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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

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COVARIANT BETHE–SALPETER WAVE FUNCTIONS FOR HEAVY HADRONS *

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1. BETHE–SALPETER AMPLITUDES FOR HEAVY HADRONS

In recent years the dynamics of heavy mesons and baryons has been considerably simplified by the development of the so–called heavy quark effective theory, HQET [1–3] where the heavy quark mass is taken to be infinite. In a series of recent papers [2, 4–8], we have presented a covariant formulation of heavy meson and heavy baryon decays in the leading order of the HQET. This method is based on a Bethe–Salpeter (BS) formulation in the limit of the heavy quark mass going to infinity. In this talk, I would like to briefly review this approach.

The starting point of our investigation was the demonstration that the equal velocity assumption, arising from the heavy quark limit, could be formulated in a covariant manner using the spin–parity projectors developed by Delbourgo, Salam and Strathdee (SDS) [4, 9]. In the zeroth order of HQET, the heavy quark is free and moves with the same four velocity as the hadron, of which it is a constituent:

$$v = \frac{p_Q}{m_Q} = \frac{P}{M} . \tag{1}$$

Since the heavy quark is "free" we can embed the spin of the heavy quark in the usual way in a four–dimensional space of Dirac indices, i.e., the $\left[\left(\frac{1}{2},0\right) \oplus \left(0,\frac{1}{2}\right)\right]$ representation of the Lorentz group.

For example, a meson is represented by a two–index wavefunction $\Phi_\alpha^\beta(P, p_Q, k)$, where the lower index α, is the heavy quark index and the upper index represents the rest of the light Lorentz structure and $P = p_Q + k$, where P is the momentum of the meson, p_Q the momentum of the heavy quark and k is the momentum of the light quark.

$\Phi_\alpha{}^\beta(P, p_Q, k)$ is the Fourier transform of the BS amplitude

$$\Phi_\alpha{}^\beta(X, P) = <0|T\left(\psi_\alpha{}^h\left(\frac{x}{2}\right)\ \bar{\psi}_\ell^\beta\left(-\frac{x}{2}\right)|P, M\right)> . \tag{2}$$

Since the heavy quark is "free" and on–shell we have a Dirac equation on the lower (heavy) index

$$\left(\frac{\not{p}_Q}{m_Q} - 1\right)_\alpha^{\alpha'} \Phi_{\alpha'}{}^\beta = 0 \tag{3}$$

but since $\frac{p_Q}{m_Q} = \frac{P}{M} = v$, this equation becomes

$$(\not{v} - 1)_\alpha{}^{\alpha'}\, \Phi_{\alpha'}{}^\beta = 0 . \tag{4}$$

This is the well-known Bargmann–Wigner equation [10]. It is more fashionable nowadays to call it the "velocity super selection rule".

Eq.(4) implies that, in leading order in HQET, all heavy meson wavefunctions are of the form

$$\Phi_\alpha{}^\beta(v,k) = \frac{1}{2}(\not v + 1)_\alpha{}^{\alpha'}\,\Phi'_{\alpha'}{}^\beta(v,k)\ . \tag{5}$$

This suggests immediately that $\Phi_\alpha{}^\beta$ should be of the form

$$\Phi_\alpha{}^\beta(v,k) = \chi_\alpha{}^{\alpha'}\,A_{\alpha'}{}^\beta(v,k) \tag{6}$$

where $\chi_\alpha{}^{\alpha'}$ are the spin projection operators developed by SDS. We can prove this by considering an interpolating field in the LSZ framework. Here I will just give a simple diagrammatic argument.

In general, in the quark model, we can decompose the BS amplitude as

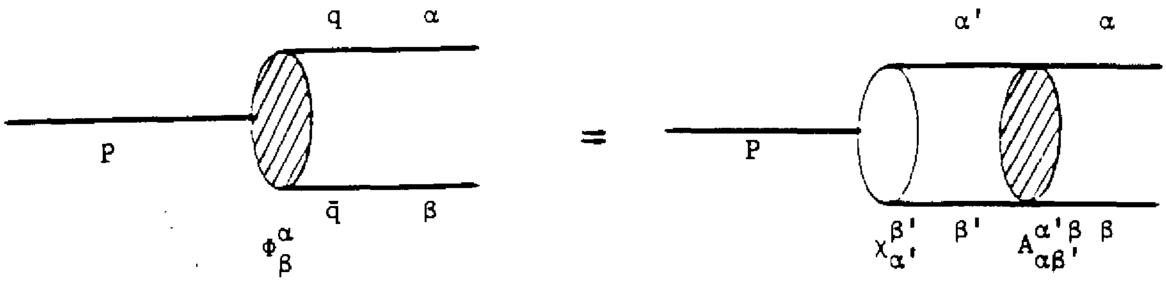


Fig.1. Decomposition of meson Bethe–Salpeter amplitude

that is

$$\Phi_\alpha{}^\beta = \chi_{\alpha'}{}^{\beta'}\,A^{\alpha'\,\beta}_{\alpha\,\beta'} \tag{7}$$

where $\chi_{\alpha'}{}^{\beta'}$ projects out the particular particle (spin) state from the two–body scattering amplitude $A^{\alpha'\,\beta}_{\alpha\,\beta'}$.

Now the picture for the heavy meson is clear. The heavy quark leg is free

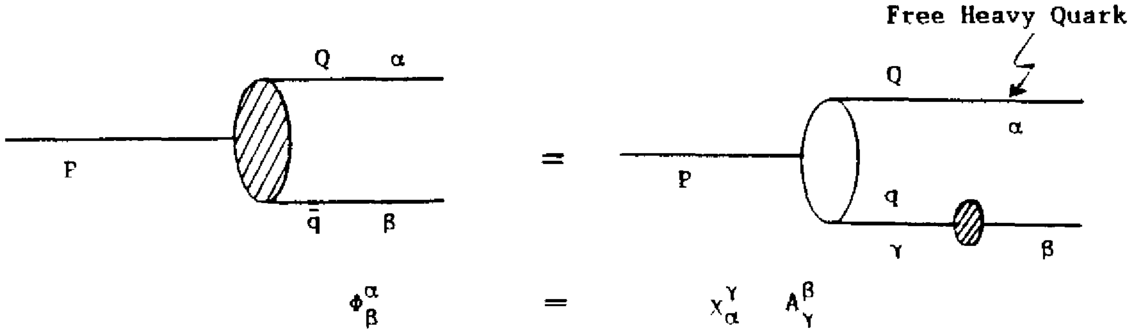
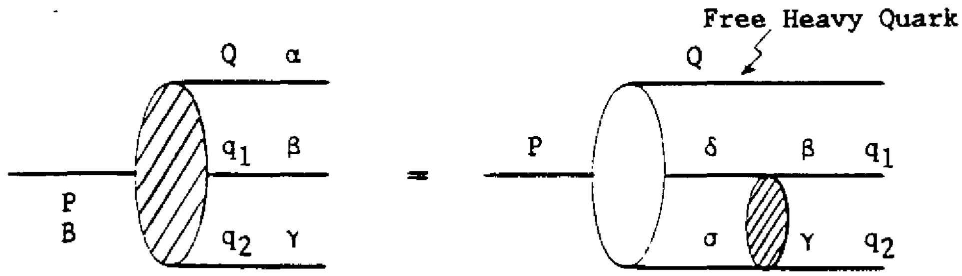


Fig.2. Ansatz for heavy meson B–S amplitude.

that is $A^{\alpha'\,\beta}_{\alpha\,\beta'} \rightarrow \delta^{\alpha'}_\alpha\,A^\beta_{\beta'}$.

Similarly for heavy hadrons:



$$B_{\alpha\beta\gamma}(v,q_1,q_2) = \chi_{\alpha\delta\sigma}(v)\, A^{\delta\sigma}_{\beta\gamma}(q_1,q_2)$$

Fig.3. Ansatz for heavy baryon B–S amplitude.

The spin projection operators χ are nothing but the Bargmann–Wigner wavefunctions. In their classic paper [10], Bargmann and Wigner showed that all types of spinning particles could be described by multispinor fields, that is, products of the fundamental spin $\frac{1}{2}$ representation of the Lorentz group.

The BW wavefunctions are multispinor wavefunctions of some given rank and symmetry type $\chi^{\beta_1 \dots \beta_m}_{\alpha_1 \dots \alpha_n}(v)$ which satisfy the Bargmann–Wigner equations on all the labels

$$\begin{aligned} (\not v - 1)_{\alpha_1}{}^{\alpha_1'}\, \chi^{\beta_1 \dots \beta_m}_{\alpha_1' \dots \alpha_n}(v) &= 0 \\ \chi^{\beta_1' \dots \beta_m}_{\alpha_1 \dots \alpha_n}(v)\, (\not v + 1)_{\beta_1'}{}^{\beta} &= 0 \, . \end{aligned} \tag{8}$$

There is a simple reason why these χ's describe particles of a given spin. In the rest frame, the Dirac equation is a projection by the operator $(1 + \gamma_0)$. Since γ_0 is the parity operator for spin $\frac{1}{2}$, all that the Dirac equation does is to specify the parity of the fundamental representation. This means that the spin (Dirac) labels take on not four but two independent values. This reduces $SO(4)$ to $SU(2)$. On imposing the Dirac equation on all the labels we get a reducible product representation of $SU(2)$. On fixing to a given symmetry type we get an irreducible representation of $SU(2)$ describing a particle of a given spin.

Example: Consider $\chi(v)$ a totally symmetric spinor of rank 2s. This is the wavefunction of a particle of spin s and parity $(+1)^{2s}$, if they are all lower indices and $(-1)^{2s}$ if all upper indices. For a multispinor, the parity operator is a tensor product of γ_0 's.

As a particular example, a rank two symmetric bispinor describes spin one particles. The gamma matrices split into symmetric and antisymmetric matrices as follows:

symmetric : $\gamma_\mu\, C, \sigma_{\mu\nu}\, C,$
antisymmetric : $C, \gamma_5\, C, \gamma_\mu\, \gamma_5\, C$.

We can thus write the symmetric bispinor as

$$\chi_{\{\alpha\beta\}} = (\gamma_\mu\, C)_{\alpha\beta}\, \phi^\mu + (\sigma_{\mu\nu}\, C)_{\alpha\beta}\, \phi^{\mu\nu} \, . \tag{9}$$

The BW equations reduce to the constraints

$$v_\mu\, \phi^\mu = 0 \, , \quad \phi^{\mu\nu} = v^\mu\, \phi^\nu - v^\nu\, \phi^\mu \, . \tag{10}$$

Identifying ϕ^μ with the polarization vector $\in^\mu$, the spin projector takes the form

$$\chi_{\{\alpha\beta\}} = \left[(1 + \not v) C \not \in \right]_{\alpha\beta} \, . \tag{11}$$

Similarly, the antisymmetric bispinor can be written as

$$\chi_{[\alpha\beta]} = \left[(1+\not{v})\,\gamma_5\,C\right]_{\alpha\beta} . \tag{12}$$

$\chi_{\{\alpha\beta\}}$ and $\chi_{[\alpha\beta]}$ have quark number 2 and parity 1.

Similar arguments for quark number zero wavefunctions leads to

$$\begin{aligned} \Phi_\alpha{}^\beta &= \left[(\not{v}+1)\,\gamma_5\right]_\alpha{}^\beta, \quad \text{spin zero, parity} = -1 \\ \Phi_\alpha{}^\beta &= \left[(\not{v}+1)\,\not{\epsilon}\right]_\alpha{}^\beta, \quad \text{spin one, parity} = -1 . \end{aligned} \tag{13}$$

For baryons consider rank three spinors $\chi_{\alpha\beta\gamma}(u)$. After the BW equations are imposed each label has two values. The total degrees of freedom are thus eight. We wish to decompose down to its irreducible parts $\left(\frac{1}{2} \oplus \frac{1}{2} \oplus \frac{3}{2}\right)$.

Put

$$\chi_{\alpha\beta\gamma} = \chi_{[\alpha\beta]\gamma} + \chi_{\{\alpha\beta\}\gamma} . \tag{14}$$

If BW equations are not imposed then $\chi_{[\alpha\beta]\gamma}$ is reducible. One would have to project out the totally antisymmetric part by

$$\chi_{[\alpha\beta]\gamma} + \chi_{[\gamma\alpha]\beta} + \chi_{[\beta\gamma]\alpha} = 0 . \tag{15}$$

However, after BW equations are imposed this condition is identically satisfied. $\chi_{[\alpha\beta]\gamma}$ represents spin $\frac{1}{2}$ particles. However, $\chi_{\{\alpha\beta\}\gamma}$ is reducible even after the imposition of the BW equations. We project away the totally symmetric part by

$$\chi_{\{\alpha\beta\}\gamma} + \chi_{\{\beta\gamma\}\alpha} + \chi_{\{\gamma\alpha\}\beta} = 0 . \tag{16}$$

This then also represents a spin $\frac{1}{2}$ particle. One can check easily that this has just two degrees of freedom. The complete decomposition is now

$$\chi_{\alpha\beta\gamma} = \chi_{[\alpha\beta]\gamma} + \chi_{\{\alpha\beta\}\gamma} + \chi_{\{\alpha\beta\gamma\}} . \tag{17}$$

Degrees of freedom = 2+2+4=8 where $\chi_{\{\alpha\beta\gamma\}}$ is the totally symmetric part.

This decomposition allows a rather nice projection for s–wave heavy baryons.

1. $\chi_{\{\alpha\beta\gamma\}}$ projects spin $\frac{3}{2}$, Σ_Q^*, Ω_Q^* particles.

2. For Λ_Q and Ξ_Q, the two light quarks are in an antisymmetric $s=0$ state. Therefore, if we let γ be the heavy quark label, then $\chi_{[\alpha\beta]\gamma}$ is the projector we need. It has the form

$$\chi_{[\alpha\beta]\gamma} = \left[(1+\not{v})\,\gamma_5\,C\right]_{\alpha\beta}\,u_\gamma, \quad (\not{v}-1)u = 0 . \tag{18}$$

3. For the heavy Σ_Q and Ω_Q baryons, the light quarks are in a symmetric $s=1$ state. With γ the heavy label, the other spin $\frac{1}{2}$ χ, $\chi_{\{\alpha\beta\}\gamma}$ is the correct projector.

There are two ways to get an explicit form for $\chi_{\{\alpha\beta\}\gamma}$.

(i) Expand $\chi_{\{\alpha\beta\}\gamma}$ in terms of symmetric gamma matrices, in α and β, and impose the BW equations. This gives

$$\chi_{\{\alpha\beta\}\gamma} = \left[(1+\not{v})\phi_\mu\,u\right]_\gamma \left[(1+\not{v})\gamma^\mu\,C\right]_{\alpha\beta} . \tag{19}$$

On now imposing tracelessness conditions we find

$$\begin{aligned} \chi_{\{\alpha\beta\}\gamma} &= \left[(1+\not{v})\gamma_\mu\gamma_5 u\right]_\gamma \left[(1+\not{v})\gamma^\mu\,C\right]_{\alpha\beta} \\ &= \left[(\gamma_\mu + v_\mu)\gamma_5 u\right]_\gamma \left[(1+\not{v})\gamma^\mu\,C\right]_{\alpha\beta} . \end{aligned} \tag{20}$$

The first square bracket on the right–hand side of this equation is the wavefunction proposed by Georgi [11].

(ii) Alternatively start with the observation that the trace condition is easily solved in terms of antisymmetric objects

$$\chi_{\{\alpha\beta\}\gamma} = \phi_{[\alpha\gamma]}\, \phi_\beta + \phi_{[\beta\gamma]}\, \phi_\alpha \tag{21}$$

which on imposing BW equations yields

$$\chi_{\{\alpha\beta\}\gamma} = \left[(1+\not v)\gamma_5\, C\right]_{\alpha\gamma} u_\beta + \left[(1+\not v)\gamma_5\, C\right]_{\beta\gamma} u_\alpha\,. \tag{22}$$

This is the form proposed by us [7, 8].

Of course, the two forms Eq.(20) and Eq.(22) must be identical as they represent the same Lorentz structure. It is a matter of some gamma algebra to explicitly show this [8].

Upto now I was only considering S–wave states. We can apply the same techniques to P–wave and higher orbital states. Only the algebra is a bit more complicated. For mesons we now list the complete S–wave and P–wave BS amplitudes incorporating these projectors [12]:

S–wave:

$$\begin{aligned}
&\Phi_\alpha{}^\beta(v,k) = \\
&{}^1S_0(0^{-+}) \;:\; \frac{1}{2}\sqrt{M}\,(\not v+1)\,\gamma_5\,\phi_s(v,k) \\
&{}^3S_1(1^{-+}) \;:\; \frac{1}{2}\sqrt{M}\,(\not v+1)\,\not\in\,\phi_s(v,k)
\end{aligned} \tag{23}$$

P–wave:

$$\begin{aligned}
&\Phi_\alpha{}^\beta(v,k) = \\
&{}^3P_0(0^{++}) \;:\; -\frac{1}{2\sqrt{3}}\sqrt{M}\,(\not v+1)(\not k - k\cdot v)\phi_p(v,k) \\
&{}^3P_1(1^{++}) \;:\; -\frac{i}{2\sqrt{2}}\sqrt{M}\,(\not v+1)\;\epsilon^{\mu\nu\rho\sigma}\; v_\mu\, \in_\nu k_\rho\, \gamma_\sigma\, \phi_p(v,k) \\
&{}^3P_2(2^{++}) \;:\; \frac{1}{2}\sqrt{M}\,(\not v+1)\,\gamma^\mu\;\in_{\mu\nu}\; k^\nu\,\phi_p(v,k) \\
&{}^1P_1(1^{+-}) \;:\; \frac{1}{2}\sqrt{M}\,(\not v+1)\,\gamma_5\, k\cdot\in\;\phi_p(v,k)\,.
\end{aligned} \tag{24}$$

In Eqs.(23) and (24) we have factored out the heavy mass scale $\sqrt{M}$. ϕ_s and ϕ_p are the S–wave and P–wave orbital functions. These, in general, will be bispinors.

These states were constructed using the analogue of the LS–coupling scheme, in which charge conjugation parity is obvious. This is O.K. for heavy $Q\bar{Q}$ quarkonium states but no longer useful for $Q\bar{q}$ states. Here the appropriate degenerate heavy light states are determined by coupling orbital angular momentum L with the spin of the light quark just as in the hydrogen atom. For P–wave states, with $\ell = 1$, one has two degenerate multiplets

$$J^P = \left(0^+, 1^+_{1/2}\right), \qquad J^P = \left(1^+_{3/2}, 2^+\right)\,.$$

Subscript on 1^+ states labels the total angular momentum of the light quark system. The $J^P = 1^+$ mesonic P–wave states $|1^+_{1/2}>$ and $|1^+_{3/2}>$ are linear combinations of the $J = 1$, P–wave spin states:

$$\begin{aligned}
|1^+_{1/2}> &= \sqrt{\frac{1}{3}}\,|1^{+-}> + \sqrt{\frac{2}{3}}\,|1^{++}> \\
|1^+_{3/2}> &= \sqrt{\frac{2}{3}}\,|1^{+-}> - \sqrt{\frac{1}{3}}\,|1^{++}>\,.
\end{aligned} \tag{25}$$

where $|1^+_{1/2}>$ and $|1^+_{3/2}>$ are not C–eigenstates.

2. WEAK TRANSITIONS

Given the wavefunctions we can now compute current induced transitions between heavy mesons. The general form of the matrix element is [12]:

$$\begin{aligned} M_\mu &= < M_2(v_2)|J_\mu^{V-A}|M_1(v_1) > \\ &= \int d^4\, k_1\, d^4\, k_2\, Tr\left\{\bar{\Phi}_2(v_2,k_2)\, \mathcal{T}_\mu(k_1,k_2;v_1,v_2)\, \Phi_1(v_1,k_1)\right\} \end{aligned} \tag{26}$$

with $\bar{\Phi} = \gamma_0\, \Phi^+\, \gamma_0$.

To lowest order in perturbation theory we get the Mandelstam–Nishijima [13] formula for transitions between bound states with

$$\mathcal{T}_\mu(k_1,k_2;v_1,v_2) = (\not{k} - m) \otimes \gamma_\mu(1-\gamma_5)\delta(k_1 - k_2) \tag{27}$$

In the heavy quark limit, one can go beyond perturbation theory. One finds quite generally [12, 14] that in this limit

$$\mathcal{T}_\mu(k_1,k_2;v_1,v_2) = \mathcal{T}(k_1,k_2;v_1,v_2) \otimes \gamma_\mu(1-\gamma_5)\ . \tag{28}$$

$\mathcal{T}$ connect light quark legs whereas the weak current connects the heavy quarks, Fig.4

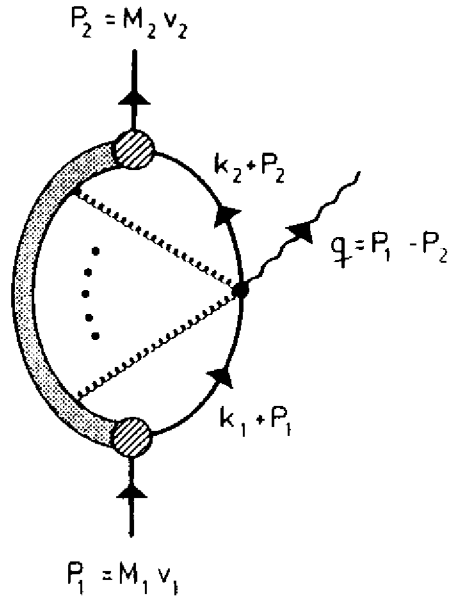


Fig.4. Current induced transitions between heavy mesons in the HQET. The curly lines represent the Wilson line glue contributions that arise on transforming to free heavy quarks.

Because of this factorization, the matrix element can always be written as

$$M_\mu = Tr\left\{\bar{\chi}_2\ \gamma_\mu(1-\gamma_5)\chi_1 \int d^4\ k_2\ d^4\ k_2\ A_1(k_1)T(k_1,k_2;v_1,v_2)\bar{A}_2(k_2)\right. \ . \tag{29}$$

Eq.(29) is obtained by substituting Eq.(28) in Eq.(26) and recalling that $\Phi = \chi A$.

S-wave $\to$ S-wave transitions

$\Phi = \chi A_S$ with $\chi = (1+\not{v})\gamma_5$ or $(1+\not{v})\not{\epsilon}$, but with A_S the same for both 0^- and 1^- states.

Evidently, for all the transitions $0^- \to 0^-, 0^- \to 1^-$ and $1^- \to 1^-$ we have the same integral

$$\int d^4\, k_1\, d^4\, k_2\, A_1(k_1)\, T\tilde{A}_2(k_2)\,.$$

This is a scalar Dirac matrix. It can only be proportional to $1, \not{v}_1, \not{v}_2$ or $\not{v}_1\not{v}_2$. However, because of the projectors χ all of these collapse to $\mathbb{1}$. Hence

$$M = Tr\{\bar{\chi}_2\, \gamma_\mu(1-\gamma_5)\chi_1\}\; F(q^2)\,. \tag{30}$$

There is a single universal form factor $F(q^2)$ describing all S-wave to S-wave transtions. This was first suggested in 1962 by Durand, de Celles and Marr [15]. Nowadays, this is written as $\xi(w)$, with $w = v_1 \cdot v_2$.

S-wave $\to$ P-wave transitions

Writing $\Phi = \chi_\nu\, k^\nu\, A_P$ for P-waves one finds for $S \to P$-wave transitions

$$M_\mu = Tr\left\{\bar{\chi}_2^\nu\, \gamma_\mu(1-\gamma_5)\chi_1(F_j(\omega)\, v_{1\nu} + F_j'(\omega)\, \gamma_\nu)\right\} \tag{31}$$

where the index $j = \frac{1}{2}$ and $\frac{3}{2}$ for the degenerate multiplets $(0^+, 1^+_{1/2})$ and $(1^+_{3/2}, 2^+)$ respectively. In fact, it turns out that there are only two linear combinations of these F_j's which describe the S to P-wave transitions. We denote the two reduced form factor functions that describe the $(0^-, 1^-) \to (0^+, 1^+_{1/2})$ and $(0^-, 1^-) \to (1^+_{3/2}, 2^+)$ transitions by $\xi^*_{1/2}(\omega)$ and $\xi^*_{3/2}(\omega)$, respectively.

$$\begin{aligned} \xi^*_{1/2}(\omega) &= (\omega+1)\; F_{1/2}(\omega) - 3F'_{1/2}(\omega) \\ \xi^*_{3/2}(\omega) &= F_{3/2}(\omega)\,. \end{aligned} \tag{32}$$

We now collect together the results for all the S-wave to S-wave and S-wave to P-wave transitions, for example, in $b \to c$ transitions.

(i)

$$\begin{pmatrix} 0^- \\ 1^- \end{pmatrix} \to \begin{pmatrix} 0^- \\ 1^- \end{pmatrix}$$

$$M_\mu = \frac{1}{4}\sqrt{M_1 M_2}\; Tr\left(\begin{pmatrix} \gamma_5 \\ \not{\epsilon}_2^* \end{pmatrix}(\not{v}_2+1)\;\gamma_\mu(1-\gamma_5)(\not{v}_1+1)\begin{pmatrix} \gamma_5 \\ \not{\epsilon}_1 \end{pmatrix}\right)\xi(\omega) \tag{33}$$

(ii)

$$\begin{pmatrix} 0^- \\ 1^- \end{pmatrix} \to \begin{pmatrix} 0^+ \\ 1^+_{1/2} \end{pmatrix}$$

$$M_\mu = \frac{1}{\sqrt{3}}\sqrt{M_1 M_2}\;\frac{1}{4}\; Tr\left(\begin{pmatrix} 1 \\ \not{\epsilon}_2^*\,\gamma_5 \end{pmatrix}(\not{v}_2+1)\;\gamma_\mu(1-\gamma_5)(\not{v}_1+1)\begin{pmatrix} \gamma_5 \\ \not{\epsilon}_1 \end{pmatrix}\right)\xi^*_{1/2}(\omega) \tag{34}$$

(iii)

$$\begin{pmatrix} 0^- \\ 1^- \end{pmatrix} \to \begin{pmatrix} 1^+_{3/2} \\ 2^+ \end{pmatrix}$$

$$M_\mu = \frac{1}{4}\sqrt{M_1 M_2}\, Tr\left(\begin{pmatrix} -\sqrt{\frac{1}{6}}\left((\omega+1)\, \not\epsilon_2^* + 3v_1\cdot \epsilon_2^*\right)\gamma_5 \\ v_{1\nu}\ \epsilon_2^{*\nu\nu'}\ \gamma_{\nu'} \end{pmatrix} \times \right.$$

$$\left. \times\, (\not v_2 + 1)\,\gamma_\mu(1-\gamma_5)(\not v_1 + 1)\begin{pmatrix}\gamma_5 \\ \not\epsilon_1\end{pmatrix}\right)\xi^*_{3/2}(\omega)\ .$$

3. CONCLUSIONS

In leading order in the HQET, i.e., in the limit $m_b \to \infty, m_c \to \infty$ one finds a dramatic reduction in the number of reduced form factors or independent amplitudes describing the transitions.

For $(0^-, 1^-) \to (0^-, 1^-)$ transitions, one has a reduction from 20 to 1 independent amplitude. Similarly, for $(0^-, 1^-) \to (0^+, 1^+_{1/2})$ there is a reduction from 20 to 1 and for $(0^-, 1^-) \to (1^+_{3/2}, 2^+)$ from 30 to 1.

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