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ABSTRACT

We present in detail a Hopf superalgebra $U_q(su(2,2/1))$ which is a q - deformation of the conformal superalgebra $su(2,2/1)$. The superalgebra $U_q(su(2,2/1))$ contains as a subalgebra a q - deformed super-Poincaré algebra and as Hopf subalgebras two conjugate 16-generator q-deformed super-Weyl algebras, which are which are q-deformation of parabolic subalgebras of $su(2,2/1)$. We use several (anti-) involutions, including the standard Cartan involution and a $*$-antiinvolution under which which the super-Weyl algebras are $*$-subalgebras of $U_q(su(2,2/1))$. The q - deformed Lorentz algebra is Hopf subalgebra of both Weyl algebras and is preserved by all (anti-) involutions considered.

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1. Introduction

The Poincaré and conformal algebras, and other non-compact Lie algebras and their supersymmetric extensions play a very important role in physics. It is important to apply the notion of quantum group as a noncocommutative Hopf algebra [1], [2], [3], [4], [5] for the deformation of these (super-) algebras. Recently, the deformations of $D = 4$ space-time (super-) symmetries were studied in [6-17]. In particular, q-deformations of the Lorentz group [6-8], Lorentz algebra [9-12], Poincaré algebra [13,11,12,14], conformal algebra [11] were studied (for more details see also [15],[16]). A q - conformal superalgebra containing a q-super-Poincaré subalgebra was announced in [11], while the way of obtaining q-super-Poincaré algebra by means of contraction was announced in [17].

In [11] a general procedure for the q - deformation of real (noncompact) simple Lie (super-) algebras $\mathcal{G}$ was proposed. The q - deformations obtained by this procedure are consistent with the Bruhat decompositions [18] of $\mathcal{G}$:

$$\mathcal{G} = \tilde{\mathcal{N}} \oplus \mathcal{A} \oplus \mathcal{M} \oplus \mathcal{N}, \tag{1}$$

where the subalgebras in this decomposition in the case of the conformal algebra $\mathcal{G} = su(2,2)$ are: *Lorentz* subalgebra $\mathcal{M} \cong so(3,1)$, subalgebra $\tilde{\mathcal{N}}$ of *translations*, subalgebra $\mathcal{N}$ of *special conformal transformations*, *dilatations* subalgebra $\mathcal{A}$, (the general meaning of (1) is explained in [11] (see also [15]). In particular, q-deformation of $\mathcal{M}$, $\mathcal{A}$, and of the parabolic subalgebras [18]:

$$\tilde{\mathcal{P}} = \tilde{\mathcal{N}} \oplus \mathcal{A} \oplus \mathcal{M}, \quad \mathcal{P} = \mathcal{A} \oplus \mathcal{M} \oplus \mathcal{N}, \tag{2}$$

i.e., $U_q(\mathcal{M}), U_q(\mathcal{A}), U_q(\tilde{\mathcal{P}})$ and $U_q(\mathcal{P})$ are Hopf subalgebras of $U_q(\mathcal{G})$ [11]. In the example of the conformal algebra $\mathcal{G} = su(2,2)$ it follows that the q - deformations of the Lorentz subalgebra, of the dilatation subalgebra and of the two (conjugate) Weyl algebras $\tilde{P}$ and $\mathcal{P}$ are Hopf subalgebras of $U_q(su(2,2))$ [11], (for $U_q(\tilde{\mathcal{P}})$ cf. also [12]). However, the q - Poincaré algebra is not Hopf subalgebra of $U_q(su(2,2))$ [11], [12]. The q - Poincaré algebra obtained recently in [14] from diffrential calculus on q - deformed Minkowski space has also eleven generators and is another example of q - deformed Weyl algebra. While the two q - deformed Weyl algebras above are conjugated under the Cartan involution, the interesting question of a self-conjugated Weyl subalgebra $U_q(\tilde{\mathcal{P}})$ was addressed in [12] where a $*$ - antiinvolution was found under which $U_q(\tilde{\mathcal{P}})$ is a $*$ - Hopf algebra.

In the present paper we present in detail the q - deformation of the conformal superalgebra $U_q(su(2,2/1))$ as announced in [11]. We have the following chain of Hopf subalgebras:

$$U_q(su(2,2/1)) \supset U_q(\tilde{\mathcal{P}}^S) \supset U_q(\mathcal{M}^S \oplus \mathcal{A}) \supset U_q(\mathcal{M}^S) \supset U_q(\mathcal{M}) , \qquad (3a)$$

where the 16-generator maximal parabolic subalgebra $\tilde{\mathcal{P}}^S$, which may be called q - deformed Weyl superalgebra, is given by

$$\tilde{\mathcal{P}}^S = \mathcal{M}^S \oplus \mathcal{A} \oplus \tilde{\mathcal{N}}^S , \quad \mathcal{M}^S = \mathcal{M} \oplus u(1) , \qquad (3b)$$

and $\tilde{\mathcal{N}}^S$ is the algebra of translations and supertranslations. Thus in terms of the super-Poincaré algebra $\mathcal{SP}$ we have

$$\tilde{\mathcal{P}}^S = \mathcal{SP} \oplus \mathcal{A} \oplus u(1) , \qquad (3c)$$

however, $\mathcal{SP}$ is only a commutation subalgebra, and not a Hopf subalgebra. We discuss also in detail the possible (anti-) involutions which preserve (up to conjugation) the sequence of real Hopf (super-)algebras (3a). In particular, we extend the $*$ - antiinvolution of [12] to $U_q(\tilde{\mathcal{P}}^S)$ making it a $*$ - Hopf superalgebra. Similar statement is true if we replace $\tilde{\mathcal{P}}^S$ with $\mathcal{P}^S$ which is a conjugated under the Cartan involution parabolic subalgebra.

The paper is organized as follows. In Section 2. we present (following [19]) the Cartan-Weyl basis of $U_q(sl(4/1;\mathbb{C}))$. In Section 3. we present explicitly $U_q(su(2,2/1))$. In Section 4. we discuss briefly $U_q(su(2,2/N))$.

2. $U_q(sl(4/1;\mathbb{C}))$

Let $\mathcal{G}^{\mathbb{C}} = sl(4/1;\mathbb{C})$. We have several choices of Dynkin diagrams [20] (see also [21],[22]) which will give different results after q - deformation. We choose the following symmetric Dynkin diagram

$$\bigcirc \!-\!-\!-\!-\! \otimes \!-\!-\!-\!-\! \otimes \!-\!-\!-\!-\! \bigcirc \qquad (4)$$

The corresponding symmetrized Cartan matrix has the elements: $a^s_{11} = a^s_{44} = 2$, $a^s_{22} = a^s_{33} = 0$, $a^s_{j,j+1} = a^s_{j+1,j} = -1$ for $j = 1,3$, $a^s_{23} = a^s_{32} = 1$, all other elements are zero. Consistently the products between the simple roots are $(\alpha_i, \alpha_j) = a^s_{ij}$. The root system is given by: $\Delta^{\pm} = \{\pm\alpha_{jk} = \pm(\alpha_j + \alpha_{j+1} + \cdots + \alpha_k) \mid 1 \le j < k \le 5, \alpha_{jj+1} = \alpha_j\}$. The roots $\pm\alpha_{jk}$ with $j = 3$

or $k = 3$ are odd, the rest are even. The Cartan-Weyl generators of $U_q(sl(4/1;\mathbb{C}))$ corresponding to nonsimple roots are defined inductively following the normal ordering prescription of [19] (see also [24], [11]) :

$$X^+_{jk} \equiv X^+_{jj+1} X^+_{j+1k} - (-1)^{\deg X^+_{jj+1} \deg X^+_{j+1k}} q^{(\alpha_j,\alpha_{j+1k})} X^+_{j+1k} X^+_{jj+1} , \quad 1 \le j < k-1 \le 4 , \qquad (5a)$$

$$X^-_{jk} \equiv X^-_{j+1k} X^-_{jj+1} - (-1)^{\deg X^-_{jj+1} \deg X^-_{j+1k}} q^{-(\alpha_j,\alpha_{j+1k})} X^-_{jj+1} X^-_{j+1k} , \quad 1 \le j < k-1 \le 4 , \qquad (5b)$$

where $\deg X^{\pm}_{jk} \doteq \bar{0}$, for the even generators, $\deg X^{\pm}_{jk} = \bar{1}$, for the odd generators; $X^{\pm}_{jj+1} \equiv X^{\pm}_j$ denote the simple root generators. Let us denote by H_j, $j = 1,2,3,4$ the generators of the Cartan subalgebra $\mathcal{H}$ corresponding to the simple roots α_j. Then the Cartan generator corresponding to the root α_{jk} is $H_{jk} = H_j + H_{j+1} + \cdots + H_k$.

In writing down the commutation relations between the generators of $U_q(sl(4/1;\mathbb{C}))$ we shall use the standard supercommutator between homogeneous elements of a superalgebra (cf. [19]):

$$[Y, Z] \equiv YZ - (-1)^{\deg Y \deg Z} ZY , \qquad (6a)$$

where $\deg H = \bar{0}$, for $H \in \mathcal{H}$, and also the deformed supercommutator :

$$[Y, Z]_q \equiv YZ - q(-1)^{\deg Y \deg Z} ZY , \qquad (6b)$$

The supercommutators between the simple root Cartan-Chevalley generators are standard [19],[23]:

$$[H_i , H_j] = 0, \quad [H_i , X^{\pm}_j] = \pm a^s_{ij} X^{\pm}_j , \qquad (7a)$$

$$[X^+_i , X^-_j] = \delta_{ij} (-1)^{\delta_{i3}} \frac{q^{H_i} - q^{-H_i}}{q - q^{-1}} = \delta_{ij} (-1)^{\delta_{i3}} [H_i]_q , \qquad (7b)$$

Now the commutation relations between Cartan-Weyl generators of $U_q(sl(4/1;\mathbb{C}))$ corresponding to nonsimple roots follow from (5) and (6). We have :

$$[X^+_{jk}, X^-_{jk}] = \epsilon_{jk} [H_{jk}] , \quad \epsilon_{jk} = (-1)^{\delta_{j3}\delta_{k4} + \delta_{j3}\delta_{k5}} ,$$

$$[X^+_{ij}, X^+_{kl}] = \tilde{\lambda} X^+_{kj} X^+_{il}, \text{ for } i < k < j < l , \quad \tilde{\lambda} \equiv (q - q^{-1})$$

$$[X^-_{ij}, X^+_{li}] = -(-1)^{deg(X_{ij}) deg(X_{li})} q^{H_j + \ldots + H_{i-1}} X^+_{ij} \text{ for } i > j > l \qquad (8)$$

$$[X^-_{ji}, X^+_{ik}] = q^{H_i + \ldots + H_{k-1}} X^-_{jk} \text{ for } j > k > i$$

$$[X^-_{ij}, X^+_{lk}] = \tilde{\lambda} q^{(H_j+\ldots+H_{k-1})} X^+_{lj} X^-_{ik} \quad \text{for } i > k > j > l$$

plus analogous expressions obtained from previous four by conjugation given [24]. The coproducts of the Cartan-Chevalley generators are standard [19] :

$$\delta(H_j) = H_j \otimes 1 + 1 \otimes H_j , \quad \delta(X_j^\pm) = X_j^\pm \otimes q^{H_j/2} + q^{-H_j/2} \otimes X_j^\pm , \tag{9a}$$

$$\varepsilon(H_j) = \varepsilon(X_j^\pm) = 0 , \tag{9b}$$

$$\gamma(H_j) = -H_j , \quad \gamma(X_j^\pm) = -q^{\pm 1} X_j^\pm . \tag{9c}$$

Now the coalgebra operations for the Cartan-Weyl generators follow from (5), (6) and (7), and the complex Hopf superalgebra $U_q(sl(4/1;\mathbb{C}))$ can be given explicitly.

In this superalgebra one may consider several (anti-) involutions. First we have an involution ω and an antiinvolution $\mathcal{I}$ which exchange positive and negative root vectors [24]:

$$\omega(X_\beta) = a_\beta(q) X_{-\beta} , \quad \omega(X_{-\beta}) = a_\beta(q^{-1})(-1)^{\deg X_\beta} X_\beta , \quad \omega(H_\beta) = -H_\beta , \quad \beta \in \Delta^+ , \tag{10a}$$

$$a_{\alpha_{ij}}(q) \equiv (-q^{-1})^{j-i-1} (-1)^{\sum_{l=1}^{j-2} deg X^+_{i,i+1} deg X^+_{i+1,j}} \tag{10b}$$

$$\omega(q) \mapsto q , \quad \omega(a \otimes b) = \omega(b) \otimes \omega(a) , \quad \omega(\delta(X_\beta)) = a_\beta(q) \delta(\omega(X_b)) . \tag{10c}$$

Note that $a_\beta(q) = 1$ if β is a simple root, i.e., $\beta \in \Pi$.

$$\mathcal{I} : X_\beta \mapsto X_{-\beta} , \quad H_\beta \mapsto H_\beta , \quad q \mapsto q^{-1} , \quad \beta \in \Delta , \tag{11a}$$

$$\mathcal{I}(a \otimes b) = \mathcal{I}(b) \otimes \mathcal{I}(a) , \quad \mathcal{I}(\delta(b)) = \delta(\mathcal{I}(b)) . \tag{11b}$$

Next we have (anti-) involutions which preserve $\Delta^\pm$ and are consistent with the chosen normal ordering. We achieve this goal by introducing the transformations that map X_{12} into X_{45} and X_{23} into X_{34}. At this point we have a freedom to choose the following Hopf algebra (anti–)involutions [1)]

[1)] For the definition of the automorphisms $\oplus$ and $\circledast$ see [25]

i) nongraded $\oplus$–antiinvolution Φ [25] such that

$$\begin{aligned} \Phi(X_j^\pm) &= X^\pm_{5-j} , \quad \Phi(H_j) = -H_{5-j} , \quad \Phi(q) = q , \\ \Phi(A \otimes B) &= \Phi(B) \otimes \Phi(A) \\ \delta(\Phi(A)) &= \Phi(\delta(A)) \\ \Phi \circ \gamma &= \gamma \circ \Phi \end{aligned} \tag{12}$$

Note that this antiinvolution exchanges the two simple even roots $\alpha_1 \equiv \alpha_{12} \longleftrightarrow \alpha_4 \equiv \alpha_{45}$ and the two simple odd roots $\alpha_2 \equiv \alpha_{23} \longleftrightarrow \alpha_3 \equiv \alpha_{34}$. This is the reason for our choice of Dynkin diagram (with other choices of Dynkin diagrams our (anti)involutions would map simple into nonsimple roots). Note that the generators $X^\pm_{24}, X^\pm_{15}$ are self-congugated. This is the extension of the $\oplus$–antiinvolution used in [12] under which the q–Weyl–Hopf algebra is seld-conjugated.

The antiinvolutions (11) and (12) are antiautomorphisms of the superalgebra $U_q(sl(4/1;\mathbb{C}))$. The involution ω should be supplemented with an *involution* acting as (12) (except on q and on $\otimes$), then it is an automorphism of $U_q(sl(4/1;\mathbb{C}))$ (similar automorphism was used in [11] for $U_q(sl(4;\mathbb{C}))$).

ii) We also have a graded $*$–antiinvolution Ψ such that $\Psi^2(X) = (-1)^{deg(X)} X$ [26]

$$\begin{aligned} \Psi(X_j^\pm) &= (-1)^{\delta_{j3}} X^\pm_{5-j} , \quad \Psi(H_j) = -H_{5-j} , \quad \Psi(q) = q , \\ \Psi(A \otimes B) &= \Psi(B) \otimes \Psi(A) \\ \delta(\Psi(A)) &= \Psi(\delta(A)) \\ \omega \circ S &= S \circ \omega \end{aligned} \tag{13}$$

We recall that $X_2^\pm, X_3^\pm$, are odd.

3. U_q(su(2,2/1))

The Lie superalgebra $\mathcal{G}^S \equiv su(2,2/1)$ [13] is a real noncompact form of $\mathcal{G}^{\mathbb{C}} = sl(4/1;\mathbb{C})$ with Cartan decomposition and splitting into even and odd parts: $\mathcal{G}^S = \mathcal{K}^S + \mathcal{P}^S = \mathcal{G}^S_{(0)} + \mathcal{G}^S_{(1)}$ such that $\mathcal{G}^S_{(0)} \cong su(2,2) \oplus u(1)$, $\mathcal{K}^S_{(0)} \cong u(2) \oplus u(2)$, $\dim_R \mathcal{P}^S_{(0)} = 8$, $\dim_R \mathcal{K}^S_{(1)} = \dim_R \mathcal{P}^S_{(1)} = 4$.

The parabolic subalgebras of $\mathcal{G}^S$ are determined by the parabolic subalgebras of the noncompact subalgebra $su(2,2)$ of the even part $\mathcal{G}^S_{(0)}$. Following [11] we consider only a q - deformation of $\mathcal{G}$ consistent with the maximal parabolic subalgebra $\mathcal{P}^S_{max} = \mathcal{M}^S \oplus \mathcal{A}^S \oplus \mathcal{N}^S$, where [11]

$$\mathcal{A}^S = \mathcal{A}^S_{(0)} = l.s.\{D\} \cong \mathcal{A} , \tag{14a}$$

$$\mathcal{M}^S = \mathcal{M}^S_{(0)} \cong \mathcal{M} \oplus u(1)\ ,\quad \mathcal{M} \cong so(3,1)\ , \tag{14b}$$

$$\mathcal{N}^S = \mathcal{G}_1^- \oplus \mathcal{G}_2^-\ ,\quad \mathcal{G}_k^- \equiv \mathcal{G}_{-\lambda_k}\ ,\quad \mathcal{N}^S_{(0)} = \mathcal{G}_2^- \cong \mathcal{N}\ , \tag{14c}$$

$$\tilde{\mathcal{N}}^S = \mathcal{G}_1^+ \oplus \mathcal{G}_2^+\ ,\quad \mathcal{G}_k^+ \equiv \mathcal{G}_{\lambda_k} = \theta\mathcal{G}_k^-\ ,\quad \tilde{\mathcal{N}}^S_{(0)} = \mathcal{G}_2^+ \cong \tilde{\mathcal{N}}, \tag{14d}$$

$$\lambda_1(D) = 1/2\ ,\ \lambda_2 = 2\lambda_1\ ,\ \dim \mathcal{G}_1^\pm = 4\ ,\ \dim \mathcal{G}_2^\pm = 4\ , \tag{14e}$$

where the $\mathcal{A}, \mathcal{M}, \tilde{\mathcal{N}}, \mathcal{N}$ are the $su(2,2)$ subalgebras given in (1), $\pm\lambda_k \in \mathcal{A}^*$, $k = 1,2$, form the (restricted) root system with respect to $\mathcal{A}$, θ denotes the Cartan involution. The Cartan subalgebra $\mathcal{H}^S \subset \mathcal{G}^S_{(0)}$ is chosen as follows:

$$\mathcal{H}^S = \mathcal{H} \oplus u(1)\ , \tag{15}$$

where $\mathcal{H}$ is the Cartan subalgebra of $su(2,2)$. We express the generators of $U_q(\mathcal{G}^S)$ in terms of those of $U_q(\mathcal{G}^{\square})$. For the Cartan generators we have (see also [21],[11]):

$$M^3 = (1/2)(H_1 + H_4)\ ,\quad N^3 = (1/2)(H_4 - H_1)\ , \tag{16a}$$

$$D = (1/2)(H_1 + H_4) + H_2 + H_3\ , \tag{16b}$$

$$E = (i/2)(H_1 - H_4) + i(H_2 - H_3) \tag{16c}$$

where M^3, N^3 are Cartan generators of the Lorentz subalgebra, D is the generator of dilatations, E is the $u(1)$ generator. For the Lorentz generators besides $H, \tilde{D}$ we choose as in [11]:

$$\begin{aligned} M^\pm &\equiv -M_{23} \pm iM_{13} = X_1^\pm + X_4^\pm\ , \\ N^\pm &\equiv N^\pm \equiv -iM_{10} \pm M_{20} = X_1^\pm - X_4^\pm \end{aligned} \tag{17}$$

where $M^\pm, N^\pm$, resp., are the raising and lowering generators of rotations, boosts, resp. Classically $M^\pm, M^3$ form the rotation subalgebra which, however, is not preserved under deformation and in fact we shall give most relations in terms of the generators $X_1^\pm \equiv X_{12}^\pm$, $X_4^\pm \equiv X_{45}^\pm$.

The translation and special conformal transformation generators are chosen as in [11],[12] (up to signs):

$$P_0 = i(X_{14}^+ + X_{25}^+)\ ,\ P_1 = i(X_{15}^+ + X_{24}^+)\ ,\ P_2 = X_{15}^+ - X_{24}^+\ ,\ P_3 = i(X_{25}^+ - X_{14}^+)\ , \tag{18}$$

$$K_0 = -i(X_{14}^- + X_{25}^-),\ K_1 = i(X_{15}^- + X_{24}^-),\ K_2 = X_{24}^- - X_{15}^-,\ K_3 = i(X_{25}^- - X_{14}^-)\ . \tag{19}$$

Next we have to express the 8 generators of $\mathcal{G}^S_{(1)}$. Let us denote the generators of $\tilde{\mathcal{N}}^S_{(1)} = \mathcal{G}_1^+$ by Q_a, $a = 1,\ldots,4$, and of $\mathcal{N}^S_{(1)} = \mathcal{G}_1^-$ by S^a, $a = 1,\ldots,4$. Then we have

$$\begin{aligned} Q_1 &= X_{13}^+ + X_{35}^+\ , Q_2 = X_{23}^+ + X_{34}^+ \\ Q_3 &= i(X_{23}^+ - X_{34}^+)\ , Q_4 = i(X_{13}^+ - X_{35}^+) \end{aligned} \tag{20}$$

$$\begin{aligned} S_1 &= X_{13}^- + X_{35}^-\ , S_2 = X_{23}^- + X_{34}^- \\ S_3 &= i(X_{23}^- - X_{34}^-)\ , S_4 = i(X_{13}^- - X_{35}^-) \end{aligned} \tag{21}$$

Now we are ready to give the commutation relations of the q - deformed conformal superalgebra:

1) Lorentz sector ($q = e^{h/2}$):

$$[M^3, M^\pm] = \pm M^\pm\ ,\ [M^+, M^-] = 2[M^3]\cosh(N^3 h/2)\ , \tag{22a}$$

$$[M^3, N^\pm] = \pm N^\pm\ ,\ [N^+, N^-] = 2[M^3]\cosh(N^3 h/2)\ , \tag{22b}$$

$$[N^3, M^\pm] = \pm N^\pm\ ,\ [N^3, N^\pm] = \pm M^\pm\ , \tag{22c}$$

$$[M^\pm, N^\mp] = \pm 2[N^3]\cosh(M^3 h/2) \tag{22d}$$

2) translations :

$$[P_a, P_0 \pm P_3]_{q^{\pm 1}} = 0\ ,\ a = 1,2;\quad [P_1, P_2] = 0,\quad [P_0 - P_3, P_0 + P_3] = \tilde{\lambda}(P_1 + iP_2)(P_1 - iP_2) \tag{23}$$

3) special conformal transformations :

$$[K_a, K_0 \pm K_3]_{q^{\mp 1}} = 0\ ,\ a = 1,2;\ [K_1, K_2] = 0,\ [K_0 - K_3, K_0 + K_3] = -\tilde{\lambda}(K_1 + iK_2)(K_1 - iK_2) \tag{24}$$

4) covariance relations :

$$[M^3, P_1 \pm iP_2] = \pm(P_1 \pm iP_2)\ ,\ [M^3, P_3 \pm P_0] = 0\ , \tag{25a}$$

$$[N^3, P_3 \pm P_0] = \mp(P_3 \pm P_0)\ ,\ [N^3, P_1 \pm iP_2] = 0\ , \tag{25b}$$

$$[M^3, K_1 \pm iK_2] = \pm(K_1 \pm iK_2)\ ,\ [M^3, K_3 \pm K_0] = 0\ , \tag{26a}$$

$$[N^3, K_3 \pm K_0] = \mp(K_3 \pm K_0)\ ,\ [N^3, K_1 \pm iK_2] = 0\ , \tag{26b}$$

$$\begin{aligned} &[X_1^+, P_3 + P_0]_{q^{-1}} = P_1 + iP_2\ ,\ [X_1^+, P_0 - P_3]_q = 0\ , \\ &[X_1^+, P_1 + iP_2]_q = 0\ ,\ [X_1^+, P_1 - iP_2]_{q^{-1}} = P_3 - P_0\ , \end{aligned} \tag{27a}$$

$$[X_4^+, P_0+P_3]_{q^{-1}} = 0, \quad [X_4^+, P_0-P_3]_q = -q(P_1+iP_2)$$
$$[X_4^+, P_1+iP_2]_{q^{-1}} = 0, \quad [X_4^+, P_1-iP_2]_q = -q(P_0-P_3), \tag{27b}$$

$$[X_1^-, P_3+P_0] = 0 \quad [X_1^-, P_0-P_3] = -(P_1-iP_2)q^{N^3-M^3},$$
$$[X_1^-, , P_1+iP_2] = (P_3+P_0)q^{N^3-M^3}, \quad [X_1^-, P_1-iP_2] = 0, \tag{27c}$$

$$[X_4^-, P_0+P_3] = q^{M^3+N^3}(P_1-iP_2), \quad [X_4^-, P_3-P_0] = 0$$
$$[X_4^-, P_1+iP_2] = q^{M^3+N^3}(P_3-P_0), \quad [X_4^-, P_1-iP_2] = 0, \tag{27d}$$

The covariance relations for K_μ may be obtained from formulae (27) by the following changes: $M^\pm \mapsto M^\mp$, $N^+ \mapsto -N^-$, $M^3 \mapsto -M^3$, $N^3 \mapsto N^3$, $P_\mu \mapsto (-1)^{\delta_{\mu 1}} K_\mu$, $q^{1/2} \mapsto q^{-1/2}$. These follow from the automorphism of $U_q(\mathcal{G}^Q)$: $X_1^\pm \longleftrightarrow X_4^\mp$, $H_1 \longleftrightarrow -H_4$, $X_2^\pm \mapsto -X_2^\mp$, $H_2 \mapsto -H_2$, $q^{1/2} \mapsto q^{-1/2}$, (then $X_{14}^\pm \longleftrightarrow -X_{25}^\mp$, $X_{15}^\pm \mapsto -X_{15}^\mp$). [The commutation relations between $N^\pm$ and P_μ, K_μ may be obtained from those between $M^\pm$ and P_μ by the changes $M^\pm \longleftrightarrow N^\pm$, $P_0 \longleftrightarrow P_3$, $P_1 \longleftrightarrow iP_2$ and from those between $M^\pm$ and K_μ by the changes $M^\pm \longleftrightarrow -N^\pm$, $K_0 \longleftrightarrow K_3$, $K_1 \longleftrightarrow -iK_2$.]

5) Dilatation + chiral sector

$$\begin{aligned} [D, P_\mu] &= P_\mu & [E, P_\mu] &= 0 \\ [D, K_\mu] &= -K_\mu & [E, K_\mu] &= 0 \\ [D, M_{\mu\nu}] &= 0 & [E, M_{\mu\nu}] &= 0 \\ [D, Q_j] &= \frac{1}{2}Q_j & [E, Q_j] &= (-1)^{\delta_{j1}+\delta_{j2}}\frac{3}{2}Q_{5-j} \\ [D, S_j] &= -\frac{1}{2}S_j & [E, S_j] &= (-1)^{\delta_{j3}+\delta_{j4}}\frac{3}{2}S_{5-j} \\ [D, E] &= 0 \end{aligned} \tag{28}$$

6) For the commutation relations between translations and special conformal transformations we have:

$$\begin{aligned} [P_3 \pm P_0, K_3 \pm K_0] &= \pm\bar{\lambda}\, q^{\pm(M^3-D)}(M^+ \mp N^+)(M^- \pm N^-), \\ [P_3 \pm P_0, K_3 \mp K_0] &= 4[\mp N^3 - D] \\ [P_1 \pm iP_2, K_1 \mp iK_2] &= 4[\mp M^3 - D], \\ [P_1 \pm iP_2, K_1 \pm iK_2] &= 0, \\ [P_3 + P_0, K_1 + iK_2] &= 2(M^+ - N^+)\, q^{M^3-D}, \\ [P_3 + P_0, K_1 - iK_2] &= -2(M^- + N^-)\, q^{-(D+N^3)}, \\ [P_3 - P_0, K_1 + iK_2] &= 2\, q^{D-M^3}(M^+ + N^+), \\ [P_3 - P_0, K_1 - iK_2] &= -2\, q^{D-N^3}(M^- - N^-), \\ [P_1 - iP_2, K_3 + K_0] &= 2(M^- + N^-)\, q^{M^3-D}, \\ [P_1 + iP_2, K_3 + K_0] &= -2(M^+ - N^+)\, q^{N^3-D}, \\ [P_1 - iP_2, K_3 - K_0] &= 2\, q^{D-M^3}(M^- - N^-), \\ [P_1 + iP_2, K_3 - K_0] &= -2\, q^{D+N^3}(M^+ + N^+). \end{aligned} \tag{29}$$

7) In the supersymmetric sector we use the following notation for the supertranslations: $Q_\pm \equiv iQ_1 \pm Q_4$, $\tilde{Q}_\pm \equiv iQ_2 \pm Q_3$, i.e., $Q_+ = 2iX_{13}^+$, $Q_- = 2iX_{35}^+$, $\tilde{Q}_+ = 2iX_{23}^+$, $\tilde{Q}_- = 2iX_{34}^+$, and analogously for the special superconformal generators : $S_\pm \equiv iS_1 \pm S_4$, $\tilde{S}_\pm \equiv iS_2 \pm S_3$, i.e., $S_+ = 2iX_{13}^-$, $S_- = 2iX_{35}^-$, $\tilde{S}_+ = 2iX_{23}^-$, $\tilde{S}_- = 2iX_{34}^-$. Then we have:

$$\begin{aligned} &[Q_\pm, Q_\pm] = 0, \quad [\tilde{Q}_\pm, \tilde{Q}_\pm] = 0, \quad [Q_\pm, \tilde{Q}_\pm]_{q^{\mp 1}} = 0, \\ &[Q_+, Q_-]_q = 2i(P_1 + iP_2), \quad [\tilde{Q}_+, \tilde{Q}_-]_q = 2i(P_1 - iP_2) \\ &[Q_+, \tilde{Q}_-]_q = 2i(P_0 - P_3), \quad [Q_-, \tilde{Q}_+]_{q^{-1}} = 2iq^{-1}(P_0 + P_3) \end{aligned} \tag{30}$$

$$\begin{aligned} &[S_\pm, S_\pm] = 0, \quad [\tilde{S}_\pm, \tilde{S}_\pm] = 0, \quad [S_\pm, \tilde{S}_\pm]_{q^{\mp 1}} = 0 \\ &[S_+, S_-]_q = 2iq(K_1 - iK_2), \quad [\tilde{S}_+, \tilde{S}_-]_q = 2iq(K_1 + iK_2), \\ &[S_+, \tilde{S}_-]_q = -2iq(K_0 + K_3), \quad [S_-, \tilde{S}_+]_{q^{-1}} = 2i(K_3 - K_0) \end{aligned} \tag{31}$$

$$\begin{aligned} &[M^3, Q_\pm] = \frac{1}{2}Q_\pm, \quad [M^3, \tilde{Q}_\pm] = -\frac{1}{2}\tilde{Q}_\pm, \\ &[N^3, Q_\pm] = \mp\frac{1}{2}Q_\mp, \quad [N^3, \tilde{Q}_\pm] = \pm\frac{1}{2}\tilde{Q}_\pm, \end{aligned} \tag{32a}$$

$$\begin{aligned} &[M^3, S_\pm] = -\frac{1}{2}S_\pm, \quad [M^3, \tilde{S}_\pm] = \frac{1}{2}\tilde{S}_\pm, \\ &[N^3, S_\pm] = \mp\frac{1}{2}S_\mp, \quad [N^3, \tilde{S}_\pm] = \mp\frac{1}{2}\tilde{S}_\pm, \end{aligned} \tag{32b}$$

$$\begin{aligned} &[X_1^+,Q_+]_q = 0\,,\ [X_1^+,Q_-] = 0\,, \\ &[X_1^+,\tilde{Q}_+]_{q^{-1}} = Q_+\,,\ [X_1^+,\tilde{Q}_-] = 0\,, \end{aligned} \tag{33a}$$

$$\begin{aligned} &[X_4^+,Q_+] = 0\,,\ [X_4^+,Q_-]_{q^{-1}} = 0\,, \\ &[X_4^+,\tilde{Q}_+] = 0\,,\ [X_4^+,\tilde{Q}_-]_q = -qQ_-\,, \end{aligned} \tag{33b}$$

$$\begin{aligned} &[X_1^-,\tilde{Q}_\pm] = 0\,,\ [X_1^-,Q_-] = 0 \\ &[X_1^-,Q_+] = \tilde{Q}_+ q^{N^3-M^3}\,, \end{aligned} \tag{33c}$$

$$\begin{aligned} &[X_4^-,\tilde{Q}_\pm] = 0\,,\ [X_4^-,Q_+] = 0\,, \\ &[X_4^-,Q_-] = -q^{M^3+N^3}\tilde{Q}_-\,, \end{aligned} \tag{33d}$$

$$\begin{aligned} &[X_1^+,\tilde{S}_\pm] = 0\,,\ [X_1^+,S_-] = 0\,, \\ &[X_1^+,S_+] = -q^{M^3-N^3}\tilde{S}_+\,, \end{aligned} \tag{34a}$$

$$\begin{aligned} &[X_4^+,\tilde{S}_\pm] = 0\,,\ [X_4^+,S_+] = 0\,, \\ &[X_4^+,S_-] = \tilde{S}_- q^{-(M^3+N^3)}\,, \end{aligned} \tag{34b}$$

$$\begin{aligned} &[X_1^-,S_+]_q = 0\,,\ [X_1^-,S_-] = 0\,, \\ &[X_1^-,\tilde{S}_+]_{q^{-1}} = -q^{-1}S_+\,,\ [X_1^-,\tilde{S}_-] = 0\,, \end{aligned} \tag{34c}$$

$$\begin{aligned} &[X_4^-,S_+] = 0\,,\ [X_4^-,S_-]_{q^{-1}} = 0\,, \\ &[X_4^-,\tilde{S}_+] = 0\,,\ [X_4^-,\tilde{S}_-]_q = S_-\,, \end{aligned} \tag{34d}$$

$$\begin{aligned} &[Q_+,P_0+P_3]_q = \tilde{\lambda}\tilde{Q}_+(P_1+iP_2)\,,\ [Q_+,P_0-P_3]_q = 0\,, \\ &[Q_+,P_1+iP_2]_q = 0\,,\ [Q_+,P_1-iP_2] = \tilde{\lambda}\tilde{Q}_+(P_0-P_3)\,, \end{aligned} \tag{35a}$$

$$\begin{aligned} &[P_0+P_3,Q_-]_q = 0\,,\ [P_0-P_3,Q_-] = \tilde{\lambda}\tilde{Q}_-(P_1+iP_2)\,, \\ &[P_1+iP_2,Q_-]_q = 0\,,\ [P_1-iP_2,Q_-] = \tilde{\lambda}\tilde{Q}_-(P_0+P_3)\,, \end{aligned} \tag{35b}$$

$$\begin{aligned} &[\tilde{Q}_+,P_0+P_3]_q = 0\,,\ [\tilde{Q}_+,P_0-P_3] = 0\,, \\ &[\tilde{Q}_+,P_1+iP_2] = 0\,,\ [\tilde{Q}_+,P_1-iP_2]_q = 0\,, \end{aligned} \tag{35c}$$

$$\begin{aligned} &[\tilde{Q}_-,P_0+P_3] = 0\,,\ [P_0-P_3,\tilde{Q}_-]_q = 0\,, \\ &[P_1-iP_2,\tilde{Q}_-]_q = 0\,,\ [\tilde{Q}_-,P_1+iP_2] = 0\,, \end{aligned} \tag{35d}$$

$$\begin{aligned} &[S_+,P_0+P_3] = -i\tilde{\lambda}Q_-(M^-+N^-)q^{(M^3-N^3-D+iE)/2}\,, \\ &[S_+,P_0-P_] = 2i\tilde{Q}_- q^{(N^3-M^3-D+iE)/2}\,, \\ &[S_+,P_1+iP_2] = 2iQ_- q^{(N^3-M^3-D+iE)/2}\,, \\ &[S_+,P_1-iP_2] = -i\tilde{\lambda}\tilde{Q}_-(M^-+N^-)q^{(M^3-N^3-D+iE)/2}\,, \end{aligned} \tag{36a}$$

$$\begin{aligned} &[P_0+P_3,S_-] = 2iq^{(D+N^3+M^3+iE)/2}\tilde{Q}_+\,, \\ &[P_0-P_3,S_-] = -i\tilde{\lambda}q^{(D-N^3-M^3+iE)/2}Q_+(M^--N^-)\,, \\ &[P_1+iP_2,S_-] = 2iq^{(D+N^3+M^3+iE)/2}Q_+\,, \\ &[P_1-iP_2,S_-] = -i\tilde{\lambda}q^{(D-N^3-M^3+iE)/2}\tilde{Q}_+(M^--N^-)\,, \end{aligned} \tag{36b}$$

$$\begin{aligned} &[\tilde{S}_+,P_0+P_3] = 2iQ_- q^{(M^3-D-N^3+iE)/2}\,,\ [\tilde{S}_+,P_0-P_3] = 0\,, \\ &[\tilde{S}_+,P_1+iP_2] = 0\,,\ [\tilde{S}_+,P_1-iP_2] = 2i\tilde{Q}_- q^{(M^3-D-N^3+iE)/2}\,, \end{aligned} \tag{36c}$$

$$\begin{aligned} &[\tilde{S}_-,P_0+P_3] = 0\,,\ r[P_0-P_3,\tilde{S}_-] = 2iq^{(D-N^3-M^3+iE)/2}Q_+\,, \\ &[P_1-iP_2,\tilde{S}_-] = 0\,,\ [\tilde{S}_-,P_1+iP_2] = -2iq^{(D-N^3-M^3+iE)/2}\tilde{Q}_+\,, \end{aligned} \tag{36d}$$

$$\begin{aligned} &[Q_+,K_0+K_3] = 2iq^{(M^3+D-N^3-iE)/2}\tilde{S}_-\,, \\ &[Q_+,K_0-K_3] = i\tilde{\lambda}q^{(D-M^3+N^3-iE)/2}(M^++N^+)S_-\,, \\ &[Q_+,K_1+iK_2] = -i\tilde{\lambda}q^{(D-M^3+N^3-iE)/2}(M^++N^+)\tilde{S}_-\,, \\ &[Q_+,K_1-iK_2] = -2iq^{(M^3-N^3+D-iE)/2}S_-\,, \end{aligned} \tag{37a}$$

$$\begin{aligned} &[K_0+K_3,Q_-] = -i\tilde{\lambda}(M^3-N^3)S_+q^{(N^3+M^3-D-iE)/2}\,, \\ &[K_0-K_3,Q_-] = 2i\tilde{S}_+q^{-(D+N^3+M^3+iE)/2}\,, \\ &[K_1+iK_2,Q_-] = -i\tilde{\lambda}(M^+-N^+)\tilde{S}_+q^{(N^3+M^3-D-iE)/2}\,, \\ &[K_1-iK_2,Q_-] = -2i\tilde{S}_+q^{-(D+N^3+M^3+iE)/2}\,, \end{aligned} \tag{37b}$$

$$\begin{aligned} &[\tilde{Q}_+,K_0+K_3] = 0\,,\ [\tilde{Q}_+,K_0-K_3] = 2iq^{(D-M^3+N^3+iE)/2}S_-\,, \\ &[\tilde{Q}_+,K_1+iK_2] = -2iq^{(D-M^3+N^3-iE)/2}\tilde{S}_-\,,\ [\tilde{Q}_+,K_1-iK_2] = 0\,, \end{aligned} \tag{37c}$$

$$\begin{aligned} &[\tilde{Q}_-,K_0+K_3] = -2iS_+q^{(N^3+M^3-D-iE)/2}\,,\ [K_0-K_3,\tilde{Q}_-] = 0\,, \\ &[K_1-iK_2,\tilde{Q}_-] = 0\,,\ [\tilde{Q}_-,K_1+iK_2] = 2i\tilde{S}_+q^{(N^3+M^3-D-iE)/2}\,, \end{aligned} \tag{37d}$$

$$\begin{aligned} &[S_+,K_0+K_3]_q = 0\,,\ [S_+,K_0-K_3] = -\tilde{\lambda}(K_1-iK_2)\tilde{S}_+\,, \\ &[S_+,K_1+iK_2] = -\tilde{\lambda}(K_0+K_3)\tilde{S}_+\,,\ [S_+,K_1-iK_2]_q = 0\,, \end{aligned} \tag{38a}$$

$$\begin{aligned} &[K_0+K_3,S_-] = -\tilde{\lambda}(K_1-iK_2)\tilde{S}_-\,,\ [K_0-K_3,S_-]_q = 0\,, \\ &[K_1+iK_2,S_-] = -\tilde{\lambda}(K_0-K_3)\tilde{S}_-\,,\ [K_1-iK_2,S_-]_q = 0\,, \end{aligned} \tag{38b}$$

$$\begin{aligned} &[\tilde{S}_+,K_0+K_3] = 0\,,\ [\tilde{S}_+,K_0-K_3]_q = 0\,, \\ &[\tilde{S}_+,K_1+iK_2]_q = 0\,,\ [\tilde{S}_+,K_1-iK_2] = 0\,, \end{aligned} \tag{38c}$$

$$\begin{aligned} &[\tilde{S}_-,K_0+K_3]_q = 0\,,\ [K_0-K_3,\tilde{S}_-] = 0\,, \\ &[K_1-iK_2,\tilde{S}_-] = 0\,,\ [\tilde{S}_-,K_1+iK_2]_q = 0\,, \end{aligned} \tag{38d}$$

$$
\begin{aligned}
[Q_+, S_+] &= -4(M^3 + D - N^3 - iE)_q \,, \\
[Q_-, S_-] &= 4(D + N^3 + M^3 + iE)_q \,, \\
[\tilde{Q}_+, \tilde{S}_+] &= -4(D - M^3 + N^3 - iE)_q \,, \\
[\tilde{Q}_-, \tilde{S}_-] &= 4(D - N^3 - M^3 + iE)_q \,, \\
[Q_+, S_-] &= 0 \,, \; [\tilde{Q}_+, \tilde{S}_-] = 0 \,, \\
[Q_+, \tilde{S}_+] &= -2\, q^{(D-M^3+N^3-iE)/2} (M^+ + N^+) \,, \\
[Q_-, \tilde{S}_-] &= -2(M^+ - N^+) q^{(N^3+M^3-D-iE)/2} \,, \\
[Q_+, \tilde{S}_-] &= 0 \,, \; [Q_-, \tilde{S}_+] = 0 \,,
\end{aligned}
\tag{39}
$$

The coproducts of the 24 generators of q–deformed conformal superalgebra are given by the formulae:

$$
\begin{aligned}
\delta(M^+ + N^+) &= (M^+ + N^+) \otimes q^{(M^3-N^3)/2} + q^{-(M^3-N^3)/2} \otimes (M^+ + N^+) \\
\delta(M^- + N^-) &= (M^- + N^-) \otimes q^{(M^3-N^3)/2} + q^{-(M^3-N^3)/2} \otimes (M^- + N^-) \\
\delta(M^+ - N^+) &= (M^+ - N^+) \otimes q^{(M^3+N^3)/2} + q^{-(M^3+N^3)/2} \otimes (M^+ - N^+) \\
\delta(M^- - N^-) &= (M^- - N^-) \otimes q^{(M^3+N^3)/2} + q^{-(M^3+N^3)/2} \otimes (M^- - N^-)
\end{aligned}
\tag{40}
$$

$$
\begin{aligned}
\delta(M^3) &= M^3 \otimes 1 + 1 \otimes M^3 \\
\delta(N^3) &= N^3 \otimes 1 + 1 \otimes N^3
\end{aligned}
$$

$$
\begin{aligned}
\delta(P_0 - P_3) = {} & (P_0 - P_3) \otimes q^{(D-N^3)/2} + q^{-(D-N^3)/2} \otimes (P_0 - P_3) \\
& + \frac{1}{2}\bar{\lambda} q^{-1/2} (P_1 - iP_2) q^{(N^3-M^3)/2} \otimes (M^+ + N^+) q^{(D-M^3)/2} \\
& + \frac{i}{2}\bar{\lambda} q^{1/2} \tilde{Q}_- q^{-(D+M^3-iE-N^3)/4} \otimes Q_+ q^{(D-M^3+iE-N^3)/4}
\end{aligned}
\tag{41a}
$$

$$
\begin{aligned}
\delta(P_3 + P_0) = {} & (P_3 + P_0) \otimes q^{(D+N^3)/2} + q^{-(D+N^3)/2} \otimes (P_3 + P_0) \\
& + \frac{1}{2}\bar{\lambda} q^{-1/2} (M^+ - N^+) q^{(-D+M^3)/2} \otimes (P_1 - iP_2) q^{(M^3+N^3)/2} \\
& + \frac{i}{2}\bar{\lambda} q^{1/2} Q_- q^{-(D-M^3+N^3-iE)/4} \otimes \tilde{Q}_+ q^{(D+M^3+N^3+iE)/4}
\end{aligned}
\tag{41b}
$$

$$
\begin{aligned}
\delta(P_1 + iP_2) = {} & (P_1 + iP_2) \otimes q^{(D+M^3)/2} + q^{-(D+M^3)/2} \otimes (P_1 + iP_2) \\
& + \frac{1}{2}\bar{\lambda} q^{-1/2} (P_3 + P_0) q^{(N^3-M^3)/2} \otimes (M^+ + N^+) q^{(D+N^3)/2} \\
& + \frac{1}{2}\bar{\lambda} q^{-1/2} (M^+ - N^+) q^{-(D-N^3)/2} \otimes (P_0 - P_3) q^{(M^3+N^3)/2} \\
& + \frac{i}{2}\bar{\lambda} q^{1/2} Q_- q^{-(D+M^3-N^3-iE)/4} \otimes Q_+ q^{(D+M^3+N^3+iE)/4}
\end{aligned}
\tag{41c}
$$

$$
\begin{aligned}
\delta(P_1 - iP_2) = {} & (P_1 - iP_2) \otimes q^{(D-M^3)/2} + q^{-(D-M^3)/2} \otimes (P_1 - iP_2) \\
& + \frac{i}{2}\bar{\lambda} q^{1/2} \tilde{Q}_- q^{(-D+M^3-N^3+iE)/4} \otimes \tilde{Q}_+ q^{(D-M^3+iE-N^3))/2}
\end{aligned}
\tag{41d}
$$

$$
\begin{aligned}
\delta(K_0 + K_3) = {} & (K_0 + K_3) \otimes q^{(D-N^3)/2} + q^{-(D-N^3)/2} \otimes (K_0 + K_3) \\
& - \frac{1}{2} q^{-1/2} \bar{\lambda} q^{-(D-M^3)/2} (M^- + N^-) \otimes q^{(M^3-N^3)/2} (K_1 + iK_2) \\
& + \frac{i}{2} q^{-1/2} \bar{\lambda} q^{-(D-M^3-N^3+iE)/4} S_+ \otimes q^{(D+M^3-N^3-iE)/4} \tilde{S}_-
\end{aligned}
\tag{42a}
$$

$$
\begin{aligned}
\delta(K_3 - K_0) = {} & q^{-(D+N^3)/2} \otimes (K_3 - K_0) + (K_3 - K_0) \otimes q^{(D+N^3)/2} \\
& - \frac{1}{2}\bar{\lambda} q^{-(M^3+N^3)/2} (K_1 + iK_2) \otimes q^{(D-M^3)/2} (M^- - N^-) \\
& + \frac{i}{2} q^{-1/2} \bar{\lambda} q^{-(D+M^3+N^3+iE)/4} \tilde{S}_+ \otimes q^{(D+N^3-M^3-iE)/4} S_-
\end{aligned}
\tag{42b}
$$

$$
\begin{aligned}
\delta(K_1 + iK_2) = {} & (K_1 + iK_2) \otimes q^{(D-M^3)/2} + q^{-(D-M^3)/2} \otimes (K_1 + iK_2) \\
& + \frac{i}{2} q^{-1/2} \bar{\lambda} q^{-(D-M^3-N^3+iE)/4} \tilde{S}_+ \otimes q^{(D+N^3-M^3-iE)/4} \tilde{S}_-
\end{aligned}
\tag{42c}
$$

$$
\begin{aligned}
\delta(K_1 - iK_2) = {} & (K_1 - iK_2) \otimes q^{D+M^3} + q^{-(D+M^3)} \otimes (K_1 - iK_2) \\
& - \frac{1}{2} q^{1/2} \bar{\lambda} q^{-(D+N^3)} (M^- + N^-) \otimes q^{(M^3-N^3)/2} (K_3 - K_0) \\
& + \frac{1}{2} q^{1/2} \bar{\lambda} q^{-(M^3+N^3)/2} (K_0 + K_3) \otimes q^{(D-N^3)/2} (M^- - N^-) \\
& + \frac{i}{2} q^{-1/2} \bar{\lambda} q^{-(D+M^3+N^3+iE)/4} S_+ \otimes q^{(D+M^3-N^3-iE)/4} S_-
\end{aligned}
\tag{42d}
$$

$$
\delta(D) = D \otimes 1 + 1 \otimes D \qquad \delta(E) = E \otimes 1 + 1 \otimes E \tag{43}
$$

$$
\begin{aligned}
\delta(Q_+) &= Q_+ \otimes q^{(D+M^3-N^3-iE)/4} + q^{-(D+M^3-N^3-iE)/4} \otimes Q_+ \\
&\quad + \frac{1}{2}\bar{\lambda} q^{-1/2} \tilde{Q}_+ q^{(N^3-M^3)/2} \otimes (M^+ + N^+) q^{(D-M^3+N^3-iE)/4} \\
\delta(Q_-) &= Q_- \otimes q^{(D+M^3+N^3+iE)/4} + q^{-(D+M^3+N^3+iE)/4} \otimes Q_- \\
&\quad - \frac{1}{2}\bar{\lambda} q^{-1/2} (M^+ - N^+) q^{(-D+M^3-iE+N^3)/4} \otimes \tilde{Q}_- q^{(M^3+N^3)/2} \\
\delta(\tilde{Q}_+) &= \tilde{Q}_+ \otimes q^{(D-M^3+N^3-iE)/4} + q^{(-D+M^3-N^3+iE)/4} \otimes \tilde{Q}_+ \\
\delta(\tilde{Q}_-) &= \tilde{Q}_- \otimes q^{(D-M^3+iE-N^3)/4} + q^{-(D-M^3+iE-N^3)/4} \otimes \tilde{Q}_-
\end{aligned}
\tag{44}
$$

$$
\begin{aligned}
\delta(S_+) &= S_+ \otimes q^{D+M^3-N^3-iE)/4} + q^{-(D+M^3-N^3-iE)/4} \otimes S_+ \\
&\quad - \frac{1}{2} q^{1/2} \bar{\lambda} q^{-(D+N^3-M^3-iE)/4} (M^- + N^-) \otimes q^{(M^3-N^3)/2} \tilde{S}_+ \\
\delta(S_-) &= S_- \otimes q^{(D+M^3+N^3+iE)/4} + q^{-(D+M^3+N^3+iE)/4} \otimes S_- \\
&\quad - \frac{1}{2} q^{1/2} \bar{\lambda} q^{-(M^3+N^3)/2} \bar{S}_- \otimes q^{(D-M^3-N^3+iE)/4} (M^- - N^-) \\
\delta(\bar{S}_+) &= \bar{S}_+ \otimes q^{(D-M^3+N^3-iE)/4} + q^{-(D-M^3+N^3-iE)/4} \otimes \bar{S}_+ \\
\delta(\bar{S}_-) &= \bar{S}_- \otimes q^{(D-M^3+iE-N^3)/4} + q^{-(D-M^3-N^3+iE)/4} \otimes \bar{S}_-
\end{aligned}
\tag{45}
$$

$$
\begin{aligned}
\gamma(M^+ + N^+) &= -q(M^+ + N^+) \\
\gamma(M^+ - N^+) &= -q(M^+ - N^+) \\
\gamma(M^- + N^-) &= -q^{-1}(M^- + N^-) \\
\gamma(M^- - N^-) &= -q^{-1}(M^- - N^-) \\
\gamma(M^3) &= -M^3 \\
\gamma(N^3) &= -N^3
\end{aligned}
\tag{46}
$$

$$
\begin{aligned}
\gamma(P_0 + P_3) &= -q^3(P_0 + P_3) + \frac{1}{2} q^3 \bar{\lambda}(M^+ - N^+)(P_1 - iP_2) \\
&\quad - \frac{i}{2} q^3 \bar{\lambda} Q_- \bar{Q}_+ + \frac{i}{4} q^3 \bar{\lambda}^2 (M^+ - N^+) \bar{Q}_- \bar{Q}_+
\end{aligned}
\tag{47a}
$$

$$
\begin{aligned}
\gamma(P_0 - P_3) &= -q^3(P_0 - P_3) + \frac{1}{2} q^3 \bar{\lambda}(P_1 - iP_2)(M^+ + N^+) \\
&\quad - \frac{i}{2} q^3 \bar{\lambda} \bar{Q}_- Q_+ \\
&\quad + \frac{i}{4} q^3 \bar{\lambda}^2 \bar{Q}_- \bar{Q}_+ (M^+ + N^+)
\end{aligned}
\tag{47b}
$$

$$
\begin{aligned}
\gamma(P_1 + iP_2) &= -q^3(P_1 + iP_2) + \frac{1}{2} q^3 \bar{\lambda}(P_0 + P_3)(M^+ + N^+) \\
&\quad + \frac{1}{2} q^3 \bar{\lambda}(M^+ - N^+)(P_0 - P_3) - \frac{i}{2} q^3 \bar{\lambda} Q_- Q_+ \\
&\quad + \frac{i}{4} q^3 \bar{\lambda}^2 Q_- \bar{Q}_+ (M^+ + N^+) \\
&\quad + \frac{i}{4} q^3 \bar{\lambda}^2 (M^+ - N^+) \bar{Q}_- Q_+ \\
&\quad - \frac{i}{8} q^3 \bar{\lambda}^3 (M^+ - N^+) \bar{Q}_- \bar{Q}_+ (M^+ + N^+)
\end{aligned}
\tag{47c}
$$

$$
\gamma(P_1 - iP_2) = -q^3(P_1 - iP_2) - \frac{i}{2} q^3 \bar{\lambda} \bar{Q}_- \bar{Q}_+
\tag{47d}
$$

$$
\begin{aligned}
\gamma(K_0 + K_3) &= -q^{-3}(K_0 + K_3) + \frac{1}{2} q^{-3} \bar{\lambda}(M^- + N^-)(K_1 + iK_2) \\
&\quad - \frac{1}{2} q^{-3} \bar{\lambda} S_+ \bar{S}_- - \frac{i}{4} q^{-3} \bar{\lambda}^2 (M^- + N^-) \bar{S}_+ \bar{S}_-
\end{aligned}
\tag{48a}
$$

$$
\begin{aligned}
\gamma(K_0 - K_3) &= -q^{-3}(K_0 - K_3) + \frac{1}{2} q^{-3} \bar{\lambda}(K_1 + iK_2)(M^- - N^-) \\
&\quad - \frac{i}{2} q^{-3} \bar{\lambda} \bar{S}_+ S_- - \frac{1}{4} q^{-3} \bar{\lambda}^2 \bar{S}_+ \bar{S}_- (M^- - N^-)
\end{aligned}
\tag{48b}
$$

$$
\gamma(K_1 + iK_2) = -q^{-3}(K_1 + iK_2) + \frac{i}{2} q^{-3} \bar{\lambda} \bar{S}_+ \bar{S}_-
\tag{48c}
$$

$$
\begin{aligned}
\gamma(K_1 - iK_2) &= -q^{-3}(K_1 - iK_2) - \frac{1}{2} q^{-3} \bar{\lambda}(M^- + N^-)(K_3 - K_0) \\
&\quad + \frac{1}{2} q^{-3} \bar{\lambda}(K_0 + K_3)(M^- - N^-) + \frac{i}{2} q^{-3} \bar{\lambda} S_+ S_- \\
&\quad + \frac{i}{4} q^{-3} \bar{\lambda}^2 (M^- + N^-) \bar{S}_+ S_- + \frac{i}{4} q^{-3} \bar{\lambda}^2 S_+ \bar{S}_- (M^- - N^-) \\
&\quad + \frac{i}{8} q^{-3} \bar{\lambda}^3 (M^- - N^-) \bar{S}_+ \bar{S}_- (M^- - N^-)
\end{aligned}
\tag{48d}
$$

$$
\begin{aligned}
\gamma(Q_+) &= -qQ_+ + \frac{1}{2} q \bar{\lambda} \bar{Q}_+ (M^+ + N^+) \\
\gamma(Q_-) &= -qQ_- + \frac{1}{2} q \bar{\lambda} (M^+ - N^+) \bar{Q}_- \\
\gamma(\bar{Q}^\pm) &= -q \bar{Q}^\pm
\end{aligned}
\tag{49}
$$

$$
\begin{aligned}
\gamma(S_+) &= -q^{-1} S_+ - \frac{1}{2} q^{-1} \bar{\lambda}(M^- + N^-) \bar{S}_+ \\
\gamma(S_-) &= -q^{-1} S_- - \frac{1}{2} q^{-1} \bar{\lambda} \bar{S}_- (M^- - N^-) \\
\gamma(\bar{S}_\pm) &= -q^{-1} \bar{S}_\pm
\end{aligned}
\tag{50}
$$

$$
\gamma(D) = -D \qquad \gamma(E) = -E
\tag{51}
$$

4. $U_q(su(2,2/N))$

The structure of the N-extended conformal superalgebra $U_q(su(2,2/N))$ was announced in [11]. We have the analogue of (3a) :

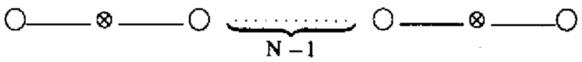
$$
U_q(su(2,2/N)) \supset U_q(\tilde{\mathcal{P}}^S) \supset U_q(\mathcal{M}^S \oplus \mathcal{A}) \supset U_q(\mathcal{M}^S) \supset U_q(\mathcal{M}) \,,
\tag{24a}
$$

where the maximal parabolic subalgebra or N–extended super-Weyl algebra $\tilde{\mathcal{P}}^S = \mathcal{M}^S \oplus \mathcal{A} \oplus \tilde{\mathcal{N}}^S$ is $(N^2 + 4N + 11)$-dimensional, $\mathcal{M}^S = \mathcal{M}^S_{(0)} \cong \mathcal{M} \oplus u(N)$, (compare with (14b)), $\mathcal{A}^S = \mathcal{A}$,

$\mathcal{N}^S_{(0)} = \mathcal{N}$, $\dim \mathcal{N}^S_{(1)} = 4N$. In terms of the N-extended super-Poincaré algebra $\mathcal{SP}$ with ten Poincaré and $4N$ supercharge generators one has

$$\tilde{\mathcal{P}}^S = \mathcal{SP} \oplus \mathcal{A} \oplus u(N) \tag{24b}$$

In order to describe the superalgebra $U_q(su(2,2/N))$ in more detail we use complexification $U_q(sl(4/N;\mathbb{C}))$ with the following choice of the Dynkin diagram:



(25)

The choice (25) of the Dynkin diagram leads to a Drinfeld–Jimbo q–deformation which allows all involutions introduced here for $N = 1$ and also easy generalizations of the other formulae.

Let us mention finally that it would be interesting to classify the (anti)involutions permitting to introduce the sequences (3a) and (25) as the sequences of real (self–conjugate) Hopf sub(super)algebras.

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