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NONLINEAR RADIATION OF WAVES AT COMBINATION FREQUENCIES DUE TO RADIATION-SURFACE WAVE INTERACTION IN PLASMAS

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Abstract.

Electromagnetic waves radiated with combination frequencies from a semi-bounded plasma due to nonlinear interaction of radiation with surface wave (both of P-polarization) has been investigated. Waves are radiated both into vacuum and plasma are found to be P-polarized. We take into consideration the continuity at the plasma boundary of the tangential components of the electric field of the waves. The case of normal incidence of radiation and rarefied plasma layer is also studied.

Introduction

Excitation of surface waves in plasmas, their propagation, and the nonlinear phenomena accompanying their interaction with electromagnetic waves has shown recently a great interest in connection with, e.g., the generation and amplification of radio waves, the appearance of new methods for optical spectroscopy of condensed media intended for research on surfaces, interfaces and thin films [1,2].

One of the simplest examples of nonlinear interaction of slow waves in a plasma is the interaction of electromagnetic radiation, incident onto plasma layer, with surface waves. A semi-bounded plasma is one of the simplest models of a plasma wave-guide. Interaction of surface waves at the boundary of a semi-bounded plasma has been investigated by many authors, e.g., [3,4]. In these works, the authors assumed that the tangential components of the electric field of waves with combination frequencies were continuous at the plasma boundary. However, as it follows from the results in [5,6], in general case, these electric field components are discontinuous at the plasma boundary and the results in [5,6] can be valid only by an order of magnitude.

The problem of nonlinear interaction of two P-polarized surface waves at the boundary of a semi-bounded plasma in a "homogeneous plasma-vacuum" transition layers has been investigated in [6]. While in [7] the author considered the wave generation due to nonlinear interaction of two S-polarized waves (surface wave with an incident radiation) at a narrow inhomogeneous plasma layer.

In the present work we consider the same problem of [6,7] but with one of the interacting waves to be a radiation of P-polarization and solving accurately the equations for the fields at combination frequencies. Also we calculate the amplitude for generated waves at combination frequencies for normal incidence of radiation and plasma with low density.

STRUCTURE OF THE FUNDAMENTAL FIELDS

Let a P-polarized radiation fall from a vacuum on a semi-bounded plasma of width d to interact with a surface wave along the y-axis. Field components of interacting waves are :

$$\left\{\vec{E}_{1,2}\ ,\ \vec{H}_{1,2}\right\} \equiv \left\{ E_{(1,2)x}\ ,\ E_{(1,2)y}\ ,\ H_{(1,2)z} \right\}$$

where, $1,2$ indicate surface wave and radiation respectively.

The unperturbed plasma density n_o will have the profile

$$n_o = \begin{cases} o & ;\ \text{at} \quad x < o \\ n_o(x) & ;\ \text{at} \quad o < x < d \\ const. & ;\ \text{at} \quad x > d \end{cases} \qquad (1)$$

For cold plasma, and in the hydrodynamics approximation the wave propagation is described by:

$$\vec{\nabla} \times \vec{E} = -\frac{1}{C}\frac{\partial \vec{H}}{\partial t}\ ,\qquad \vec{\nabla} \times \vec{B} = \frac{1}{C}\frac{\partial \vec{E}}{\partial t} + \frac{4\Pi}{C}\vec{J} \qquad (2)$$

$$\vec{J} = -e\,(n_o + n)\,\vec{V}\ ,\qquad \frac{\partial n}{\partial t} + \nabla\left[(n_o + n)\,\vec{V}\right] = 0 \qquad (3)$$

$$\frac{\partial \vec{V}}{\partial t} + (\vec{V}.\vec{\nabla})\,\vec{V} = -\frac{e}{m}\left[\vec{E} + \frac{1}{C}(\vec{V} \times \vec{H})\right] \qquad (4)$$

where all terms have their usual meanings.

In the linear approximation, the electromagnetic fields of the interacting waves can be represented by:

$$\left\{\vec{E}(\vec{r},t)\ ;\ \vec{H}(\vec{r},t)\right\} \equiv \left\{E_1(x)\ ;\ H_1(x)\right\} \exp\left[i(k_1 y - \omega_1 t)\right] +$$

$$\left\{E_2(x)\ ;\ H_2(x)\right\} \exp\left[i(k_2 y - \omega_2 t)\right] + C.C.\ , \qquad (5)$$

and from (2-4) we get the following field solutions in the linear approximation :

$$E_{(1,2)x} = \frac{N_{1,2}}{\epsilon_{1,2}} H_{(1,2)z}\ ,\ E_{(1,2)y} = -i\frac{C}{\omega_{1,2}\epsilon_{1,2}} H'_{(1,2)z}\ , \qquad (6)$$

$$H_{1z}(x) = H_{10}\exp(\varkappa_{10}x) \;;\; H_{2z}(x) = H_{20}\left[\exp(i\varkappa_{20}x) - \alpha\exp(-i\varkappa_{20}x)\right], \quad \text{at } x \le 0 \qquad (7)$$

$$E_{1x} = \frac{N_1}{\epsilon_1}H_{10}\,, \quad E_{1y} = -i\frac{c\varkappa_{10}}{\omega_1}H_{10} \qquad \text{at } 0 \le x \le a \qquad (8)$$

$$E_{2x} \cong -2n_0\frac{\epsilon_{2a}}{\epsilon_2}\frac{\varkappa_{20}}{\varkappa_t}H_{20}\,, \quad E_{2y} = 2N_2\frac{\varkappa_{2a}\varkappa_{20}}{k_2\varkappa_t}H_{20} \qquad \text{at } 0 \le x \le a \qquad (9)$$

where,

$$N_{1,2} = \frac{k_{1,2}c}{\omega_{1,2}}\,, \quad \epsilon_{1,2} = 1 - \frac{\Omega^2}{\omega_{1,2}^2}\,, \quad \Omega^2 = \frac{4\pi e^2 n_0(x)}{m_e}$$

$$\varkappa_{10}^2 = k_1^2 - \frac{\omega_1^2}{c^2}\,, \quad \varkappa_{2a}^2 = -\left(k_2^2 - \frac{\omega_2^2}{c^2}\epsilon_{2a}\right), \quad \varkappa_{20}^2 = -\left(k_2^2 - \frac{\omega_2^2}{c^2}\right),$$

$$\varkappa_t = \varkappa_{2a} + \epsilon_{2a}\varkappa_{20}\,, \quad \alpha = \frac{\varkappa_{2a} - \epsilon_{2a}\varkappa_{20}}{\varkappa_t}\,, \quad \epsilon_{2a} = \epsilon_{1,2}(x \ge a)\,,$$

$$H'_{(1,2)z} = \frac{\partial}{\partial x}H_{(1,2)z}\,, \quad \frac{\partial}{\partial x}\left\{H_{(1,2)0}, \epsilon_{2a}, \varkappa_{(1,2)0}, \varkappa_{(1,2)a}\right\} = 0,$$

H_{10} and H_{20} are the amplitude of surface wave and incident radiation respectively, and α is the reflection coefficient of the incident radiation from the plasma layer.

To obtain the above relations we considered that the characteristic lengths of the waves under investigation are large compared to the width of the layer, i.e., $\varkappa_{1,2}d \ll 1$.

WAVE RADIATION AT COMBINATION FREQUENCIES ($\omega = \omega_1 + \omega_2$)

The magnetic field of waves radiated with combination frequencies $\omega = \omega_1 + \omega_2$ in vacuum ($x \le 0$) and in the region of the plasma homogeneity ($x \ge d$) looks like :

$$H_z(\mathbf{r},t) = H'_0\exp(\varkappa'_0 x + ky - \omega t) + C.C.\,, \qquad \text{at } x \le 0 \qquad (10)$$

$$H_z(\mathbf{r},t) = H'_a\exp(\varkappa'_a x + ky - \omega t) + C.C.\,, \qquad \text{at } x \ge d \qquad (11)$$

where,

$$\varkappa_a'^2 = \frac{\omega^2}{c^2}\epsilon'_a - k^2\,, \quad \varkappa_0'^2 = \frac{\omega^2}{c^2} - k^2\,, \quad k = k_1 + k_2$$

$$\epsilon' = 1 - \frac{\Omega^2}{\omega^2}\,, \quad \epsilon'_a = \epsilon'(x \ge a)\,,$$

H'_0 and H'_a are constant w.r.t $\mathbf{r}, t$ and represent the amplitudes of generated waves at regions $x \le 0$ and $x \ge d$ respectively. Following the methods used in [5,6], the values of these amplitudes are obtained:

$$H'_0 = \frac{1}{\varkappa'_t}\left[i\epsilon'_a\int_0^a P(x)\,dx + \varkappa'_a\int_0^a \epsilon'_a dx\int_0^x P(x')\,dx'\right] \qquad (12)$$

$$H'_a = H'_0 - \int_0^a \epsilon'_a dx\int_0^x P(x')\,dx' \qquad (13)$$

where, $\varkappa'_t = \varkappa'_a + \epsilon'_a\varkappa'_0$ and

$$P(x) = \frac{1}{c}\left[curl\,\frac{4\pi\vec{J}}{\epsilon'_a}\right]_z; \qquad (14)$$

$$4\pi\vec{J} = -i\frac{e}{m\,\omega_1\omega_2}\left[-\frac{\Omega^2}{\omega}grad(\vec{E}_1\cdot\vec{E}_2) + \frac{\vec{E}_1}{\omega_2}div(\Omega^2\vec{E}_2) + \frac{\vec{E}_2}{\omega_1}div(\Omega^2\vec{E}_1)\right]_z \qquad (15)$$

It is clear from (11)-(15) that waves with combination frequencies are radiated, with P-polarization, into vacuum when $\frac{K^2c^2}{\omega^2} < 1$, and into plasma and vacuum when $\frac{K^2c^2}{\omega^2} < \epsilon'_a$ provided that $\omega_{1,2} > 0$ and $k_1k_2 < 0$.

Using (8),(9),(14),(15) into (12),(13) we obtain the following expressions for H'_0 and H'_a :

$$H'_0 = A_1(A_2 + \varkappa'_a A_3)H_{10}H_{20}\,, \qquad H'_0 = A_1(A_2 - \varkappa'_0\epsilon'_a A_3)H_{10}H_{20}\,, \qquad (16)$$

where,

$$A_1 = \left(\frac{2\,e\,c\,k\varkappa_{20}}{m\,\omega_1^2\,\omega_2^2}\right)\left(\frac{1}{\varkappa'_t\,\varkappa_t}\right)$$

$$\mathcal{A}_2 = \frac{\Omega^2(a)}{\omega}\left[\varkappa_{10}\varkappa_{2a} + i\,\frac{k_1 k_2}{\epsilon_{1a}}\right] - i\,k_1\,k_2\,\epsilon'_a\epsilon_{2a}\,\tilde{\Omega}\,,$$

$$\mathcal{A}_3 = \frac{\Omega^2(a)}{k}\left[\frac{\varkappa_{10}\,k_3}{\omega_2} + i\,\frac{k_1\,\varkappa_{2a}}{\epsilon_{1a}\,\omega_1}\right]. \tag{17}$$

where,

$$\tilde{\Omega} = \frac{\Omega^2(a)\,\omega}{\omega_1\omega_2\epsilon_{1a}\epsilon_{2a}} - \frac{\omega_1^3\,\ln\epsilon_{1a}}{(2\omega_1+\omega_2)(\omega_1-\omega_2)} + \frac{\omega_2^3\,\ln\epsilon_{2a}}{(2\omega_2+\omega_1)(\omega_2-\omega_1)} +$$

$$+ \frac{\omega^3\,\ln\epsilon'_a}{(2\omega_1+\omega_2)(\omega_1+2\omega_2)}$$

If we consider the case of normal incidence of P-polarized waves ($k_z = 0$), and for a rarefied plasma,

$$\left(\frac{\Omega}{\omega_{1,2}}\,;\,\frac{\Omega}{\omega}\right) \ll 1,$$

we can derive the following expressions for the amplitudes of generated waves radiated at vacuum and plasma :

$$H'_0 = \mathcal{A}_4\left[\varkappa_{10}\left(\frac{\omega_2}{\omega}\right)+i\left(\frac{\omega_2}{\omega_1}\right)\right]H_{10}H_{20}, \quad H'_a = \mathcal{A}_4\left[\varkappa_{10}\left(\frac{\omega_2}{\omega}\right)-i\left(\frac{\omega_2}{\omega_1}\right)\right]H_{10}H_{20} \tag{18}$$

where,

$$\mathcal{A}_4 = \frac{e\,k_1\,\Omega^2(a)}{2\,m\,\omega_1^2\omega_2^2\,(\omega^2/c^2 - k_1^2)^{1/2}}$$

CONCLUSIONS

In our treatment we considered a semi-bounded plasma, which represent the simplest model for a plasma wave-guide. In this model plasma is inhomogeneous in some regions along x-direction ($0 \le x \le d$), and accordingly the wave vector projection onto this direction falls into a continuous value, i.e., the concept of wave

vector looses, strictly speaking, its sense. This increases the possibility of nonlinear interaction of waves. This also can be applied to the case of slow waves propagating along plasma waveguides.

In this work, as seen from (16) and (19), due to nonlinear wave interaction, a P-polarized waves are radiated with higher intensity in the vacuum than in the plasma ($H'_0 > H'_a$) , even for normal incidence of radiation. On the otherhand, waves are not emitted for normal incidence of S-polarized waves [7].

It is also clear from (16)-(18) that the amplitudes of generated waves at combination frequencies are strongly dependent on the plasma density distribution over the layer thickness.

It should be mentioned here that, in cases when the surface waves are caused by the initial perturbation and not sustained by an external source, their energy turns into the energy of the plasma's longitudinal oscillations in the region where $\epsilon_1(x) \simeq 0$ during time of order $(k_1 d\,\omega)^{-1}$ [4]. Besides the changing of amplitudes of surface waves at the expense of wave radiation at combination frequencies, under the conditions of applicability of a weakly nonlinear approximation [5], is much slower. Therefore, in our case, as in [6], it makes no sense to study the dependence of the amplitudes of the surface waves on time associated with the nonlinear interaction of these waves.

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