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Exotic Fermions in the Left-Right Symmetric Model

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A systematic study is made of non-standard fermion multiplets in left-right symmetric models with gauge group $SU(3)_C\otimes SU(2)_L\otimes SU(2)_R\otimes U(1)_{B-L}$. Constraints from gauge anomaly cancellation and invariance of Yukawa coupling terms are used to define interesting classes of exotic fermions. We identify the standard quark-lepton spectrum of left-right symmetric models as the simplest member of an infinite class. Phenomenological implications of the next simplest member of this class are then studied. We also consider classes of exotic fermions which may couple to the standard fermions through doublet Higgs bosons. We then show that some of these exotics may be used to induce a generalised universal see-saw mechanism.

I. INTRODUCTION

The left-right symmetric model (LRSM) with gauge group G_{LR} where

$$G_{LR} = SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \tag{1.1}$$

provides an attractive extension to the standard model (SM)[1]. In it, the standard quarks and leptons are placed symmetrically under G_{LR} and transform as

$$\begin{aligned} \psi_L &= \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \sim (1,2,1)(-1), \quad & \psi_R &= \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_R \sim (1,1,2)(-1) \\ Q_L &= \begin{pmatrix} u \\ d \end{pmatrix}_L \sim (3,2,1)(1/3), \quad & Q_R &= \begin{pmatrix} u \\ d \end{pmatrix}_R \sim (3,1,2)(1/3) \end{aligned} \tag{1.2}$$

per generation. Note that the fermion spectrum of the SM fits extremely smoothly into this left-right symmetric generalisation (the only drastic change being the presence of the right-handed neutrino). The purpose of this paper is to systematically study fermion structures in the LRSM. During this analysis we will compare and contrast the properties of exotic fermions as motivated by the SM with those motivated by left-right symmetry. In this way we hope to provide new insights into the relationship between these theories. Before commencing this study, however, it will be useful to briefly review the essential features of left-right symmetric gauge theory.

Usually, the Lagrangian of the model is made exactly parity invariant through the imposition of the left-right discrete symmetry

$$\psi_L \leftrightarrow \psi_R, \quad Q_L \leftrightarrow Q_R, \quad W_L \leftrightarrow W_R, \tag{1.3}$$

where $W_{L,R}$ are the gauge bosons of $SU(2)_{L,R}$. This parity invariance is then spontaneously broken through the nonzero vacuum expectation values (VEV's) of Higgs fields. This produces the observed disparity between left- and right-handed quarks and leptons in weak interactions. Phenomenologically, we know that the spontaneous breakdown of both parity and $SU(2)_R$ happens at a scale significantly higher than the ordinary electroweak scale (the lower bound is of the order of a few TeV's).

Spontaneous symmetry breaking (SSB) may be implemented in a number of ways in the left-right symmetric model. The generalisation of the SM Higgs doublet is obtained through the Yukawa Lagrangian $\mathcal{L}^{(1)}_{Yuk}$ where

$$\mathcal{L}^{(1)}_{Yuk} = h_1\overline{\psi}_L\psi_R\phi + \tilde{h}_1\overline{\psi}_L\psi_R\tilde{\phi} + h_2\overline{Q}_L Q_R\phi + \tilde{h}_2\overline{Q}_L Q_R\tilde{\phi} + \text{H.c.} \tag{1.4}$$

The Higgs field ϕ is classified by

$$\phi \sim (1,2,2)(0) \tag{1.5}$$

under G_{LR}, and $\tilde{\phi}$ is its charge-conjugate field given by $\tilde{\phi} = \tau_2\phi^*\tau_2$. Under left-right symmetry,

$$\phi \leftrightarrow \phi^\dagger. \tag{1.6}$$

A nonzero VEV for ϕ generates masses for the quarks and leptons, but cannot induce a high enough breaking scale for parity or $SU(2)_R$, and it obviously cannot break $U(1)_{B-L}$. We therefore have to expand the Higgs sector.

Two main ways of doing this have been explored in the literature. The most common scenario uses triplet Higgs multiplets[2] defined via the Lagrangian $\mathcal{L}^{(2)}_{Yuk}$ where

$$\mathcal{L}^{(2)}_{Yuk} = \lambda(\overline{\psi^c_L}\psi_L\Delta_L + \overline{\psi^c_R}\psi_R\Delta_R) + \text{H.c.} \tag{1.7}$$

The quantum numbers of the Higgs fields $\Delta_{L,R}$ are

$$\Delta_L \sim (1,3,1)(2), \qquad \Delta_R \sim (1,1,3)(2), \tag{1.8}$$

and under left-right symmetry

$$\Delta_L \leftrightarrow \Delta_R. \tag{1.9}$$

Nonzero VEV's for $\Delta_{L,R}$ separate the breaking scales of $SU(2)_{L,R}$, induce $U(1)_{B-L}$ breaking and generate nonzero Majorana masses for the neutrinos. Phenomenologically, the hierarchy $\langle\Delta_L\rangle \ll \langle\phi\rangle \ll \langle\Delta_R\rangle$ is necessary, which also leads to the standard see-saw form for the neutrino mass matrices.

An alternative Higgs sector is provided through the fields $\chi_{L,R}$ where[3]

$$\chi_L \sim (1,2,1)(-1), \qquad \chi_R \sim (1,1,2)(-1), \qquad \chi_L \leftrightarrow \chi_R. \tag{1.10}$$

A priori, it seems attractive to consider the introduction of just the fields $\chi_{L,R}$ (i.e. suppose no ϕ or $\Delta_{L,R}$ multiplets are used). They appear to be the minimal multiplets needed to achieve G_{LR} breaking. There are, however, two problems with this. First, if the discrete symmetry $\chi_L \leftrightarrow \chi_R$ is exact in the Higgs potential, then the parity breaking minima have the VEV of either χ_L or χ_R as exactly zero[3, 4]. Second, these fields have the wrong quantum numbers to couple to quarks and leptons. One consequence of this is that fermion masses remain zero. The first problem may be overcome by either enlarging the Higgs sector[3, 4] or by not demanding an exactly parity invariant Lagrangian. The second problem can only be overcome by introducing additional, and hence exotic, fermions [5]. If appropriately chosen, these exotic fermions could also be used to give nonzero masses to quarks and leptons through mixing. This possibility has in fact been studied within the so-called "universal see-saw (US) mechansim" scenario[6]. Later on in this paper we will do a complete analysis of exotic fermions which can couple to ordinary quarks and leptons through $\chi_{L,R}$. We will defer discussion of the universal see-saw mechanism until then.

The main aim of this paper is, as stated previously, to do a thorough analysis of possible exotic fermions in the LRSM. The problem of how to couple quarks and leptons to $\chi_{L,R}$ Higgs bosons in this theory is just one of the motivations for this study. Since we have very little theoretical knowledge about the fermion spectrum itself (in either the SM or the LRSM), the possible existence of exotic fermions is an interesting topic in its own right.

The remainder of this paper is structured as follows: Section II briefly reviews the exotic fermion story in the SM. Sections III and IV then examine exotic fermions in the LRSM. Section III studies direct generalisations of the standard left-right symmetric quark and lepton multiplets. We will compare the exotic fermions occuring here to those occuring in the SM. Section IV contains a systematic analysis of exotic fermions motivated through coupling to $\chi_{L,R}$. Emphasis will be placed on the mass spectra produced by having such exotic fermions present. Do they reproduce the qualitative features of the observed fermion mass spectra? We look at ways to generalise the universal see-saw mechanism by incorporating multiplets other than singlets, and show that this is possible with certain combinations of exotics with standard fermions. We conclude in Section V.

II. REVIEW OF EXOTIC FERMIONS IN THE SM

Consider the standard quark-lepton generation in the SM, with gauge group $G_{SM} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$:

$$(1,2)(-1)_L \oplus (1,1)(-2)_R \oplus (3,2)(1/3)_L \oplus \begin{Bmatrix} (3,1)(4/3)_R \\ (3,1)(-2/3)_R \end{Bmatrix}. \qquad (2.1)$$

This is the minimal set of representations needed to account for the currently known fermion spectrum (we restrict ourselves to one family here for simplicity). Equation (2.1) has several important features:

(i) Gauge anomalies cancel within this generation - in particular, the $SU(2)_L \otimes U(1)_Y$ anomalies of the leptons cancel with those of the quarks.

(ii) A single Higgs doublet $\delta \sim (1,2)(1)$ is sufficient to couple LH and RH leptons together, and LH and RH quarks together. These Yukawa couplings produce fermion masses after SSB.

We can adopt these as the conditions under which any exotic fermion is introduced. Then the structure of a general fermion generation under condition (ii) is:

$$(N_c, N)(y)_R \oplus \begin{Bmatrix} (N_c, N-1)(y+1)_L \\ (N_c, N-1)(y-1)_L \end{Bmatrix} \oplus \begin{Bmatrix} (N_c, N-2)(y+2)_R \\ (N_c, N-2)(y)_R \\ (N_c, N-2)(y-2)_R \end{Bmatrix} \oplus \cdots$$

$$\cdots \oplus \begin{cases} (N_c, 2)(y+N-2)_{L or R} \\ (N_c, 2)(y+N-4)_{L or R} \\ \cdot \\ \cdot \end{cases} \tag{2.2}$$

where $N_c = 1$ or 3. Note that multiplets in adjacent columns can couple to each other in the Yukawa Lagrangian if their hypercharges differ by +1 or -1, through δ or $\tilde{\delta}$ [where $\tilde{\delta} = i\tau_2\delta^* \sim (1,2)(-1)$]. Also note that the first multiplet $(N_c, N)(y)_R$ could equally be chosen as $(N_c, N)(y)_L$ - as long as the adjacent columns alternate in chirality.

Next we impose condition (i), keeping in mind that anomaly cancellations may occur either within the leptons and within the quarks separately (leptonlike and quarklike generations respectively), or that anomalies of the leptons may cancel those of the quarks (quark-leptonlike generation). Reference [7,8] shows the details of the procedure in cancelling anomalies, so we only quote those results relevant to the discussions that follow. This will allow for easier comparison between the SM case and the LRSM case.

In reference [7], it was shown that the simplest nontrivial example of a leptonlike generation that is free of gauge and global anomalies is given by

$$(1,3)(0)_{L,R} \oplus \left\{ \begin{matrix} (1,2)(+1)_{R,L} \\ (1,2)(-1)_{R,L} \end{matrix} \right\} \oplus (1,1)(0)_{L,R} \tag{2.3}$$

(ie. anomalies cancel completely within this generation). This is a member of a series defined by

$$(1,N)(0)_{L,R} \oplus \left\{ \begin{matrix} (1,N-1)(+1)_{R,L} \\ (1,N-1)(-1)_{R,L} \end{matrix} \right\} \oplus (1,N-2)(0)_{L,R} \tag{2.4}$$

Note that the N=2 case has a global anomaly[9]. By making the fermions in Eq. (2.4) triplets under $SU(3)_c$ one can similarly define a general quarklike generation.

The generalized quark-leptonlike family is given by

$$(1,N)(Y_l)_{L,R} \oplus \left\{ \begin{matrix} (1,N-1)(Y_l+1)_{R,L} \\ (1,N-1)(Y_l-1)_{R,L} \end{matrix} \right\} \oplus (1,N-2)(Y_l)_{L,R}$$

$$\oplus(3,N)(Y_q)_{L,R} \oplus \left\{ \begin{matrix} (3,N-1)(Y_q+1)_{R,L} \\ (3,N-1)(Y_q-1)_{R,L} \end{matrix} \right\} \oplus (3,N-2)(Y_q)_{R,L}. \tag{2.5}$$

Cancellation of the $[SU(2)_L]^2U(1)_Y$ anomaly requires $Y_q = -Y_l/3$.

When $N = 2$, and one of the two possible chirality assignments is chosen, this case gives essentially the standard quarks and leptons with a right-handed neutrino:

$$(1,2)(Y_l)_L \oplus \left\{ \begin{matrix} (1,1)(Y_l+1)_R \\ (1,1)(Y_l-1)_R \end{matrix} \right\} \oplus (3,2)(-Y_l/3)_L \oplus \left\{ \begin{matrix} (3,1)(+1-Y_l/3)_R \\ (3,1)(-1-Y_l/3)_R \end{matrix} \right\} \tag{2.6}$$

This spectrum yields precisely the standard quarks and leptons (together with a right-handed neutrino), provided $Y_l = -1$. Note the freedom one has in assigning the value of Y_l. This just reflects the hypercharge quantization problem of a quark-lepton generation which includes a RH neutrino. If one were to remove the RH neutrino, then anomaly-cancellation would enforce the $Y_l = -1$ condition[7].

The next simplest quark-lepton generation is obtained by putting $N = 3$ in Eq. (2.5) with the condition $Y_q = -Y_l/3$. The result is:

$$(1,3)(Y_l)_{L,R} \ni \begin{Bmatrix} (1,2)(Y_l+1)_{R,L} \\ (1,2)(Y_l-1)_{R,L} \end{Bmatrix} \oplus (1,1)(Y_l)_{L,R}$$

$$\oplus(3,3)(-Y_l/3)_{L,R} \oplus \begin{Bmatrix} (3,2)(-Y_l/3+1)_{R,L} \\ (3,2)(-Y_l/3-1)_{R,L} \end{Bmatrix} \oplus (3,1)(-Y_l/3)_{L,R} \tag{2.7}$$

We will compare this generation with its equivalent in the LRSM in the following section.

In summary, by having only one Higgs doublet and imposing anomaly cancellation within a generation [conditions (i) and (ii)], the standard fermion spectrum of Eq. (2.1) can be shown to be a special case of a more general class, of which Eq. (2.7) is the next simplest case.

III. GENERALISED STANDARD GENERATION

A. Anomaly cancellation

Recall that the general condition for cancellation of triangle anomalies is

$$Tr(T_L^a\{T_L^b, T_L^c\}) - Tr(T_R^a\{T_R^b, T_R^c\}) = 0 \tag{3.1}$$

where $\{T_L^a\}$ are the generators of the group for LH multiplets, and $\{T_R^a\}$ are the generators of the same group for RH multiplets.

In the LRSM, the discrete L↔R symmetry puts the following constraints which are relevant to anomaly cancellations:

(i) The number of a particular $(N, n_1, n_2)(l)_L$ multiplet always equals the number of its RH partner, $(N, n_2, n_1)(l)_R$. For instance, if there are five copies of $(1,3,2)(-1)_L$ present, there are also five copies of $(1,2,3)(-1)_R$ present.

(ii) The $U(1)_{B-L}$ quantum number of a LH multiplet always equals its RH partner.

These two factors ensure that all triangle anomalies cancel trivially, *except* the $[SU(2)_L]^2U(1)_{B-L}$ and $[SU(2)_R]^2U(1)_{B-L}$ anomalies. To find the conditions for cancellation of these anomalies, we first write down a general quark-lepton multiplet as follows:

$$(1, n_1, n_2)(l)_L + (1, n_2, n_1)(l)_R + (3, m_1, m_2)(b)_L + (3, m_2, m_1)(b)_R. \tag{3.2}$$

The $[SU(2)_L]^2U(1)_{B-L}$ anomaly coefficient [ie. LHS of Eq. (3.1)] of Eq. (3.2) is then given by:

$$A_{n_1,n_2,l} + 3A_{m_1,m_2,b} = (q_{n_1}n_2 - q_{n_2}n_1)l + 3(q_{m_1}m_2 - q_{m_2}m_1)b \tag{3.3}$$

where q_n is an SU(2) factor defined as $Tr(T^a_n T^b_n) \equiv q_n\delta^{ab}$ for an n dimensional representation under SU(2), and is proportional to $(n+1)n(n-1)$. The $[SU(2)_R]^2U(1)_{B-L}$ anomaly coefficient for the same multiplets is simply given by $A_{n_2,n_1,l} + 3A_{m_2,m_1,b}$. Eq. (3.3) implies that the simplest non-trivial leptonlike generation that is anomaly free is given by

$$(1,3,1)(0)_{R,L} + (1,1,3)(0)_{L,R}. \tag{3.4}$$

[Note that $(1,2,1)(0)_{R,L}+(1,1,2)(0)_{L,R}$ has global anomaly.] This is the LRSM analog of Eq. (2.3). Of course we can also have the quarklike generation $(3,3,1)(0)_{R,L}$ + $(3,1,3)(0)_{L,R}$. We will be looking at these multiplets in Section IV.

Substituting for q_n in Eq. (3.3) gives

$$A_{n_1,n_2,l} + 3A_{m_1,m_2,b} = n_1n_2[(n_1+1)(n_1-1) - (n_2+1)(n_2-1)]l$$
$$+3m_1m_2[(m_1+1)(m_1-1) - (m_2+1)(m_2-1)]b. \tag{3.5}$$

Hence the standard lepton multiplet of $\psi_L + \psi_R$ gives $A_{2,1,-1} = -6$, which is cancelled out by the standard quark multiplet of $Q_L + Q_R$. Now similar anomaly cancellation features result for the general quark-lepton generation of

$$(1,n,n-1)(l)_L + (1,n-1,n)(l)_R + (3,n,n-1)(-l/3)_L + (3,n-1,n)(-l/3)_R \tag{3.6}$$

We may call this the "generalised standard generation". Note the following points:

(i) It is the LRSM analog of Eq. (2.5) with $b = -l/3$ put in, so that the anomalies of the leptons cancel those of the quarks.

(ii) The simplest case of $n = 2$ and $l = -1$ gives the standard generation (SG).

(iii) The LH fields can couple to its RH partner through the bidoublet Higgs ϕ because $\overline{\Psi_L}\Psi_R \sim (1,2,2)(0)$ [where $\Psi_L \sim (1,n,n-1)(l)_L, \Psi_R \sim (1,n-1,n)(l)_R$ or their quark equivalents].

B. Exotic-standard fermion mixing

Now we look in some detail at the next simplest case of $n = 3$ in Eq. (3.6). Let's call this the triplet-doublet generation (TDG). As was the case in the SM, the chirality of the leading multiplet in Eq. (3.6) is arbitrary, so we choose it to be RH, anticipating the coupling of this TDG to the SG through ϕ. This will produce non-diagonal mass terms after SSB. Invariance of such Yukawa coupling terms under G_{LR} also forces us to choose $l = -1$. Of course, if it exists, the TDG may have $l \neq -1$ and not couple to the SG (ignoring any other more complicated scenarios), but we choose to explicitly analyse the above most interesting case. At any rate, it will be easy to apply the work that follows to any value of l.

1. Lepton sector

Consider first the leptonic part of the TDG. Let $\Psi_R \sim (1,3,2)(-1)$ and $\Psi_L \sim$

$(1,2,3)(-1)$. The first interesting point is that as

$$SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \rightarrow SU(2)_L \otimes U(1)_Y,$$

where $Y/2 = I_{3R} + (B-L)/2$. $\Psi_R + \Psi_L$ reduces to

$$\begin{aligned} (1,3,2)(-1)_R &\rightarrow (1,3)(0)_R \oplus (1,3)(-2)_R \\ (1,2,3)(-1)_L &\rightarrow (1,2)(1)_L \oplus (1,2)(-1)_L \oplus (1,2)(-3)_L. \end{aligned} \tag{3.7}$$

This looks very different to the analogous generation in the SM of $(1,3)(-1)_R \oplus (1,2)(0)_L \oplus (1,2)(-2)_L \oplus (1,1)(-1)_R$, as given in Eq. (2.7). This is in contrast to the standard lepton generation of $(1,2,1)(-1)_L + (1,1,2)(-1)_R$ which resembles its SM analog of Eq. (2.6) when it is viewed under $SU(2)_L \otimes U(1)_Y$. This is interesting because it manifests a difference between the LRSM and the SM at the fermion spectrum level when guided by anomaly free generalisations of the ordinary fermions. LRSM is only one of many possible extensions of the SM, and perhaps future experiments comparing the likelihood of the TDG's existence against its SM analog of Eq. (2.7) may give us some indication whether we are on the right track with the LRSM or not.

Next we look at the mass relations amongst the members of the TDG. The exotic leptons $\Psi_R + \Psi_L$ may couple to each other via ϕ or $\tilde{\phi} \sim (1,2,2)(0)$. Remembering that the electric charge assignment under G_{LR} is given by $Q = I_{3L} + I_{3R} + \frac{B-L}{2}$, we can let Ψ_L be such that

$$\Psi_{L1_j}{}^k = \text{(j,k) element of} \begin{pmatrix} N_1^0/\sqrt{2} & f^+ \\ E_1^- & -N_1^0/\sqrt{2} \end{pmatrix}_L$$

$$\Psi_{L2_j}{}^k = \text{(j,k) element of} \begin{pmatrix} E_2^-/\sqrt{2} & N_2^0 \\ p^{--} & -E_2^-/\sqrt{2} \end{pmatrix}_L . \tag{3.8}$$

Hence under G_{LR},

$$\Psi_{L a j}{}^{k} \rightarrow U_{L a}{}^{b} U_{R j}{}^{m} U^{\dagger}_{R n}{}^{k} \Psi_{L b m}{}^{n}. \tag{3.9}$$

Also

$$\phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} \text{ and } \phi_a{}^{k} \rightarrow U_{L a}{}^{b} \phi_b{}^{n} U^{\dagger}_{R n}{}^{k}. \tag{3.10}$$

The Yukawa couplings between Ψ_L and Ψ_R are

$$\mathcal{L}_{\Psi\Psi} = H \overline{\Psi_L}{}^{a}{}_{jk} \phi^{bj} \Psi_R{}^{k}{}_{ab} + \tilde{H} \overline{\Psi_L}{}^{a}{}_{jk} \tilde{\phi}^{bj} \Psi_R{}^{k}{}_{ab} + \text{H.c.} \tag{3.11}$$

SSB occurs via $\langle\phi\rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix}$, upon which we obtain the following mass spectrum:

mass eigenstate fields	mass	
p^{--}	m_p	
f^{+}	m_f	(3.12)
N_1^0, N_2^0	$\frac{m_p}{4}\left\|1 \pm \sqrt{1 + \frac{m_f^2}{2m_p^2}}\right\|$	
E_1^-, E_2^-	$\frac{m_f}{4}\left\|1 \pm \sqrt{1 + \frac{2m_p^2}{m_f^2}}\right\|$	

This shows the mass relations amongst the particles explicitly. Similar relations will occur in the quark sector as well. Since no mixing with the SG has been introduced, the mass spectrum of Eq. (3.12) is valid for all values of l (with different electric charges on the fields, of course). In general, such mass relations will occur for any generalised quark-lepton generation of Eq. (3.6) if $n > 2$, since there are 2 Yukawa couplings but $n(n-1)$ lepton fields and $n(n-1)$ quark fields.

We now go on to include the Yukawa couplings of the TDG to ordinary fermions. With $\psi_L + \psi_R$ being the standard lepton doublets, the relevant Yukawa couplings in the lepton sector are now:

$$\mathcal{L}_{Yuk} = h\overline{\psi_L}\phi\psi_R + \tilde{h}\overline{\psi_L}\tilde{\phi}\psi_R + H\overline{\Psi_L}\phi\Psi_R + \tilde{H}\overline{\Psi_L}\tilde{\phi}\Psi_R +$$

$$Y(\overline{\psi_L}\Phi\Psi_R + \overline{\psi_R}\Phi\Psi_L) + \tilde{Y}(\overline{\psi_L}\tilde{\Phi}\Psi_R + \overline{\psi_R}\tilde{\Phi}\Psi_L) + \text{H.c.} \tag{3.13}$$

with indices suppressed. After SSB, we obtain

$$(\bar{\nu}\ \overline{N_1}\ \overline{N_2})_L M_N \begin{pmatrix} \nu \\ N_1 \\ N_2 \end{pmatrix}_R + (\bar{e}\ \overline{E_1}\ \overline{E_2})_L M_E \begin{pmatrix} e \\ E_1 \\ E_2 \end{pmatrix}_R \quad \text{where}$$

$$M_N = \begin{pmatrix} h\kappa + \tilde{h}\kappa' & Y\kappa + \tilde{Y}\kappa' & Y\kappa' + \tilde{Y}\kappa \\ Y\kappa + \tilde{Y}\kappa' & (H\kappa + \tilde{H}\kappa')/2 & (H\kappa' + \tilde{H}\kappa)/\sqrt{2} \\ Y\kappa' + \tilde{Y}\kappa & (H\kappa' + \tilde{H}\kappa)/\sqrt{2} & 0 \end{pmatrix} \tag{3.14}$$

$$\text{and} \quad M_E = \begin{pmatrix} h\kappa' + \tilde{h}\kappa & Y\kappa + \tilde{Y}\kappa' & Y\kappa' + \tilde{Y}\kappa \\ Y\kappa + \tilde{Y}\kappa' & 0 & (H\kappa + \tilde{H}\kappa')/\sqrt{2} \\ Y\kappa' + \tilde{Y}\kappa & (H\kappa + \tilde{H}\kappa')/\sqrt{2} & (H\kappa' + \tilde{H}\kappa)/2 \end{pmatrix} \tag{3.15}$$

Note that in the limit of $Y \sim \tilde{Y} \to 0$, we get $m_\nu \to h\kappa + \tilde{h}\kappa'$, $m_e \to h\kappa' + \tilde{h}\kappa$ and the masses of the exotics in the order of $(\kappa + \kappa')H$ (if $H \cong \tilde{H}$). In this case, as in the standard model, h and $\tilde{h}$ must be "fine-tuned" to obtain the observed mass difference between e and ν_e. A priori, there is no reason why Y or $\tilde{Y}$ should be very small, but because putting $Y, \tilde{Y} = 0$ does increase the symmetry of the Lagrangian, it is "technically natural" and experimentally indicated by the suppression of FCNCs.

To get an order of magnitude indication on the phenomenological upper bound of the Yukawa couplings within this hierarchy of $Y \ll h \ll H$, consider the mass matrix in the charged sector with all three families taken into account. Then

$$M_E \simeq k \begin{pmatrix} h_{11} & h_{12} & h_{13} & Y_1 & Y_1 \\ h_{21} & h_{22} & h_{23} & Y_2 & Y_2 \\ h_{31} & h_{32} & h_{33} & Y_3 & Y_3 \\ Y_1 & Y_2 & Y_3 & 0 & H/\sqrt{2} \\ Y_1 & Y_2 & Y_3 & H/\sqrt{2} & H/2 \end{pmatrix}, \tag{3.16}$$

where $k = \kappa + \kappa'$, $Y \cong \hat{Y}$, $H \cong \hat{H}$, and the basis is $(e', \mu', \tau', E_1', E_2')$ (primes indicating weak eigenstates). We set $h_{ij} = 0$ if $i \neq j$ for simplicity. Mass eigenvalues consistent with small mixing are $M_E^D \sim k\,diag[h_{11}, h_{22}, h_{33}, H/2, H]$. We observe that non-zero values of Y's will produce FCNCs. There is a stringent bound on $\Gamma(\mu \to 3e)/\Gamma(\mu \to all)(\lesssim 10^{-12}$[11]) and somewhat less stringent bound on $\Gamma(\tau \to 3e)/\Gamma(\tau \to all)(\lesssim 10^{-5}$[11]). Taking these into account, we obtain following estimates for the Ys:

$$Y_1 \lesssim 10^{-5} h_{11}, \quad Y_2 \lesssim 10^{-4} h_{22}, \quad Y_3 \lesssim 10^{-3} h_{33}. \tag{3.17}$$

2. Quark sector

The quark sector of the TDG comprises of the exotic fields:

$$P^{-\frac{1}{3}}, \quad F^{+\frac{2}{3}}, \quad U_1^{+\frac{2}{3}}, \quad U_2^{+\frac{2}{3}}, \quad D_1^{-\frac{1}{3}}, \quad D_2^{-\frac{1}{3}}. \tag{3.18}$$

These will couple to the standard quarks and produce mass matrices similar to Eqs. (3.14) and (3.15). An estimate for the Yukawa couplings similar to Eq. (3.17) can be made from the measurement of the mass difference between K_L and K_S[11] and gives:

$$Y_1 \lesssim 10^{-3} h_{11}, \quad Y_2 \lesssim 10^{-2} h_{22}, \quad Y_3 \lesssim 10^{-1} h_{33}. \tag{3.19}$$

C. Production and detection

The phenomenology of the TDG will depend on the masses of the exotics, their mixing with the standard fermions, and their gauge couplings. Having already made some comments on the first two factors, here we would like to show how the gauge coupling of the TDG differs from that of the standard generation. This will also enable us to point out that if they exist, the masses of the exotic fermions arising from the generalised standard generation are constrained to be heavy because of their gauge couplings.

To see this, note that for a general $(1, N, N-1)(-1)_L$ multiplet, we have:

$$D^\mu = \partial^\mu + ig_L\lambda_L.\mathbf{W}^\mu_L + ig_R\lambda_R.\mathbf{W}^\mu_R - i\frac{g'}{2}B^\mu \tag{3.20}$$

where $\lambda_{L,R}$ are the generators of $SU(2)_{L,R}$. For the standard doublet ψ_L, $\lambda = \tau/2$, where τ are the usual Pauli matrices. From this one can show that $W\bar{\psi}\psi$ vertex coupling is proportional to $\frac{g_L}{2}$. For Ψ_L of the TDG, to be consistent with the normalisation in the doublet case, we use $\lambda = \mathbf{T}$ where

$$T_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad T_2 = \frac{i}{\sqrt{2}}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \quad T_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{3.21}$$

From this one can show that the $W\bar{\Psi}\Psi$ vertex factor is $\propto g_L$ instead of $\frac{g_L}{2}$.

Because the decay widths of W and Z bosons are related to the gauge couplings by $\Gamma(W \to f_1 f_2) \propto (W f_1 f_2 \text{ vertex factor})^2$ and similarly for Γ_Z (although it is slightly more complicated due to mixing), if these exotic fermions are light enough there will be an appreciable change in the decay rates of W and Z. Such a marked increase in the widths of W and Z cannot fit the observed values, hence placing a lower limit of

~45 GeV on the masses of the exotics. The same considerations will apply to the non-singlet exotics in Section IV.

IV. HIGGS-DOUBLET MOTIVATED EXOTIC FERMIONS

As we mentioned in the introduction, having the simplest Higgs fields of χ_L (1,2,1)(-1) + χ_R (1,1,2)(-1) means we need to include exotic fermions to generate non-zero masses for ordinary quarks and leptons. In this section we will look at the mass spectra produced from couplings of such exotic fermions to the standard fermions. A brief description of the US mechanism may be in order here.

In the original US mechanism [6], the authors attempt to account for the mass hierarchy of $m_\nu \ll m_{e,u,d} \ll m_W$ within the LRSM. They introduce singlet fermions under G_{LR} with tree-level masses $\sim M$. These singlets couple to the standard fermions through χ_L and χ_R with respective VEV's of η_L and η_R. A hierarchy of $\eta_L \ll \eta_R \ll M$ implies fermion masses of $m(\nu_L) \sim \eta_L^2/M \ll m(e,u,d) \sim \eta_L\eta_R/M < m(W_L)$. This is the essence of the US mechanism, with the standard fermion masses being inversely proportional to the exotic singlet fermion masses (hence the term "see-saw").

In section III it was shown that LR discrete symmetry implies that only $[SU(2)_L]^2U(1)_{B-L}$ and $[SU(2)_R]^2U(1)_{B-L}$ anomalies are potentially non-zero. Since these anomalies cancel within a standard lepton-quark generation, the simplest anomaly free exotic fermions will have to be in multiplets of the form (N,N) under $SU(2)_L \otimes SU(2)_R$ [$n_1 = n_2$, $m_1 = m_2$ case in Eq. (3.5)] if the $B-L$ charge is non-zero. Multiplets with $B-L=0$ will obviously be anomaly free. We will look at such cases below. Table I. lists all the fermions that can couple to the standard fermions.

A. Lepton-like Generations

We choose the following four types of exotic leptons to investigate, each of which is anomaly free independant of the others (see Table I):

$$\begin{aligned} &E_{L,R} \sim (1,1,1)(-2) \qquad B_R \sim (1,2,2^*)(0) + B_L \sim (1,2^*,2)(0) \\ &N_{L,R} \sim (1,1,1)(0) \qquad T_R \sim (1,3,1)(0) + T_L \sim (1,1,3)(0) \end{aligned} \tag{4.1}$$

The leptons $B'_{L,R}$ and $T'_{L,R}$ are not included in this list because the former will be produce very similar effects as $B_{L,R}$ above, while the latter requires the presence of other multiplets to cancel its non-vanishing anomalies hence complicating the spectrum. The four types in Eq. (4.1) have the advantage that any set of them may be included simultaneously.

The most general Yukawa Lagrangian is then

$$\begin{aligned} \mathcal{L}_Y = {} & Y_N^L \overline{\psi_L} \chi_L N_R + i Y_m^L \psi_L^T C \tau_2 \tilde{\chi}_L N_L + Y_E^L \overline{\psi_L} \tilde{\chi}_L E_R + Y_B^R \overline{\psi_L} B_R \chi_R + \\ & \tilde{Y}_B^R \psi_L^T C \tilde{B}_L \tilde{\chi}_R + Y_T^R \overline{\psi_L} T_R \chi_L + (L \leftrightarrow R) + \text{ H.c.} \end{aligned} \tag{4.2}$$

where $\tilde{\chi} = i\tau_2 \chi^*$ and $\tilde{B}_{L,R} = iB_{L,R}^T \tau_2$. In addition, there are the bare mass terms:

$$\mathcal{L}_{mass} = m_N \overline{N} N + m_E \overline{E} E + m_B Tr(\overline{B} B) + m_N^L N_L^T C N_L + m_N^R N_R^T C N_R. \tag{4.3}$$

SSB occurs via $\langle \chi_L \rangle = \begin{pmatrix} \eta_L \\ 0 \end{pmatrix}$ and $\langle \chi_R \rangle = \begin{pmatrix} \eta_R \\ 0 \end{pmatrix}$ [$\eta_R > \eta_L$ provides the observed disparity between the right and left handed weak interactions. We also mention here that in Ref. [4], it is shown that an additional parity-odd Higgs field of $\sigma \sim (1,1,1)(0)$ is one way of ensuring that neither η_L nor η_R is zero. Since σ can only couple to $\overline{N} N$ and $\overline{E} E$, the VEV of σ may be thought of as being included in m_N and m_E].

Let

$$T_L = \begin{pmatrix} t_1^0/\sqrt{2} & t_2^+ \\ t_3^- & -t_1^0/\sqrt{2} \end{pmatrix}_L \qquad \text{and} \qquad B_L = \begin{pmatrix} b_1^0 & b_2^+ \\ b_3^- & b_4^0 \end{pmatrix}_L . \tag{4.4}$$

The first observation that can be made is that including T will produce a charged massless fermion, t_2^+, because in $Y_T^R \overline{\psi_L} T_R \chi_L \xrightarrow{SSB} Y_T^R \eta_L [\overline{\nu_L} t^0_{1R} + \overline{e_L} t^-_{3R}]$, a t_2^+ mass term does not occur at all. So we discard T immediately [12]. Note that the same problem does not arise for the other exotics due to the bare mass terms.

The charged lepton mass matrix for the remaining multiplets is:

$$M_c = \begin{pmatrix} 0 & Y_B^R \eta_R & \tilde{Y}_B^R \eta_R^* & Y_E^L \eta_L \\ Y_B^L \eta_L^* & m_B & 0 & 0 \\ \tilde{Y}_B^L \eta_L & 0 & m_B & 0 \\ Y_E^R \eta_R & 0 & 0 & m_E \end{pmatrix} \quad \text{in } (e\, b_3\, b_2^c\, E)_R \text{ basis,} \tag{4.5}$$

while the neutral lepton mass matrix is:

$$M_n = \begin{pmatrix} 0 & 0 & Y_m^L \eta_L^* & Y_N^L \eta_L & 0 & Y_B^R \eta_R \\ 0 & 0 & Y_N^R \eta_R^* & Y_m^R \eta_R & Y_B^L \eta_L^* & \tilde{Y}_B^L \eta_L \\ Y_m^L \eta_L^* & Y_N^R \eta_R^* & m_N^L & m_N & 0 & 0 \\ Y_N^L \eta_L & Y_m^R \eta_R & m_N & m_N^R & 0 & 0 \\ \tilde{Y}_B^R \eta_R^* & Y_B^L \eta_L^* & 0 & 0 & 0 & m_B \\ Y_B^R \eta_R & 0 & 0 & 0 & m_B & 0 \end{pmatrix} \quad \text{in } (\nu^c\, \nu\, N^c\, N\, b_1^c\, b_1)_R \text{ basis.} \tag{4.6}$$

Note that b_4 does not couple to any other field hence its mass is m_B.

In order to get a semi-quantitative feel for the mass and mixing angle pattern we will assume that $Y_N \sim Y_E \sim Y_B \sim \tilde{Y}_B$, and use the hierarchy of

$$l \equiv Y^L \eta_L \ll r \equiv Y^R \eta_R \ll m_B \sim m_E \tag{4.7}$$

for caluculations in each sector.

(i) Charged lepton sector:

In terms of mass eigenvalues, it is found that all the combinations of leptons from the set of $\{\psi, B, E\}$, namely $\{\psi, E\}$, $\{\psi, B\}$ and $\{\psi, B, E\}$, produce the US mechanism. While the $\{\psi, E\}$ case produces the usual singlet-induced US mechanism[6], the $\{\psi, B\}$ case produces mass matrix of:

$$M_c \sim \begin{pmatrix} 0 & r & r \\ l & m_B & 0 \\ l & 0 & m_B \end{pmatrix}. \tag{4.8}$$

By considering the symmetric matrix $M_c M_c^\dagger$, we find, using Eq. (4.7), that two of the its eigenvalues are $\sim m_B^2$, m_B^2 and thus, by assumption, very heavy. Let the third eigenvalue be λ_3. We observe that:

$$\begin{aligned} &(i)\lambda_3 \to 0 \text{ as } l \to 0 \\ &(ii)\lambda_3 \to 0 \text{ as } m_B \to \infty \\ &(iii)\mathrm{Det}[M_c M_c^\dagger] = \prod(\text{eigenvalues}) = 4l^2 r^2 m_B^2. \end{aligned} \tag{4.9}$$

Hence

$$\lambda_3 \sim \frac{4l^2 r^2}{m_B^2} \tag{4.10}$$

This establishes a generalised US mechanism for charged lepton masses for this case. The next case is $\{\psi, B, E\}$, which is of particular interest because it has two large scales: m_B and m_E. The mass matrix looks like:

$$M_c \sim \begin{pmatrix} 0 & r & r & l \\ l & m_B & 0 & 0 \\ l & 0 & m_B & 0 \\ r & 0 & 0 & m_E \end{pmatrix}. \tag{4.11}$$

Again by following the same analysis as the previous case, we obtain three eigenvalues of $M_c M_c^\dagger$ as $\sim m_B^2$, m_B^2, m_E^2 and the fourth one to be $\sim l^2 r^2 \left(\frac{2m_E + m_B}{m_E m_B}\right)^2$. This shows that a generalised US mechanism is once again obtained.

(ii) Neutral lepton sector

With the inclusion of the bare masses m_N^L and m_N^R, there are too many free parameters present for a coherent analysis to be performed. We may leave them out for the sake of simplicity. Then the US mechanism survives for $\{\psi, N\}$ case, with Majorana masses as shown by Davidson and Wali in Ref. [6]. With $\{\psi, B\}$, the mass matrix is

$$M_n \sim \begin{pmatrix} 0 & 0 & 0 & r \\ 0 & 0 & l & l \\ r & l & 0 & m_B \\ r & 0 & m_B & 0 \end{pmatrix} \text{ in } (\nu^c\ \nu\ b_1^c\ b_1)_R \text{ basis.} \tag{4.12}$$

Within the hierarchy of $l \ll r \ll m_N \sim m_B$, the same sort of numerical calculations as in the charged lepton case show that the smallest eigenmass is inversely proportional to the tree level exotic lepton mass m_B, establishing the US mechanism again.

Next case to be considered is $\{\psi, B, N\}$, with the mass matrix of

$$M_n \sim \begin{pmatrix} 0 & 0 & l & l & 0 & r \\ 0 & 0 & r & r & l & l \\ l & r & 0 & m_N & 0 & 0 \\ l & r & m_N & 0 & 0 & 0 \\ r & l & 0 & 0 & 0 & m_B \\ r & 0 & 0 & 0 & m_B & 0 \end{pmatrix} \text{ in } (\nu^c\ \nu\ N^c\ N\ b_1^c\ b_1)_R \text{ basis.} \tag{4.13}$$

This again results in a small eigenmass that is inversely proportional to m_N and m_B.

B. Quark-like Generations

As in the lepton sector, the simplest exotic quark-like fermions to introduce are the bidoublet and the singlets since their anomalies cancel trivially:

$$U_{L,R} \sim (3,1,1)(4/3) \quad D_{L,R} \sim (3,1,1)(-2/3) \quad \Sigma_R \sim (3,2,2^*) + \Sigma_L \sim (3,2^*,2) \tag{4.14}$$

[Again, $\Sigma'_{R,L}$ (see Table I) will produce very similar effects as $\Sigma_{R,L}$.] If we let $Q_L \sim (3,2,1)(1/3)$ be the standard quark doublet, then we know from the arguments analogous to those in the lepton sector that any combinations of Q with $\{U, D, \Sigma\}$ will induce a US mechanism. Including the triplet is more complicated in the quark sector than in the lepton sector since we cannot now simply assign its B-L charge to be zero if it were to couple to $Q_{L,R}$. Table I shows in fact that $\Lambda_{L,R}$ will require the additional combination of quarks and leptons, $\Lambda'_{L,R} + T'_{L,R}$, to be anomaly free. As was shown earlier, such models with triplets are not viable due to the presence of massless charged fermions.

V. CONCLUSION

We have seen how the generation $(1,n,n-1)(l)_L + (1,n-1,n)(l)_R + (3,m,m-1)(-l/3)_L + (3,m-1,m)(-l/3)_R$ is the generalisation of the standard generation in the LRSM. $N = 3$ and $l = -1$ gives the next simplest multiplet which looks different to the corresponding $N = 3$ case in the SM when viewed under $SU(2)_L \otimes U(1)_Y$. This suggests a way to distinguish between the two models at the fermion spectrum level. Gauge couplings constrain the masses of exotics arising from such non-standard multiplets to be heavy (ie. masses at least at the weak gauge boson mass scale).

We then examined the following exotic fermions which couple to the standard fermions via the Higgs doublets χ_L and χ_R:

$$B_{L,R} \sim (1,2,2)(0) \qquad \Sigma_{L,R} \sim (3,2,2)(4/3)$$

$$E_{L,R} \sim (1,1,1)(-2) \qquad U_{L,R} \sim (3,1,1)(4/3)$$

$$N_{L,R} \sim (1,1,1)(0) \qquad D_{L,R} \sim (3,1,1)(-2/3)$$

$$T_L \sim (1,3,1)(0) + T_R \sim (1,1,3)(0)$$

Discarding the triplet because of a massless charged fermion problem, we examined the mass spectrum produced and saw that US mechanism can be induced with any combination of the other exotics and the standard fermions.

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TABLES

Exotic fermion	Couples to	A_L	A_R
$N_R(1,1,1)(0)$	$\overline{\nu_L}\chi_L$	0	0
$N_L(1,1,1)(0)$	$\overline{\nu_R}\chi_R$	0	0
$E_R(1,1,1)(-2)$	$\overline{\nu_L}\tilde{\chi}_L$	0	0
$E_L(1,1,1)(-2)$	$\overline{\nu_R}\tilde{\chi}_R$	0	0
$B_R(1,2,2^*)(0)$	$\overline{\nu_L}\chi_R$	0	0
$B_L(1,2^*,2)(0)$	$\overline{\nu_R}\chi_L$	0	0
$B'_R(1,2,2^*)(-2)$	$\overline{\nu_L}\tilde{\chi}_R$	4	4
$B'_L(1,2^*,2)(-2)$	$\overline{\nu_R}\tilde{\chi}_L$	−4	−4
$T_R(1,3,1)(0)$	$\overline{\nu_L}\chi_L$	0	0
$T_L(1,1,3)(0)$	$\overline{\nu_R}\chi_R$	0	0
$T'_R(1,3,1)(-2)$	$\overline{\nu_L}\tilde{\chi}_L$	8	0
$T'_L(1,1,3)(-2)$	$\overline{\nu_R}\tilde{\chi}_R$	0	−8
$U_R(3,1,1)(4/3)$	$\overline{Q_L}\chi_L$	0	0
$U_L(3,1,1)(4/3)$	$\overline{Q_R}\chi_R$	0	0
$D_R(3,1,1)(-2/3)$	$\overline{Q_L}\tilde{\chi}_L$	0	0
$D_L(3,1,1)(-2/3)$	$\overline{Q_R}\tilde{\chi}_R$	0	0
$\Sigma_R(3,2,2^*)(4/3)$	$\overline{Q_L}\chi_R$	−8	8
$\Sigma_L(3,2^*,2)(4/3)$	$\overline{Q_R}\chi_L$	8	−8
$\Sigma'_R(3,2,2^*)(-2/3)$	$\overline{Q_L}\tilde{\chi}_R$	4	4
$\Sigma'_L(3,2^*,2)(-2/3)$	$\overline{Q_R}\tilde{\chi}_L$	−4	−4
$\Lambda_R(3,3,1)(4/3)$	$\overline{Q_L}\chi_L$	−16	0
$\Lambda_L(3,1,3)(4/3)$	$\overline{Q_R}\chi_R$	0	16
$\Lambda'_R(3,3,1)(-2/3)$	$\overline{Q_L}\tilde{\chi}_L$	8	0
$\Lambda'_L(3,1,3)(-2/3)$	$\overline{Q_R}\tilde{\chi}_R$	0	−8

TABLE I. List of all exotic fermions that can couple to the standard fermions via the Higgs doublets χ_L and χ_R. The third and fourth columns are the anomaly coefficients of $[SU(2)_L]^2U(1)_{B-L}$ and $[SU(2)_R]^2U(1)_{B-L}$ respectively [proportional to $n_1n_2(n_1+1)(n_1-1)/$ and $-n_1n_2(n_2+1)(n_2-1)/$ from Eq. (3.5)]