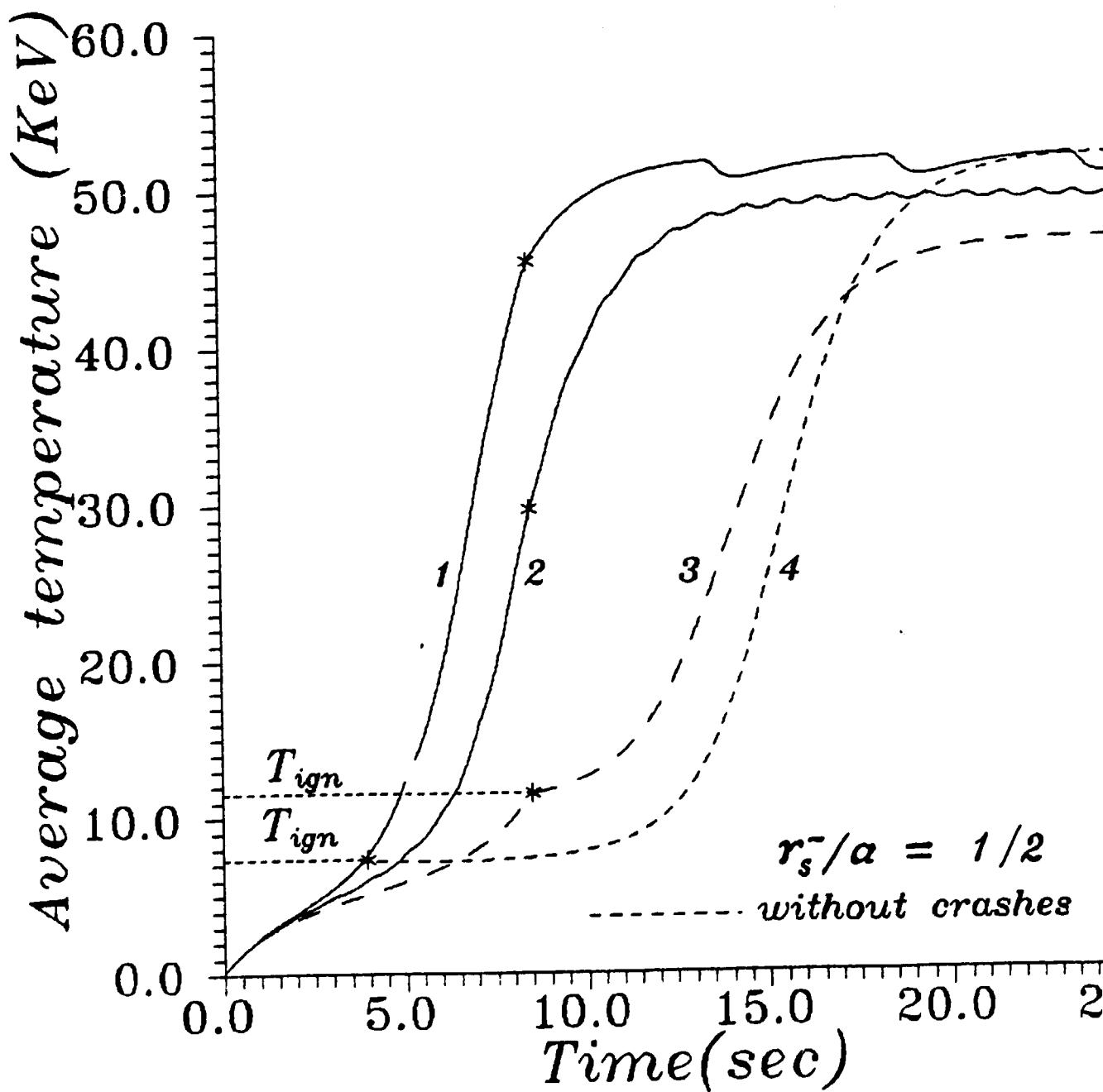


Fig. 8

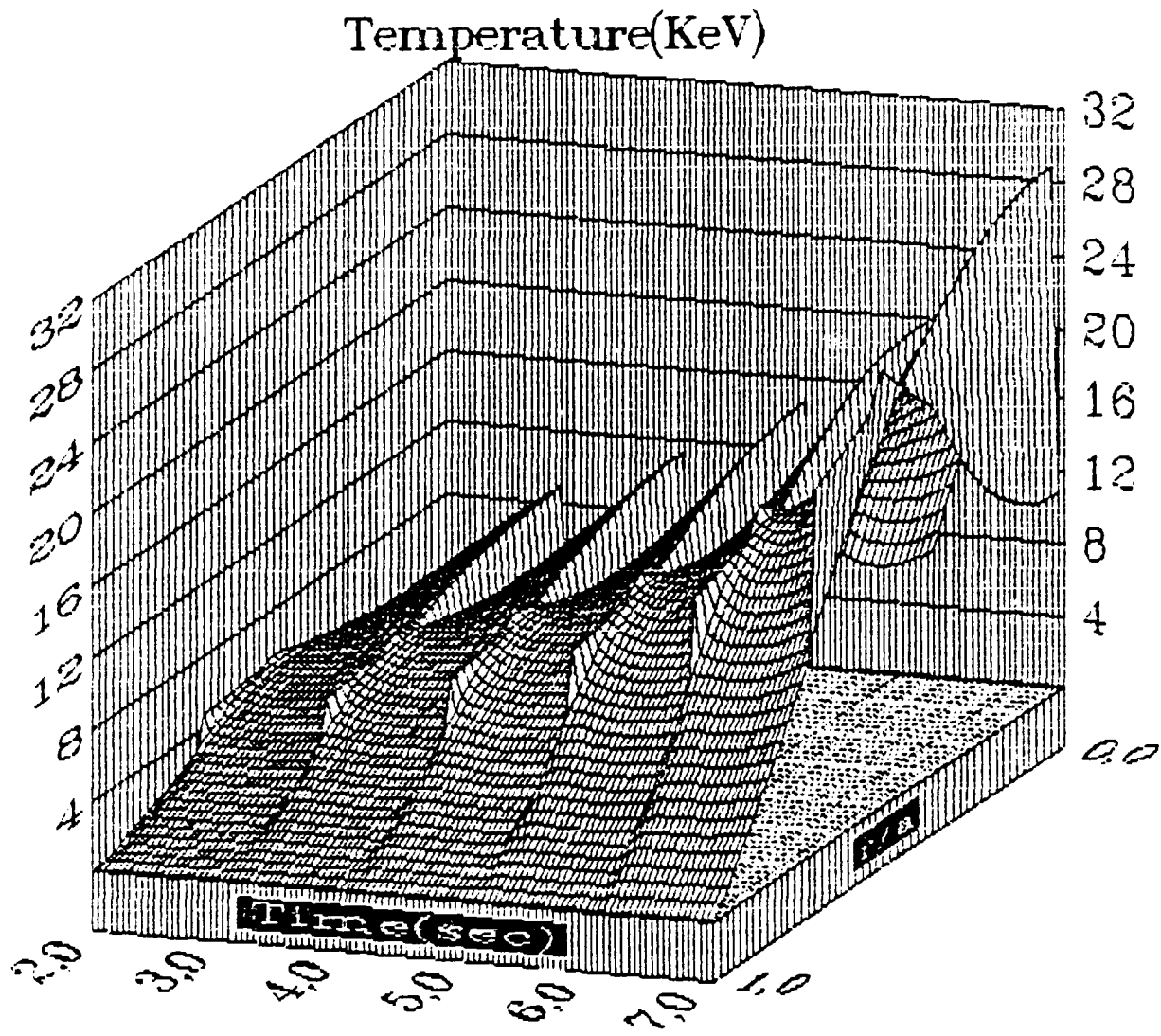


Fig. 9