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ABSTRACT

By applying the Dirac quantization method, we build the constraint that all electrons are in the lowest Landau level into the Chern–Simons field theory approach for the fractional quantum Hall system and show that the constraint can be transmuted from hierarchy to hierarchy. For a finite system, we derive that the action for each hierarchy can be split into two parts: a surface part provides the action for the edge excitations while the remaining part is precisely the bulk action for the next hierarchy. And the action for the edge could be decoupled from the bulk only at the hierarchy filling.

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Since the celebrated discovery of the fractional quantum Hall effect (FQHE) [1], a considerable progress [2] has been made in understanding for FQHE following upon the seminal paper of Laughlin [3]. Motivated by the analogies between FQHE and superfluidity [4] as well as the existence of large ring exchanges on a large length scale [5], Girvin and MacDonald [6] raised a subtle question whether there is an off-diagonal long range order (ODLRO) in FQHE ground state. By introducing a 2+1 dimensional bosonization transformation, they did find a sort of ODLRO for the bosonized Laughlin wave functions. Based on such an observation, extensive studies on the effective field theory approach for FQHE have been appeared in the literatures. Among others, the Ginzburg-Landau Chern-Simons approach (GLCS) [7,8] successively interprets a variety of properties for the FQHE system from an *ab intio* point of view and the chiral Luttinger liquid approach [9,10] for the edge excitations exhibits a deep insight for such an interesting system.

Despite the successes for the various effective field theory approaches[7-12], we still have the question that whether one should build in the constraint that all electrons are in the lowest Landau level (LLL) from the very beginning of the Chern-Simons (C-S) field theory approaches for the FQHE. As we have seen in GLCS, the "trivial Gaussian fluctuation" in fact is raised from the inter-Landau level degrees of freedom [7]. From a more basic point of view, it is known that FQHE system is essentially a 1+1 dimensional system. The 1-D nature of the FQHE system should be a direct consequence of the LLL constraint in the context of the 2+1 dimensional C-S field theory. It is also understandable that the 1-D nature might be rather crucial for the dynamics for FQHE system.

Motivated by the above arguments, in this paper, we build explicitly the LLL constraint into the C-S field theory description for FQHE system and show that the constraint can be transmuted from hierarchy to hierarchy. After a careful treatment of the partial integrations for the FQHE action of a finite system, we show further that the action for each hierarchy can be split into two parts: a surface part provides the action of the edge excitations while the remaining bulk part is exactly the action for the next hierarchy. In particular, the surface action for the edge excitations could be decoupled from the bulk only at the hierarchy filling. As the primary consequences, besides the quantization conditions for FQHE states as well as its hierarchy scheme [13] can be deduced systematically as usual, we derive the equations for the fractionally charged vortices which has an interesting form without any mass scale dependent parameters. It also does not depend on whether FQHE has a BCS type of symmetry breaking. This approach provides further a theoretical background that the vortices (quasi-particles) of each hierarchy can have only zero effective mass in the context of C-S field theory, while the "conventional" vortices often has finite effective mass contributed from the massive constituting particles.

For a 2-D N-electron system in a strong perpendicular magnetic field B, if all electrons are in the LLL, it can be described by the following Lagrangian [5] as

$$\mathcal{L} = -\frac{e}{c}\sum_i \dot{\mathbf{r}}_i(t)\cdot\mathbf{A}(\mathbf{r}_i(t)) - \sum_{i<j} V(\mathbf{r}_i - \mathbf{r}_j) \tag{1}$$

where $\mathbf{r}_i(t)$ is the 2-D coordinate for the i-th electron with $i = 1,...,N$, $\dot{\mathbf{r}}_i(t) = d\mathbf{r}_i(t)/dt$, $\mathbf{A}(\mathbf{r})$ is the vector potential for the uniform applied magnetic field, $\nabla \times \mathbf{A} = B$ and $V(\mathbf{r}_i - \mathbf{r}_j)$ is the interaction between electrons. Because $\partial\mathcal{L}/\partial\dot{\mathbf{r}}_i(t)$, $i = 1,...,N$, are independent of $\dot{\mathbf{r}}_i(t)$'s and $[\Pi_\alpha{}^i, \Pi_\beta{}^j] = -i \in_{\alpha\beta} \delta_{ij}\hbar^2\lambda^{-2}$, so that we have the second class constraints [14] as $\Pi^i_\alpha \equiv p^i_\alpha + (e/c)A^i_\alpha(\mathbf{r}_i) \doteq 0$, where

$\alpha, \beta = 1, 2$ is the component index for the 2-D vectors, $\in_{\alpha\beta}$ are the invariant second rank antisymmetric tensor, and $\lambda = (\hbar c/eB)^{\frac{1}{2}}$ the magnetic length. We then have the Hamiltonian for the N-electron in the LLL as

$$H = \sum_{i<j} V(\mathbf{r}_i - \mathbf{r}_j) \tag{2}$$

which is associated with the commutation relation $[x^\alpha_i, x^\beta_j] = i \in_{\alpha\beta} \delta_{ij}\lambda^2$, or, equivalently, with the constraint for the N-electron wave function as $\Pi_i\Psi(\mathbf{r}_1, ..., \mathbf{r}_N) = 0$, where $\Pi_i = (\Pi^i_1 - i\Pi^i_2)/\sqrt{2}$. We now introduce further the second quantization representation for N-electron system described above. It is straightforward to verify that the second quantized Hamiltonian has the form as

$$H = V[\Psi^+\Psi - \bar{\rho}] = \frac{1}{2}\int d^2x d^2x'(\Psi^+(x)\Psi(x) - \bar{\rho})V(\mathbf{x} - \mathbf{x}')(\Psi^+(x')\Psi(x') - \bar{\rho}) \tag{3}$$

where the electron wave field operator $\Psi(x)$ subjected to a constraint that

$$\Pi\hat{\Psi}(x) = 0 \tag{4}$$

and $\bar{\rho}$ is the average electron density contributed by the positive background.

We further introduce [6-8] the bosonized representation $\Phi(x)$ for the electron wave field with the help of a C-S field $\mathbf{a}(x)$. By applying the standard procedure, the Hamiltonian actually have the same form as eq. (3), $H = V[\Phi^+\Phi - \bar{\rho}]$ and the LLL constraint becomes $\tilde{\Pi}\Phi(x) = 0$, in which $\tilde{\Pi} = \partial/\partial z + iA/(a\lambda^2 B) + ia$ with $z = (x + iy)/\sqrt{2}$, $A = (A_1 - iA_2)/\sqrt{2}$ and $a = (a_1 - ia_2)/\sqrt{2}$. We have also an additional constraint for the C-S field as $\nabla \times \mathbf{a} = -2\pi m\Phi^+\Phi$, with m being an odd integer. Base upon the above discussions, the path integral representation for the Z-generating functional would have the following form as

$$Z[A] = \int \mathcal{D}\Phi\mathcal{D}\Phi^+\mathcal{D}a_\mu\delta[\tilde{\Pi}\Phi]\delta[\Phi^+\tilde{\Pi}^+]$$

$$\times exp i\{\Phi^+(i\frac{\partial}{\partial t} - e\varphi - a_0)\Phi - V[\Phi^+\Phi - \bar{\rho}] - \frac{1}{4\pi m}\in_{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda\} \qquad (5)$$

where the gauge fixing condition is understood involved implicitly. In eq.(5), μ, ν, and λ run over both the time index 0 and the spatial indices 1,2; $\in_{\mu\nu\lambda}$ is the 3-D euclidean antisymmetric tensor; a_0 is the temporal component for the C-S gauge field playing a role as a λ-multipler and $\delta[\cdots]$ is the δ–functional. From now on, we would also include an applied electric field where $\varphi(x)$ is the corresponding scalar potential. Since now we are in the boson representation, we prefer to introduce the phase $\theta(x)$ and the electron density $\rho(x)$ for the wave field as the dynamical variables: $\Phi(x) = \sqrt{\rho(x)}exp(i\theta(x))$ in which $\theta(x)$ could be split into $\theta_r(x) + \theta_s(x)$. $\theta_r(x)$ is the regular part satisfying $\in_{\alpha\beta} \partial_\alpha\partial_\beta\theta_r = 0$ and $\theta_s(x)$ the singular part satisfying $\in_{\alpha\beta} \partial_\alpha\partial_\beta\theta_s = -\rho_s/(2\pi)$. with ρ_s having the physical intuition as the density for the vortices. Taking into considerations of the expression for the $\hat{\Pi}$ and $\hat{\Pi}^+$, as a result of introducing the ρ-θ representation, the C-S field acqnires a pure gauge term as: $a_\mu \to a_\mu + \partial_\mu\theta_r$, $\mu = 0, 1, 2$, in the matter part of the action. Since the action for the C-S gauge field itself is invariant respect to the local gauge transformation up to a surface term, we may eliminate the regular part of the phase variable $\theta_r(x)$ and would come back to the resulting surface term later on. It is then straightforward to solve the constraint in terms of ρ-θ variable and further carry out the integration over the C-S field, we obtain

$$Z[A] = \int \mathcal{D}\rho\mathcal{D}\theta_s\delta(\mathcal{F}[\rho, \rho_s; B])exp i\{-\rho\dot{\theta}_s - e\rho\varphi + \frac{1}{4\pi m}\in_{\alpha\beta} a_\alpha \dot{a}_\beta - V[\rho - \bar{\rho}]\} \quad (6)$$

with

$$\mathcal{F}[\rho, \rho_s; B] \equiv \frac{1}{2}\nabla^2 \ln\rho + \frac{1}{\lambda^2} - 2\pi m\rho - 2\pi\rho_s = 0 \qquad (7)$$

and a_α being now the solution of $\nabla \times \mathbf{a} = -2\pi\rho$ in consistency with certain gauge fixing constraints. We also notice the term λ^{-2} in eq.(7) could be understood as

$(e/\hbar c)\nabla \times \mathbf{A}$. This is the first result we like to derive, i.e., the Z-generating functional for FQHE in which the constraint for the LLL has been carefully built. The LLL constraint not only makes the electrons' kinetic energy disappeared, but also manifest itself as a functional relation among ρ, ρ_s and B: $\mathcal{F}[\rho, \rho_s; B] = 0$.

Since $\in_{\alpha\beta} \partial_\alpha\partial_\beta\theta_s$ can be nonzero only at certain singular 2+1 dimensional world line, so that the vortex density should have the expression as $\rho_s(x) = \sum_j q_j\delta^2(\mathbf{x} - \mathbf{x}_j(t))$ where $q_j = \pm 1$ is the vortex charge and $\mathbf{x}_j(t)$'s are the world lines for the j-th vortex. If we take a time derivative to the vortex density, we obtain immediately a conservation theorem as $\dot{\rho}_s + \partial_\alpha j_s^\alpha = 0$, where the vortex current has the expression $j_s^\alpha(x) = \sum_j q_j\dot{x}_j^\alpha(t)\delta^2(x - x_j(t))$, or, equivalently, $j_s^\alpha(x) = \in_{\alpha\beta}(\partial_0\partial_\beta - \partial_\beta\partial_0)\theta_s/(2\pi)$.

It is straightforward to derive from the Z-functional the following equation[15]

$$\frac{1}{2}\nabla^2 < \ln\rho > -2\pi m < \rho > + \frac{1}{\lambda^2} - 2\pi < \rho_v > = 0 \qquad (8)$$

where $< \ldots >$ is the path integral averaging over the normalized Z-generating functional, i.e., average over the physical ground state. This equation had been derived directly from the constraint equations for the LLL[16]. What we have here more is to make its connection to the dynamics more explicit. For a uniform system with zero vortex, we derive the quantization condition for FQHE states, $< \rho > \equiv \bar{\rho} = (2\pi m\lambda^2)^{-1}$, immediately. For a single vortex, we can draw the conclusion easily from eq.(8) that it carries a fractional charge of $-q/m$. So this equation can be interpreted as the equation for the vortices (quasi-particles) of the first hierarchy and its mean field approximation can be solved without difficulty. We notice that, different from the usual Ginzburg-Landau type description, there is no mass scale dependent parameters appeared in eq.(8). It also does not depend conceptually on whether there is a "BCS type symmetry breaking" in the FQHE state.

It is known that the term $\nabla^2 \ln \rho/2$ plays a role to cancel the δ-function like singularities in ρ_s. Therefore, for sake of convenience, we would ignore the $\nabla^2 \ln \rho$ term in the following which should be understood that in the first quantization representation for the vortices, there is always a term $- \epsilon_{\alpha\beta}\ \partial_\beta \ln \rho/2$ associated with $\partial_\alpha \theta_s$ implicitly, while in its second quantization representation, such a term can be reasonably ignored.

Now we shall separate the surface part of the action from the bulk part for a finite FQHE system. Before going into the details, we like to introduce certain descriptions for the boundary of the system. We imagine the 2-D system being enclosed by a (spatially) one dimensional boundary Γ. We further introduce a displacement vector $\delta \mathbf{r}$ which is defined formally along the boundary as

$$\int d^2x(\rho - \bar{\rho}) \equiv -\bar{\rho} \oint_\Gamma dln_\alpha \delta r_\alpha \tag{9}$$

where dl is the linear integral along the boundary, n_α is the normal for the boundary being defined always toward outside of the system and $\bar{\rho}$ is certain average electron density to be defined. If we take $\bar{\rho} = \bar{\rho}$, the lefthand side of the equation should be zero so that we should have $\oint_\Gamma dln_\alpha \delta r_\alpha = 0$. Consequently, δr_α can be interpreted either as the displacement for the particles (electrons) passing back and forth through the boundary or as the " rippling " displacement for the boundary [10] deviating out- or inward along the boundary. Obviously, this equation is valid up to the first order of $\delta \mathbf{r}$. Comparing with the constraint equation (8), we find $\bar{\rho} = (2\pi m \lambda^2)^{-1} - m^{-1}\bar{\rho}_s$ with $\bar{\rho}_s$ being the average density of vortices. If we split θ_s into two parts: $\theta_s = \theta_s^b + \theta_s^c$, correspondingly, $\rho_s = \rho_s^b + \rho_s^c$. We have the surface part of vortex density $\rho_s^c = - \epsilon_{\alpha\beta}\ \partial_\alpha \partial_\beta \theta_s^c / 2\pi$ which is nonzero only at the boundary and has zero contribution to the $\bar{\rho}_s$ while the bulk part of vortex density $\rho_s{}^b = - \epsilon_{\alpha\beta}\ \partial_\alpha \partial_\beta \theta_s{}^b / 2\pi$ with $\bar{\rho}_s = \bar{\rho}_s^b$. By applying further eq.(7) to eq.(9), we have finally $\delta r_\alpha = -(2\pi m \bar{\rho})^{-1}[\epsilon_{\alpha\beta}\ \partial_\beta \theta_s^c]_\Gamma$ up to an arbitrary gauge transformation $\theta_s{}^c \to \theta_s{}^c + \theta$ where θ is a regular function defined along the Γ: $\oint dln_\alpha \partial_\alpha \theta = 0$ but not determined yet. Moreover, it is known that since a finite 2-D FQHE system are practically in a confining potential, it acquires a chemical potential μ in such a way that the Gibbs free energy being minimized in consistency with the spatial distribution for the electrons. Therefore, the local deviation for the applied electric potential, $-e\varphi$, from the chemical potential at the boundary bears the work done by those electron got passed through the boundary, or in another words, due to the local displacement of the boundary from its equilibrium configuration. We should then have $(-e\varphi - \mu)|_\Gamma = -e\mathbf{E} \cdot \delta \mathbf{r}|_\Gamma$.

Now for the first term of the action (see eq. (6)): $-\rho \dot{\theta}_s$, we introduce a dual gauge field $A'_\alpha = a_\alpha / m$ with $\rho = - \epsilon_{\alpha\beta}\ \partial_\alpha A'_\beta/(2\pi)$. By applying a partial integration with respect to ∂_α, we separate a surface term from the bulk action with a remaining bulk part which can be expressed in terms of A'_α and $j_{s,\alpha}$. On the meanwhile, we substitute ρ by $(2\pi m)^{-1}[\lambda^{-2} - 2\pi \rho_s{}^b + \epsilon_{\alpha\beta}\ \partial_\alpha \partial_\beta \theta_s{}^c]$ into the second term of the action: $\rho(-e\varphi - \mu)$. Noticing the constant $(2\pi m \lambda^2)^{-1}$ does not contribute to the dynamics , we perform once again a partial integration with respect to the "∂_α" for the last term, so that we separate one another surface term from the bulk action. Taking into account of the above considerations, and further utilizing the expression for δr_α, we obtain an interesting form for the Z-functional as

$$Z = \int \mathcal{D}\theta_s^b \mathcal{D}\theta_s^c \int \mathcal{D}\rho \delta[\mathcal{F}[\rho, \rho_s^b + \rho_s^c; B]]$$
$$\times expi\{\mathbf{j}_s^b \cdot \mathbf{A}' - \frac{e}{m}\rho_s^b \varphi - \frac{m}{4\pi} \epsilon_{\alpha\beta}\ A'_\alpha \dot{A}'_\beta - V - I_\Gamma[\rho, \theta_s^c]\} \tag{10}$$

with the surface action

$$I_\Gamma[\rho,\theta_s^c] = \frac{1}{2\pi m}\int dt \oint dl\{(-n_\alpha \in_{\alpha\beta} \partial_\beta\theta_s)\dot{\theta}_s - \tilde{v}_D(n_\alpha \in_{\alpha\beta} \partial_\beta\theta_s^c)^2\} \qquad (11)$$

In eq.(11), we have assumed the applied electric field is paralleled to the normal on the boundary and $\tilde{v}_D = v_D/(1-2\pi\lambda^2\bar{\rho}_s)$ with $v_D = cE/B$. For a finite system, eq.(10) means that the action for FQHE system can be divided into a bulk part and a surface part. The bulk part has the intuition that the vortices moves in a dual gauge field $\mathbf{A}'$ and bears a fractional statistics $1/m$ with fractional charge $-qe/m$. If we solve A'_α in terms of ρ, then applying eq.(7), we recover easily the form derived in [7]. Moreover, when the system is exactly in a FQHE state of the first hierarchy, *i.e.* $\rho_s{}^b = 0$, we then have $\theta_s = \theta_s{}^c$. As an interesting result, the surface action $I_\Gamma[\rho,\theta_s{}^c]$ will decouple from the bulk and describe an ensemble of independent propagating edge excitations in a form of a chiral boson action proposed in [9,10]. If we perform a gauge transformation to the whole action in eq.(10), it would also produce a surface term which may cancel the surface term left previously.

As we have noticed above, $\theta_s(x)$ has only the isolated singularities in the 2-D plane, so that $\mathcal{D}\theta_s$ integrates over only the space-time propagation of those singularities: the coordinate of vortices. Therefore

$$\int \mathcal{D}\theta_s^b expi\{\mathbf{j}_s^b\cdot\mathbf{A}'\} = \sum_{N=1}^{\infty}\int\prod_{j=1}^{N}\mathcal{D}\mathbf{r}'_j(t)expi\{\sum_j \dot{\mathbf{r}}'_j(t)\cdot\mathbf{A}'(\mathbf{r}'_j(t))\} \qquad (12)$$

where $\mathbf{r}'_j(t)$ is the coordinate for the j-th vortex. For sake of convenience, we assume in eq.(12) and the following that $q_j = +1$. This identity makes it explicit that the bulk action for the vortices in eq.(12) is essentially in a first quantization representation. Moreover, it becomes clear also that such an action has only first order time derivative of the vortex coordinates but no second order. Once again we have a

system of vortices with " zero kinetic energy " which should be described again by a second class constraint. Now the bulk action for the vortices has a form almost the same as the original action for the electrons, eq.(1). We can then run the same procedure as for the electron case, *i.e.*, the procedure from eq.(1) to eq.(11). But there are still certain delicate differences which should be carefully treated as the following: (i) Now, for the bulk action for the vortices, we have a vector potential $\mathbf{A}'$ playing a similar role as the vector potential for the magnetic field in electrons case but having a curl $\nabla\times\mathbf{A}' = -2\pi\rho$; (ii) In the application of the Dirac quantization to the vortices in the first quantization representation, we need the condition $[\Pi'^{i}_\alpha, \Pi'^{j}_\beta] = -2\pi \in_{\alpha\beta} \rho \neq 0$ to be satisfied, where Π'^{i}_α has the same form as Π^i_α with the corresponding quantities substituted by those for the vortices. Since ρ could be zero (or singular) only at the isolated locations for the vortices, in the spirit of long wave length approximation, we may reasonably take the approximation as $\rho > 0$ (finite). In fact, these singular behaviors at the vortex locations will disappear after its second quantization procedure being completed; (iii) Corresponding to the original C-S gauge field with the statistical index of odd integers m, we now introduce a C-S gauge field a'_μ with the statistical index of even integers $2p$, because the world lines for the vortex "particles" are originated from the singularities of the phase field θ_s of the bosonized electrons.

Taking into all the above considerations, following almost exactly the same procedure as those for the electrons, the Z-functional can be transformed into the following form as

$$\int \mathcal{D}\rho_s\mathcal{D}\theta'_s\delta[\mathcal{F}'[\rho_s,\rho'_s;B]]$$
$$\times expi\{-\rho_s\dot{\theta}'_s + \frac{e}{m}\rho_s\varphi - \frac{\kappa}{16p^2\pi}\in_{\alpha\beta} a'_\alpha\dot{a}'_\beta - V[\rho_s] + I_\Gamma[\rho,\theta_s]\} \qquad (13)$$

with $\kappa = m^{-1} + 2p$ and

$$\mathcal{F}'[\rho_s, \rho'_s; B] = \frac{1}{2}\nabla^2 \ln \rho_s - \frac{1}{m\lambda^2} + 2\pi\rho_s\kappa + \epsilon_{\alpha\beta}\,\partial_\alpha\partial_\beta\theta'_s \qquad (14)$$

where ρ_s, the density of the vortices, is now in a second quantization representation, i.e., the modulus of the vortex wave field, and θ'_s is the singular part for the phase field which describes the isolated "vortices" ρ'_s for the next higher hierarchy. In deriving eq.(13), we have carry out the path integral for $\mathcal{D}\rho$ so that the constraint equation (7), $\mathcal{F}[\rho, \rho_s; B] = 0$, is understood being always satisfied. Here we notice further that the application of the Dirac's quantization theory for the constrained systems to the overall space-time propagation of the vortices in form of eq.(12) provides a field-theoretical background that the vortices (quasi-particles) in FQHE have only zero effective mass while the "conventional" vortices often have finite effective mass contributed by the massive constituting particles.

For homogeneous system with $\bar{\rho}'_s$ equals to zero, we have a condensate for both electrons and vortices. Then the constraints $\mathcal{F}[\rho, \rho_s] = 0$ and $\mathcal{F}'[\rho_s, \rho'_s] = 0$ results the second hierarchy with a filling $\nu = (m + (2p)^{-1})^{-1}$[11]. For systems having isolated "vortices" in the sense of the second hierarchy, $\mathcal{F}'[\rho_s, \rho'_s] = 0$ provides the corresponding "vortex" equation. If we perform a similar procedure to split further the surface and bulk parts for the bulk action in eq.(13), we derive the corresponding edge excitations of the second hierarchy as well as bulk action for the vortex of the third hierarchy in which the "vortex" current would couple to a dual C-S gauge field as $\mathbf{j}'^b_s \cdot \mathbf{A}''$ with a C-S action $\kappa\ \epsilon_{\alpha\beta}\ A''_\alpha \dot{A}''_\beta/(4\pi)$. Now it is sufficiently convincing that by repeating the procedure developed above, we present a full dynamical description for both infinite and finite FQH systems. The action incorporated with the constraint can be transformed from hierarchy to hierarchy

in an almost universal form and has distinguished features which have not been seriously exploited before.

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