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ABSTRACT

We use the method of analytic continuation of the equation of motion including the self-fields to evaluate the radiation reaction for a classical relativistic spinning point particle in interaction with scalar, tensor and linearized gravitational fields in flat spacetime. In the limit these equations reduce to those of spinless particles. We also show the renormalizability of these theories.

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1 Introduction

There are different theories of the relativistic spinning particle[1]. But, to be a complete theory these different formulations must pass also the test of coupling to external fields, be renormalizable and contain radiation reaction terms in the equations of motion.

In this work we consider a recently proposed classical model for the spin in terms of c-number four component spinors [1] which has proved to be very valuable. Even without interaction this model exhibits many remarkable properties. Namely, in this theory the free particle performs an oscillatory motion with frequency $\omega = 2m$(m is the mass of the particle), called the Zitterbewegung, explaining the origin of the spin. The trajectory of the particle is a helix around a center of mass that moves like a relativistic spinless point particle.

In its simplest form, the free particle action in terms of the worldline invariant time parameter τ, not necessarily the proper time of the particle, reads

$$W = \int d\tau \{ i\lambda \bar{z}\dot{z} + p_\mu(\dot{x}^\mu - \bar{z}\gamma^\mu z)\} \tag{1}$$

Here the dot means derivative with respect to the invariant time τ. The variables $z(\tau)$ and $\bar{z}(\tau) = z^+(\tau)\gamma^0$ form a pair of canonicaly conjugate four component complex spinor variables that describe the internal spin degree of freedom. These, together with the position $x_\mu(\tau)$ and the momentum $p_\mu(\tau)$, form a complete set of dynamical variables. The constant λ is one of the parameters of the model and has the unit of action. Although a term with mass (another fundamental constant) does not appear in this expression, it still refers to a massive particle because the mass enters as a constant of the equations of motion, as it should. Otherwise stated, the hamiltonian $H = p_\mu \bar{z}\gamma^\mu z$ is a constant of motion and thus can be set equal to the mass of the particle. Finally, p_μ, the canonical momentum in the dynamical equations of motion (which is independent of $\dot{x}_\mu$), behaves as a Lagrange multiplier imposing the nonholonomic constraint $\dot{x}_\mu = \bar{z}\gamma_\mu z$.

Furthermore, the similarity between this classical system and its quantum theory is another remarkable feature of this model. The interaction with an electromagnetic field is given by the minimal coupling principle, just like the Dirac's quantum electron. There is also an intuitive concept of antiparticles already at a classical level. Furthermore, upon quantisation, via canonical [1], Schrodinger [2] or path integral [3] methods, one obtains precisely the Dirac equation for a relativistic quantum electron.

When coupled to an external electromagnetic field and to its self-field the renormalized equation of motion for this spinning particle [4] is

$$\dot{\pi}_\mu = eF^{ext}_{\mu\nu}v^\nu + e^2\tilde{g}_{\mu\nu}[\frac{2}{3}\frac{\ddot{v}^\nu}{v^2} - \frac{9}{4}\frac{(v\cdot\dot{v})\dot{v}^\nu}{v^4}] \tag{2}$$

with $\pi_\mu = p_\mu - eA_\mu$, $\tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{v_\nu v_\mu}{v^2}$ and $v_\mu \equiv \dot{x}_\mu$. In the spinless limit, i.e. $v^2 \rightarrow 1$, $v \cdot \dot{v} \rightarrow 0$, $\dot{\pi}_\mu \rightarrow m\ddot{x}_\mu$, eq.(2) goes over to the well known relativistic Lorentz-Dirac

[1]The literature on the theory of spinning particles is very vast. We will not attempt to list all the references.

equation with radiation reaction

$$m\ddot{x}_\mu = eF_{\mu\nu}\dot{x}^\nu + \frac{2}{3}e^2[\dddot{x}_\mu + (\ddot{x})^2\dot{x}_\mu], \tag{3}$$

where m is the renormalized mass and $F_{\mu\nu}$ is an external electromagnetic field. These are, indeed, important expressions in the classical theory of charged particles. The reason being that they are non-perturbative. Therefore, with the proviso of using the proper interpretation of the renormalization procedure [5], they are capable of describing every radiative process of charged particles in closed form.

Rather than using the original method of Dirac [6], in which the combination of retarded and advanced fields, $1/2(F_{ret} - F_{adv})$, evaluated at the position of the particle gives a finite radiation reaction term and the combination, $1/2(F_{ret} + F_{adv})$, (which is infinite at the position of the particle) is introduced into the inertial mass term which is then renormalized to the experimental mass, equation (2) was derived using exclusively retarded fields by a method of analytic continuation of the equations of motion [7]. This method not only permits to obtain radiation reaction terms in the most concise way but also to perform the renormalization of mass. This has already been shown for a particle in an electromagnetic field and for a spinless particle in interaction with a scalar, tensor and linearized gravitational fields [8].

In the same way that Lorentz-Dirac equation was generalized to include spin, in the present work we use the Zitterbewegung model for the spin and the method of analytic continuation to evaluate radiation reaction terms in the equation of motion for spinning particles in interaction with scalar, tensor and linearized gravitational fields in flat spacetime. In the spinless limit the equations reduce to those obtained by Barut and Villarroel. The renormalization of these theories is also proved by showing that the infinite term can properly be absorbed in the invariant mass term.

2 The Scalar Field

We start by generalizing the motion of a spinless particle [9] and considering a single point particle with the Zitterbewegung motion interacting with an external scalar field. The particle radiates and the radiation emitted in turn affects the motion of the particle which of course is the radiation reaction effect. The phase space action, in units $c = 1$ and $\lambda = 1$, for particle and field is

$$W = \int d\tau\{i\bar{z}\dot{z} + p_\mu(\dot{x}^\mu - \bar{z}\gamma^\mu z)\} + g\int dx d\tau\ \phi(x)\delta(x - x(\tau)) - \frac{1}{2}\int dx\ \phi_{,\mu}\phi^{,\mu} \tag{4}$$

Here g is the coupling constant to the field ϕ, which is any external field including the self-field of the particle. Variation of this action results in the equations of motion

$$\dot{x}^\mu = \bar{z}\gamma^\mu z, \quad \dot{p}^\mu = g\phi^{,\mu} \tag{5}$$
$$i\dot{z} = p_\mu\gamma^\mu z, \quad -i\dot{\bar{z}} = \bar{z}p_\mu\gamma^\mu$$

for the particle and

$$\Box\phi = -g\int d\tau\ \delta(x - x(\tau)) \tag{6}$$

for the field ϕ. Instead of working with these complex spinor equations, we shall next formulate the equations of motion using a set of real dynamical variables. These are, besides the position x_μ and the momentum p_μ, the velocity and the spin variables defined by: $v_\mu \equiv \bar{z}\gamma_\mu z$, $S_{\mu\nu} \equiv -\frac{i}{4}\bar{z}[\gamma_\mu, \gamma_\nu]z$, respectively. Hence the new set of dynamical equations is

$$\dot{x}_\mu = v_\mu, \quad \dot{v}_\mu = -4S^{\rho\mu}p_\rho$$
$$\dot{S}_{\mu\nu} = p_\nu v_\mu - p_\mu v_\nu, \quad \dot{p}_\mu = g\phi_{,\mu}$$

Equation (6) can be integrated using the retarded Green function $D_{ret}(X - x(\tau))$ as follows

$$\phi = \phi^{ext}(X) - g\int dx' d\tau D_{ret}(X - x')\delta(x' - x(\tau)) \tag{7}$$

or, $\phi^{self} = -\{g/R\}$, where

$$R = (X - x(\tau))^\sigma \dot{x}_\sigma(\tau) \tag{8}$$

is the relativistic relative distance evaluated at τ_0 and X the observation point. This field is the retarded scalar self-field, and like the Lienard-Wiechert potential in classical electrodynamics, is evaluated at τ_0 i.e. the root of $(X - x(\tau))^2$ with $X^0 > x^0(\tau_0)$. From this solution we calculate the derivative

$$\phi_{,\nu} = \frac{g}{R}\{\dot{x}_\nu(\tau) + (X - x(\tau))_\nu \frac{(Q - \dot{x}^2(\tau))}{R}\} \tag{9}$$

where

$$Q = (X - x(\tau))^\sigma \ddot{x}_\sigma(\tau) \tag{10}$$

Now, in the method of analytic continuation we first need to distinguish the field point $X = x(\tau + u)$ from the source point $x(\tau)$. By letting $u \to 0$, we then approach the position of the particle. Noting that $v_\mu v^\mu$ is different from one for the spinning particle (*τ can be interpreted as the proper time of its "center of mass"* [4]), we compute first the expansions

$$\begin{aligned}
X_\mu \equiv x_\mu(\tau + u) &= x_\mu + u\dot{x}_\mu + \frac{1}{2}u^2\ddot{x}_\mu + \frac{1}{6}u^3\dddot{x}_\mu + ... \\
\dot{x}_\mu(\tau + u) &= \dot{x}_\mu + u\ddot{x}_\mu + \frac{1}{2}u^2\dddot{x}_\mu + ... \\
\ddot{x}_\mu(\tau + u) &= \ddot{x}_\mu + u\dddot{x}_\mu + ... \\
R &= u\dot{x}^2 + \frac{1}{2}u^2\dot{x}\cdot\ddot{x} + \frac{1}{6}u^3\dot{x}\cdot\dddot{x} + ... \\
Q &= u\dot{x}\cdot\ddot{x} + \frac{1}{2}u^2\ddot{x}^2 + \frac{1}{6}u^3\ddot{x}\cdot\dddot{x} + ...
\end{aligned}$$

On the right hand side all quantities $\dot{x}, \ddot{x}...$ refer to the time τ. Substitution of these expansions into the equation of motion

$$\dot{p}_\mu(\tau + u) = g\phi^{ext}_{,\mu}(X = x(\tau + u)) + g\phi_{,\mu}(X = x(\tau + u), x(\tau)) \tag{11}$$

produces a term proportional to $\frac{1}{u}$, a constant term and terms of higher order in u. More explicitly

$$\dot{p}_\mu = g\phi^{ext}_{,\mu} + \frac{g^2}{2u\dot{x}^6}\{3\dot{x}_\mu(\dot{x}\cdot\ddot{x}) - \ddot{x}_\mu\dot{x}^2\} + g^2[\frac{\dot{x}_\mu\ddot{x}^2}{2\dot{x}^6} + \frac{\dot{x}_\mu(\dot{x}\cdot\dddot{x})}{6\dot{x}^6} + \frac{5\ddot{x}_\mu(\dot{x}\cdot\ddot{x})}{4\dot{x}^6} \tag{12}$$

$$-\frac{\dddot{x}_\mu}{6\dot{x}^4} - \frac{9\dot{x}_\mu(\dot{x}\cdot\ddot{x})^2}{4\dot{x}^8}] + O(u)$$

If we first transform the infinite term in this equation to the time $\tau + u$, bring the infinite part to the left hand side and take the limit $u \to 0$, we obtain our final equation for the scalar case

$$\dot{p}^{ren}_\mu = g^2[\frac{1}{3}\frac{\dddot{x}_\mu}{\dot{x}^4} - \frac{\dot{x}_\mu\ddot{x}^2}{\dot{x}^6} - \frac{4}{3}\frac{\dot{x}_\mu(\dot{x}\cdot\dddot{x})}{\dot{x}^6} - \frac{9(\dot{x}\cdot\ddot{x})}{4\dot{x}^8}(\ddot{x}_\mu\dot{x}^2 + \dot{x}_\mu(\dot{x}\cdot\ddot{x}))] \tag{13}$$

where we have defined $\dot{p}^{ren}_\mu$ as

$$\dot{p}^{ren}_\mu \equiv \dot{p}_\mu - \frac{g^2}{2u\dot{x}^6}\{3\dot{x}_\mu(\dot{x}\cdot\ddot{x}) - \ddot{x}_\mu\dot{x}^2\} \tag{14}$$

Note that in the spinless limit, $\dot{x}^2 = 1$, $\dot{p}^{ren}_\mu \to m_{ren}\ddot{x}_\mu$,we get

$$m_{ren}\ddot{x}_\mu = \frac{g^2}{3}(\dddot{x}_\mu + \ddot{x}^2\dot{x}_\mu) \tag{15}$$

which is the same equation as obtained by Barut and Villarroel [8].

Now let us show that the infinite term renormalizes the mass. For this purpose we make use of the formulas $H = v_\mu p^\mu - g\phi$ and $\dot{S}_{\mu\nu} = v_\mu p_\nu - v_\nu p_\mu$. It is easy to verify that the "hamiltonian", $H = p_\mu\bar{z}\gamma^\mu z - g\phi$ is a constant of the motion. It will be taken to be the mass of the particle. Thus we have the indentity

$$p_\mu = \frac{1}{v^2}\{v_\mu H + v^\alpha\dot{S}_{\alpha\mu} + gv_\mu\phi\} \tag{16}$$

This is because

$$\frac{v_\alpha\dot{S}^{\alpha\mu}}{v^2} = \frac{v_\alpha(v^\alpha p^\mu - v^\mu p^\alpha)}{v^2} = \frac{v^2p^\mu - v^\mu(v\cdot p)}{v^2} \tag{17}$$

$$= p^\mu - v^\mu\frac{(H + g\phi)}{v^2}$$

which is the same as (16), as asserted. Moreover, from $\dot{p}_\mu = \frac{d}{d\tau}(\frac{\sqrt{v^2}}{\sqrt{v^2}}p_\mu)$ or, using (16), we have

$$\dot{p}_\mu = \frac{1}{\sqrt{v^2}}\frac{d}{d\tau}(\frac{v_\mu H}{\sqrt{v^2}}) - \frac{\dot{v}\cdot v}{v^2}p_\mu + \frac{1}{\sqrt{v^2}}\frac{d}{d\tau}(\frac{gv_\mu\phi + v^\alpha\dot{S}_{\alpha\mu}}{\sqrt{v^2}}) \tag{18}$$

Since

$$\frac{1}{2u\dot{x}^6}\{3\dot{x}_\mu(\dot{x}\cdot\ddot{x}) - \ddot{x}_\mu\dot{x}^2\} = -\frac{1}{u\sqrt{\dot{x}^2}}\frac{d}{d\tau}[\frac{\dot{x}_\mu}{(\sqrt{\dot{x}^2})^3}] \tag{19}$$

we see that the renormalization term, i.e. the second term on the right hand side of (14), can be absorbed into the mass by defining

$$H_{ren} = H + \frac{g^2}{2uv^2} \tag{20}$$

Hence we have in effect a mass renormalization only.

3 The Tensor Field

In this case, the starting point is the phase space action

$$W = \int d\tau\{i\bar{z}\dot{z} + p_\mu(\dot{x}^\mu - \bar{z}\gamma^\mu z)\} - g\int d\tau dx\phi_{\mu\nu}\dot{x}^\mu\dot{x}^\nu\delta(x - x(\tau)) - \frac{1}{2}\int dx\phi_{\mu\nu,\lambda}\phi^{\mu\nu,\lambda} \tag{21}$$

leading to the equations of motion

$$\dot{x}_\mu = \bar{z}\gamma_\mu z, \quad i\dot{z} = p_\mu\gamma^\mu z, \quad i\dot{\bar{z}} = -\bar{z}p_\mu\gamma^\mu$$

and

$$\frac{d}{d\tau}(p_\mu - 2g\phi_{\mu\sigma}\dot{x}^\sigma) = -g\phi_{\alpha\beta,\mu}\dot{x}^\alpha\dot{x}^\beta \tag{17}$$

or

$$\dot{p}_\mu = -g\{\phi_{\alpha\beta,\mu}\dot{x}^\alpha\dot{x}^\beta - 2\phi_{\mu\alpha,\beta}\dot{x}^\alpha\dot{x}^\beta - 2\phi_{\mu\alpha}\ddot{x}^\alpha\} \tag{22}$$

From the equation of motion for the field $\phi_{\mu\nu}$

$$\Box\phi_{\mu\nu} = g\int d\tau\dot{x}_\mu\dot{x}_\nu\delta(x - x(\tau)) \tag{23}$$

we compute the retarded field

$$\phi_{\mu\nu} = g\frac{\dot{x}_\mu\dot{x}_\nu}{R} \tag{24}$$

where R is as given in (8). The derivative of the field is

$$\phi_{\mu\nu,\lambda} = g[\ddot{x}_\mu\dot{x}_\nu\frac{(X - x)_\lambda}{R^2} + \dot{x}_\mu\ddot{x}_\nu\frac{(X - x)_\lambda}{R^2} + \frac{\dot{x}_\mu\dot{x}_\nu\dot{x}_\lambda}{R^2} + \dot{x}_\mu\dot{x}_\nu\frac{(X - x)_\lambda(\dot{x}^2 - Q)}{R^3}] \tag{25}$$

with Q as given in (10). Using the same expansions as in the previous section one obtains the equation of motion

$$\dot{p}_\mu = g^2[\frac{3}{2u\dot{x}^2}\{\ddot{x}_\mu\dot{x}^2 + \dot{x}_\mu(\dot{x}\cdot\ddot{x})\} - \{\frac{1}{6}\dddot{x}_\mu - \frac{1}{2}\frac{\dot{x}_\mu\ddot{x}^2}{\dot{x}^2} - \frac{9}{4}\frac{\dot{x}_\mu(\dot{x}\cdot\ddot{x})^2}{\dot{x}^4} \tag{26}$$

$$-\frac{13}{6}\frac{\dot{x}_\mu(\dot{x}\cdot\dddot{x})}{\dot{x}^2} - \frac{9}{4}\frac{\ddot{x}_\mu(\dot{x}\cdot\ddot{x})}{\dot{x}^2}\}] + O(u)$$

After taking the appropiate limit, the final renormalized equation of motion for the particle in the tensor case becomes

$$p_\mu^{ren} = -g^2[\frac{5}{3}\dddot{x}_\mu + \frac{\dot{x}_\mu\ddot{x}^2}{\dot{x}^2} - \frac{2}{3}\frac{\dot{x}_\mu(\dot{x}\cdot\dddot{x})}{\dot{x}^2} - \frac{3}{4}\frac{(\dot{x}\cdot\ddot{x})}{\dot{x}^4}(\ddot{x}_\mu\dot{x}^2 + \dot{x}_\mu(\dot{x}\cdot\ddot{x})] \quad (27)$$

We also have the identity

$$\dot{p}_\mu = \frac{1}{\sqrt{v^2}}\frac{d}{d\tau}[\frac{v_\mu}{\sqrt{v^2}}(H + g\phi_{\mu\nu}\dot{x}^\nu)] + \frac{1}{\sqrt{v^2}}\frac{d}{d\tau}[\frac{v_\alpha\dot{S}^\alpha_\mu}{\sqrt{v^2}}] - \frac{v\cdot\dot{v}}{v^2}p_\mu \quad (28)$$

Here the constant of the motion $H = p_\mu\dot{x}^\mu - g\phi_{\mu\nu}\dot{x}^\mu\dot{x}^\nu$, which is different from the usual expression $H = p\cdot\dot{x} + i\bar{z}\dot{z} - L$, has been used. Now, since

$$\frac{3}{2u\dot{x}^2}\{\ddot{x}_\mu\dot{x}^2 + \dot{x}_\mu(\dot{x}\cdot\ddot{x})\} = \frac{3}{2u\sqrt{\dot{x}^2}}\frac{d}{d\tau}(\dot{x}_\mu\sqrt{\dot{x}^2}) \quad (29)$$

the renormalization of the mass is given by

$$H_{ren} = H - \frac{3}{2u}g^2v^2 \quad (30)$$

The spinless limit of (27) is

$$m_{ren}\ddot{x}_\mu = -\frac{5}{3}g^2(\dddot{x}_\mu + \dot{x}_\mu\ddot{x}^2) \quad (31)$$

in agreement with the result given in reference [8]

4 The Linearized Gravitational Field

The equation of motion for a linearized gravitational field $\psi_{\mu\nu}$ reads

$$\Box\psi_{\mu\nu} = -2j_{\mu\nu} + \eta_{\mu\nu}j \quad (32)$$

where $j = j^\alpha_\alpha$ is the trace of the current $j_{\mu\nu} = g\dot{x}_\mu\dot{x}_\nu$. The retarded field solution is therefore

$$\psi_{\mu\nu} = -\int D_{ret}(x-x')(2j_{\mu\nu} - \eta_{\mu\nu}j)dx' \quad (33)$$

Now consider a particle that moves in a flat spacetime but which interacts with a gravitational field in this approximation. The particle again produce a field that acts on itself. Following reference [10] we write down the action for the spinning particle that also gives us equation (32) for the field,

$$W = \int d\tau\{i\bar{z}\dot{z} + p_\mu(\dot{x}^\mu - \bar{z}\gamma^\mu z) + g\psi_{\mu\nu}\dot{x}^\mu\dot{x}^\nu\}$$
$$-\frac{1}{2}\int dx\{\frac{1}{2}\psi_{\mu\nu,\lambda}\bar{\psi}^{\mu\nu,\lambda} - \bar{\psi}^{,\lambda}_{\mu\lambda}\bar{\psi}^{\mu\sigma}_{,\sigma}\} \quad (34)$$

where $\bar{\psi}_{\mu\nu} \equiv \psi_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\psi^\sigma_\sigma$. Note that since we are working in flat spacetime spin connections [2] are not needed in (34). From this action the equation of motion for the particle follows

$$\dot{p}_\mu = g\{\psi_{\alpha\beta,\mu}\dot{x}^\alpha\dot{x}^\beta - 2\psi_{\alpha\mu,\beta}\dot{x}^\alpha\dot{x}^\beta - 2\psi_{\mu\alpha}\ddot{x}^\alpha\} \quad (35)$$

For the field $\psi(x)$ we obtain equation (32) if the gauge $\bar{\psi}^{\mu\nu}_{,\nu} = 0$ is used [10]. Now define new fields $\phi_{\mu\nu}$ and ϕ by

$$\phi_{\mu\nu} = \int D_{ret}(x-x')j_{\mu\nu}(x')dx',$$
$$\phi = \int D_{ret}(x-x')j(x')dx' \quad (36)$$

Therefore we have the relation $\psi_{\mu\nu} = -(2\phi_{\mu\nu} - \eta_{\mu\nu}\phi)$. Substitution of this expression into the equation of motion (35) gives

$$\dot{p}_\mu = -2g\{\phi_{\alpha\beta,\mu}\dot{x}^\alpha\dot{x}^\beta - 2\phi_{\alpha\mu,\beta}\dot{x}^\alpha\dot{x}^\beta - 2\phi_{\mu\alpha}\ddot{x}^\alpha\} + g\{\phi_{,\mu}\dot{x}^2 - 2\phi_{,\rho}\dot{x}^\rho\dot{x}_\mu - 2\phi\ddot{x}_\mu\} \quad (37)$$

In effect, a linearized gravitational field is a combination of tensor and scalar fields although the coupling to the scalar field contains $\dot{x}^2$ which is different than (4). Therefore, we need to compute the radiation reaction terms arising from the second term of (37) because the first term, except for a factor of 2, is identical to (22). Thus, now consider

$$\dot{p}_\mu = g\{\phi_{,\mu}\dot{x}^2 - 2\phi_{,\rho}\dot{x}^\rho\dot{x}_\mu - 2\phi\ddot{x}_\mu\} \quad (38)$$

In this case the retarded field obtained from (36) is given by

$$\phi = g\frac{\dot{x}^2}{R} \quad (39)$$

and the derivative of the field by

$$\phi_{,\nu} = (X-x)_\nu\frac{2g(\dot{x}\cdot\ddot{x})}{R^2} - \frac{g}{R}\{\dot{x}_\nu + (X-x)_\nu\frac{(Q-\dot{x}^2)}{R}\} \quad (40)$$

The renormalization term is the same as in (29) and the renormalized equation of motion reads

$$p_\mu^{ren} = g^2\{-\frac{1}{3}\dddot{x}_\mu + \frac{\dot{x}_\mu\ddot{x}^2}{\dot{x}^2} + \frac{4\dot{x}_\mu(\dot{x}\cdot\dddot{x})}{3\dot{x}^2} + \frac{5}{4}\frac{\ddot{x}_\mu(\dot{x}\cdot\ddot{x})}{\dot{x}^2} - \frac{11}{4}\frac{\dot{x}_\mu(\dot{x}\cdot\ddot{x})^2}{\dot{x}^4}\} \quad (41)$$

Finally the equation of motion for a particle in a linearized gravitational field follows from a linear combination of the scalar case given in this section and the tensor case from the previous section(see equation (37)). The result is

$$p_\mu^{ren} = g^2\{-\frac{11}{3}\dddot{x}_\mu - \frac{\dot{x}_\mu\ddot{x}^2}{\dot{x}^2} + \frac{8\dot{x}_\mu(\dot{x}\cdot\dddot{x})}{3\dot{x}^2} + \frac{11}{4}\frac{\ddot{x}_\mu(\dot{x}\cdot\ddot{x})}{\dot{x}^2} - \frac{5}{4}\frac{\dot{x}_\mu(\dot{x}\cdot\ddot{x})^2}{\dot{x}^4}\} \quad (42)$$

which in the spinless limit reduces to

$$m_{ren}\ddot{x}_\mu = -\frac{11}{3}(\dddot{x}_\mu + \dot{x}_\mu\ddot{x}^2)$$

Using the "hamiltonian" $H = p_\mu\dot{x}^\mu + \psi_{\mu\nu}\dot{x}^\mu\dot{x}^\nu$ (which, again, differs from the formula $H = \dot{x}\cdot p + i\bar{z}\dot{z} - L$) the renormalization of the mass is given by $H_{ren} = H - \frac{3}{2u}g^2v^2$.

5 Conclusions

The classical Zitterbewegung motion for a relativistic point particle gives rise to an integrable symplectic dynamical system with many remarkable properties. It has associated continuous spin variables, whose quantization produces the discrete quantum spin as in the Dirac equation. Therefore the trajectory of an elementary particle with spin in the classical domain cannot just trace a simple straight line. Instead, the motion of the charge is more complicated even in the absense of an external force, a helical motion exhibiting many properties of the quantum electron such as antiparticles.

In addition, when the spinning particle is coupled to external fields one can calculate renormalizable non-perturbative equations of motion. This has been shown previously [4] in the case of coupling to an electromagnetic field where the Lorentz-Dirac equation was generalized for spinning particles. In this work we have shown that the theory works equally well when the spinning particle is coupled to scalar, tensor and linearized gravitational fields in flat spacetime. At this point we would like to remark that there is still some freedom in defining the action when interactions are included. For instance, in the tensor field case, one could have started from different interaction terms, such as: $\phi_{\mu\nu}\dot{x}^{\mu}\bar{z}\gamma^{\nu}z$, or $\phi_{\mu\nu}(\bar{z}\gamma^{\mu}z)(\bar{z}\gamma^{\nu}z)$ etc. Although they look like different theories, it can happen that they are in fact equivalent from the physical point of view because of the constraint $\dot{x}_{\mu} = \bar{z}\gamma_{\mu}z$. Finally, work is in progress to evaluate the radiation reaction terms of a spinning particle in interaction with an electromagnetic field in a curved space, thereby generalizing eq.(2) to arbitrary Riemannian spaces.

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