

High Spin Studies with Radioactive Ion Beams

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ABSTRACT

The variety of new research possibilities afforded by the culmination of the two frontier areas of nuclear structure: high spin and studies far from nuclear stability (utilizing intense radioactive ion beams) are discussed. Topics presented include: new regions of exotic nuclear shape (e.g. superdeformation, hyperdeformation, and reflection-asymmetric shapes); the population of and consequences of populating exotic nuclear configurations; and complete spectroscopy (i.e. the overlap of state of the art low- and high-spin studies in the same nucleus).

I. INTRODUCTION

Probably today's two most exciting frontiers of nuclear structure are rapidly-rotating nuclei and nuclei far from stability. The current renaissance in both frontier areas is based on recent technical innovations: *to wit*, the advent of large Compton-suppressed germanium detector arrays (e.g. EUROGAM and GAMMASPHERE) based on the ability to grow large crystals of high-purity germanium,¹ and new breakthroughs in producing and accelerating high-intensity beams of radioactive ions. Therefore, it is imperative to consider the culmination of these two initiatives, high-spin research possibilities afforded by intense beams of radioactive ions.

The large angular momentum imparted to the residues of heavy-ion induced fusion-evaporation reactions produces two major nuclear structure effects: (i) a stabilization of highly-deformed nuclear shapes; and (ii) a major rearrangement of the nuclear single-particle states generated by the Coriolis and centrifugal forces acting on the nucleons due to the rotation of the nucleus. The increased stability of superdeformed nuclei is well known to this audience. The modification of the single-particle spectrum of states by rotation is illustrated in Fig. 1. The Coriolis plus centrifugal term in the single-nucleon hamiltonian⁵ ($= -\omega j_1$, where ω is the angular frequency of rotation, and j_1 is the component of the intrinsic nucleonic angular momentum along the rotational axis) can produce perturbations as large as those resulting from nuclear deformation. Such rotationally-induced single-particle effects are the bases of a variety of high-spin phenomena as diverse as "backbending" (band crossings),^{6,7} loss of pair correlations,^{8,9} the nuclear shrinking paradox,⁴ and an enhancement of residual interactions¹⁰ based on an increased overlap of nucleonic states.^{4,11}

Radioactive ion beams will provide experimental access to a variety of even more exotic shapes and combinations of single-particle orbitals in nuclei far from stability. Some examples are described in Sections 2 and 3, respectively. Likewise, the possibility and consequences of comprehensive high spin studies of nuclei, whose low angular momentum

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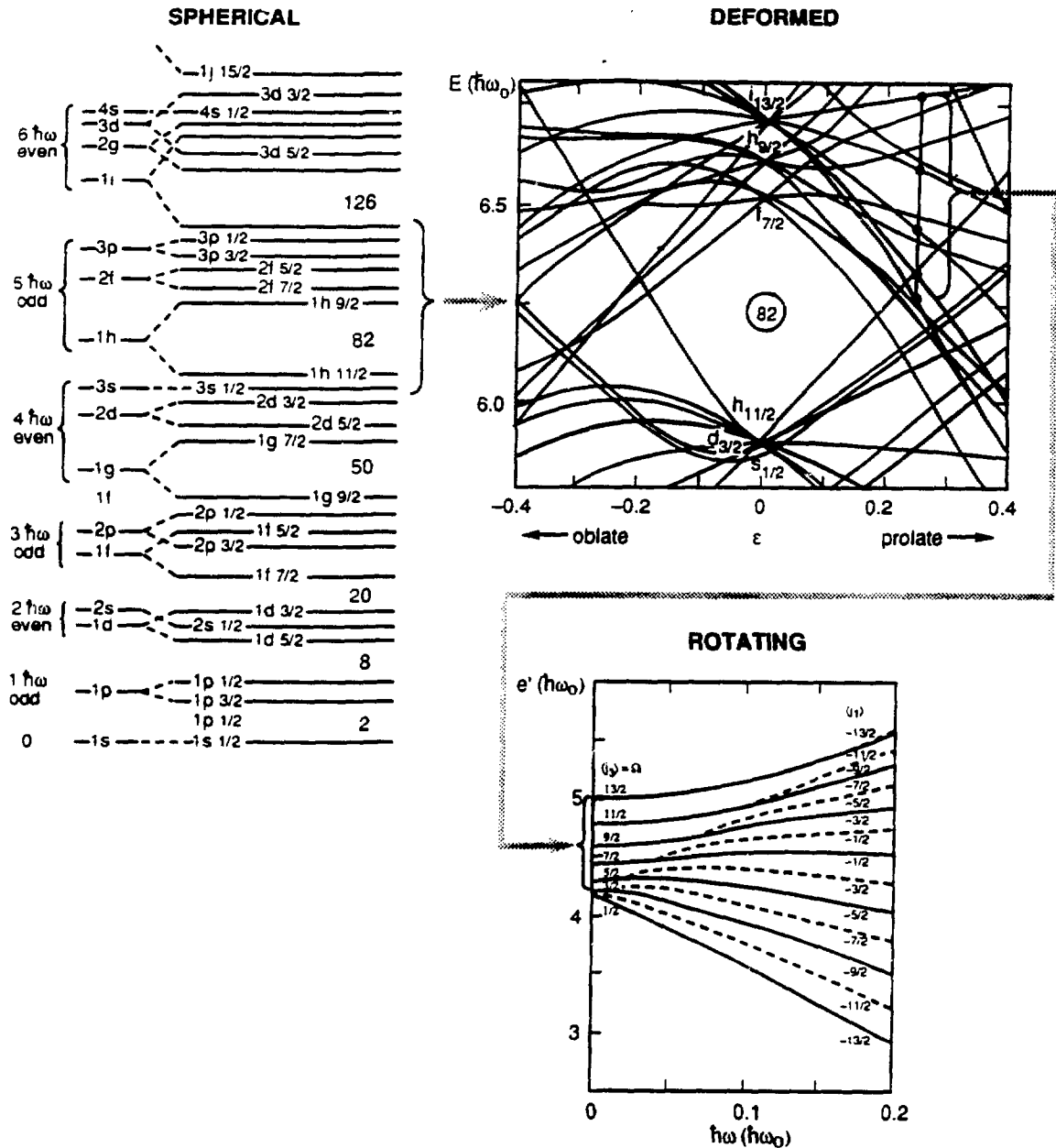


Fig. 1. Comparison of the spectra of single-particle states from spherical, deformed, and rotating deformed nuclear potentials taken from refs. 2-4, respectively (see also Figs. 4 and 6). Because of the greater complexity of the single-particle states with increasingly-complex approximations to the nuclear potentials, only the subset of states indicated in the figure is shown for the deformed and rotational cases.

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properties also are well known, are discussed in Section 4. Whereas the neutron-rich portion of this research will require large ISOL facilities, for example, the proposed IsoSpin Laboratory,¹² nearly all of the proton-rich studies also can be made using "first generation" facilities, such as the Oak Ridge RIB Facility.¹³ For a more comprehensive discussion of the structure of rapidly-rotating nuclei with radioactive beams the reader is referred to ref. 14.

II. EXOTIC NUCLEAR SHAPES ACCESSIBLE WITH RADIOACTIVE ION BEAMS

The observations of high-spin superdeformed states (2:1 ratio of the major to minor axes) in $A = 150$ and 190 nuclei (refs. 15,16 and 17,18, respectively) and deformations intermediate between normal and superdeformed in $A \approx 130$ and 186 (refs. 16, 19 and 20, respectively) have focused an intense interest on highly-deformed nuclear shapes. Such states correspond to secondary minima in the nuclear potential energy at large deformations resulting from major gaps in the single-particle levels.²¹⁻²³ This striking illustration of the interplay of single-particle and collective degrees of freedom was developed²¹ to describe isomeric fission of actinide nuclei.^{24,25} Already in 1968 it was realized that such secondary minima should also exist in lighter nuclei; indeed the proton and neutron numbers of the occurrence of such shell corrections were predicted.²⁶ However, it wasn't until 1986 that the technique of populating superdeformed minima at high spin and observing the gamma-ray cascade based on the rotating superdeformed intrinsic state succeeded¹⁵ in establishing additional superdeformed states at high spin.

The high-spin superdeformed states near $A = 150$ and 190 and the actinide fission isomers all occur at or near predicted minima in both the proton and neutron shell energy, see Fig. 2. Other regions of the table of isotopes where the proton and neutron shell energies are predicted to act coherently to produce additional minima in the potential energy at superdeformed deformations, also are indicated in this figure, as are the projected "new" nuclei that can be studied¹⁴ using the radioactive ion beams planned for the IsoSpin Laboratory.¹² Though a large number of "new" neutron-rich nuclides become accessible to study with radioactive beams, these cannot be populated by heavy-ion fusion-evaporation reactions, the traditional technique for studying high-spin superdeformed states.^{16,18} Therefore, new techniques are necessary to populate high-spin superdeformed states to the right of the valley of beta stable nuclei. For example, Coulomb excitation of neutron-rich radioactive beams can provide some high spin information, but this process has a vanishingly-small probability of populating superdeformed states. On the other hand, Coulomb excitation plus few-particle transfer may offer some chance of studying superdeformation in these neutron-rich nuclei; however, the associated cross sections also are small at very high spin. So far, the probability of populating superdeformed states has not been demonstrated by these mechanisms.

Indeed, the known $A \approx 150$ and 190 high-spin superdeformed nuclei^{16,18} are on the proton-rich side of stable nuclei, see Fig. 2. Although some of the best candidates^{28,29} of the light superdeformed nuclei, $Z = 33-35$ and $38-42$ and $N = 29-47$ are accessible with stable beams and targets and have been investigated, no discrete transitions attributed to superdeformed states have been observed.³⁰ This lack of success may be the result of the

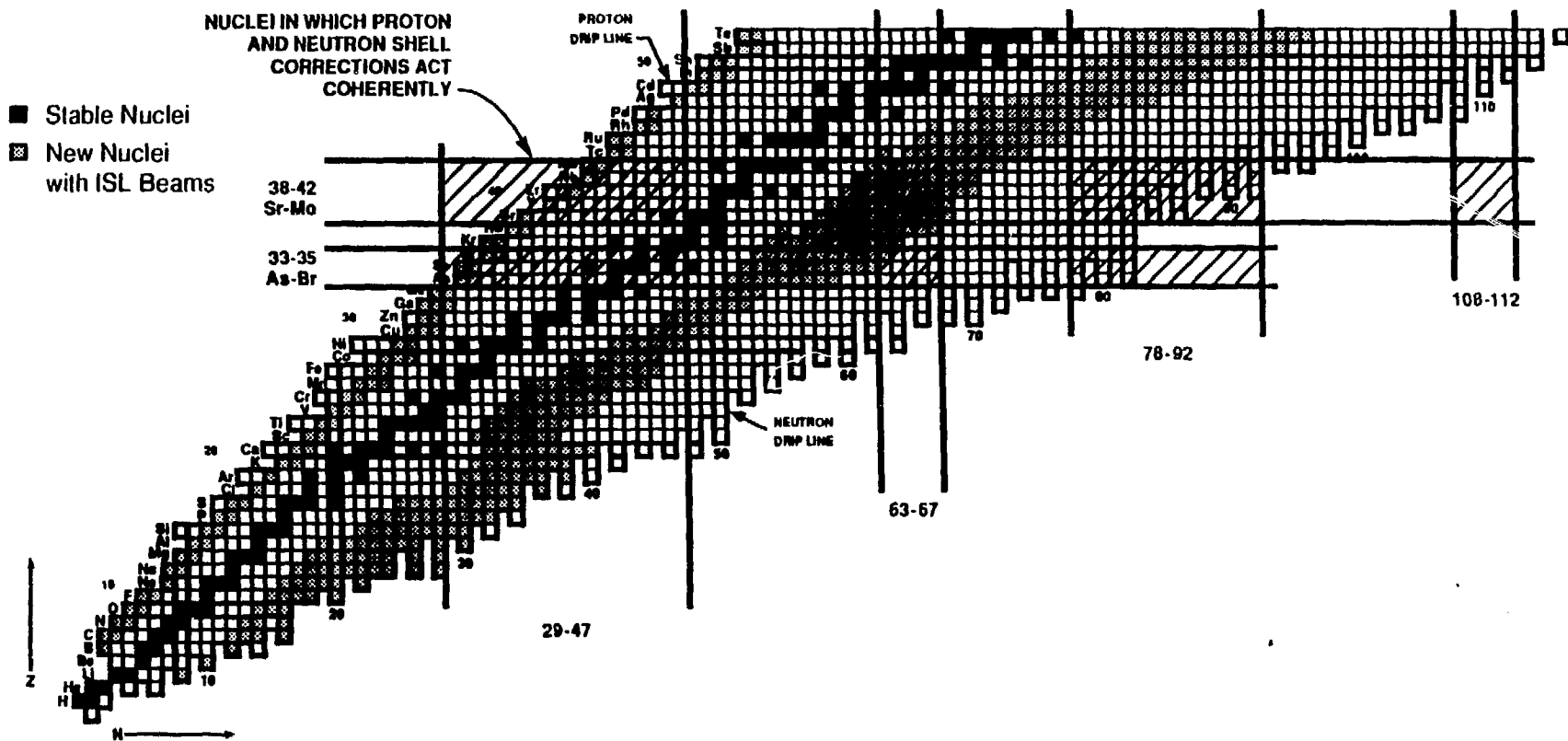
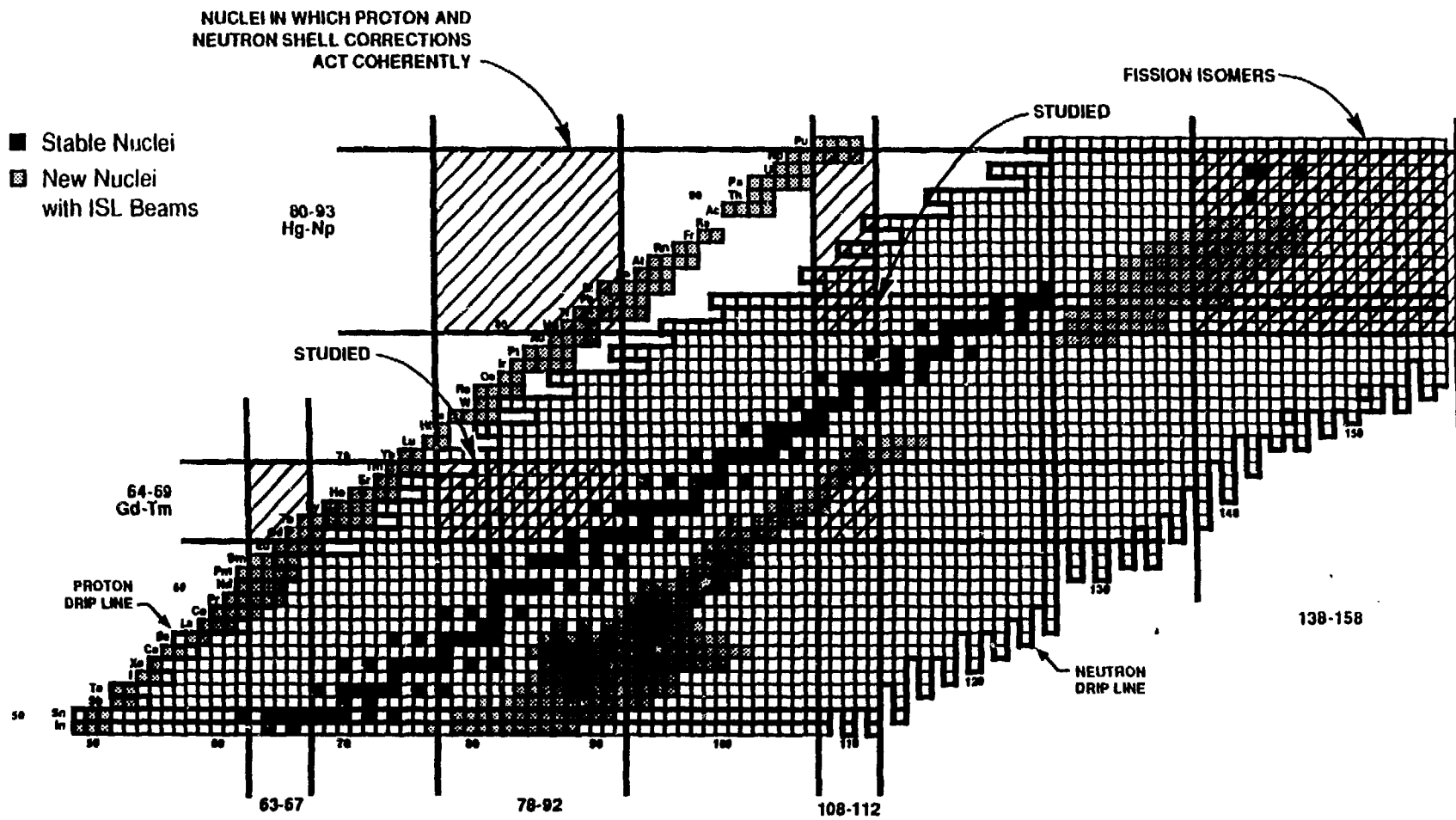


Fig. 2. Comparison of regions of nuclei in which the proton and neutron shell energy corrections act coherently at high spins producing stable prolate superdeformed shapes and the "new" nuclei which can be produced with high intensity radioactive ion beams. The proton and neutron numbers of the minimum shell energy corrections are taken from refs. 45 and 46, and the regions of overlapping proton and neutron shell energy minima are indicated by diagonal shading. The lower Z boundary of the heaviest regions has been decreased to include Hg, Tl, and Pb, since superdeformed states have been identified for isotopes of these elements.¹⁸ The predicted "new" nuclei that can be studied with radioactive ion beams, but which cannot be studied with stable beams and stable targets are estimated¹⁴ using the projected beams of the IsoSpin Laboratory.¹² Smaller "first generation" facilities, such as the Oak Ridge RIB Facility¹³ could produce most of the proton-rich nuclei indicated with $Z \leq 82$, see ref. 14. See next page for heavier nuclei.



large Doppler shift and low photopeak efficiency associated with the high-energy gamma-ray transitions between superdeformed states of light nuclei (due to the smaller moment of inertia). It also may be that the nuclei with the most stable superdeformed minimum at the lowest excitation energy (i.e. the best candidates for superdeformation) in this mass region are too proton-rich to be populated with stable beams and targets.³¹

Two unstudied proton-rich regions of superdeformation that should become accessible to experiments with radioactive ion beams are indicated by Fig. 2. These are the light ($N = 63-67$) isotopes of Gd - Tm nuclei ($Z = 64-69$) and the light ($N = 70-92$) isotopes of Hg - Np ($Z = 80-93$). Indeed, the proton and neutron numbers for the very proton-rich mercury - lead nuclei suggested to be superdeformed are the combination of the proton number of the known superdeformed mercury - lead nuclei and the neutron number of the known superdeformed europium - dysprosium nuclei. However, the larger ratio of protons to neutrons will lead to increased fission especially at high angular momentum for these very proton-rich heavy nuclei. Thus, the lighter region of very proton-rich Gd - Tm ($Z = 64-69$) isotopes perhaps is the favored new region for superdeformation studies with proton-rich radioactive ion beams.

The preceding discussion of possible new regions of superdeformation accessible to radioactive ion beams is primitive; such a discussion could have been given nearly a quarter century ago.²⁶ The complete potential energy surface, minimized with respect to a variety of deformations, must be calculated systematically for the new nuclei that can be populated at high spin using radioactive ion beams. The first results of a systematic program of such state-of-the-art calculations for the mercury isotopes³² are shown in Fig 3. Indeed, these calculations correctly predict the minima associated with the known low-lying normally-deformed prolate states ($\beta_2 \approx 0.25$) in $^{180-190}\text{Hg}$ (ref. 33) and the known superdeformed states¹⁸ in $^{189-194}\text{Hg}$ ($\beta_2 \approx 0.47$) that coexist with the oblate ground-state configuration. These superdeformed states, predicted to occur at an excitation energy of between 3 and 4 MeV for $^{188,190}\text{Hg}$ in the absence of rotation ($\omega = 0$), should become yrast at $I \approx 30-40$. An unobserved high-lying hyperdeformed (3:1 ratio of the major to minor axis) configuration ($\beta_2 \approx 0.8$, $\beta_3 \approx 0$) also is predicted for $^{188,190}\text{Hg}$. Another superdeformed configuration ($\beta_2 \approx 0.56$, $\beta_3 \approx 0$, but "soft" with respect to the reflection-assymeric, β_3 , degree of freedom is predicted to occur quite low in excitation energy for some of the light mercury isotopes that are on the verge of experimental study at high spin with stable beams and targets and which should be easily accessible with radioactive beams. This superdeformed configuration can be associated with the shell corrections shown in Fig. 2, and described in the preceding paragraphs of this section. Likewise, a hyperdeformed ($\beta_2 \approx 0.8$, $\beta_3 \approx 0.15$) configuration is predicted at an ever lower excitation energy for $^{178-184}\text{Hg}$ than for the heavier mercury isotopes.

Both the superdeformed and hyperdeformed minima predicted for the lightest mercury isotopes, see the calculations for ^{170}Hg and ^{180}Hg shown in Fig. 3, are "soft" with respect to the octupole degree of freedom. This "softness" can probably be attributed to the occurrence of both proton and neutron $1i_{13/2} - 2f_{7/2} \Delta\ell = 3$ pairs near the Fermi level for these lightest mercury isotopes at large deformation, see Fig. 4. The lowest fission barrier of these nuclei is associated with a large octupole deformation. Indeed, the predicted decrease in the fission barrier associated with the octupole degree of freedom could lead to isomeric fission for the light mercury isotopes!

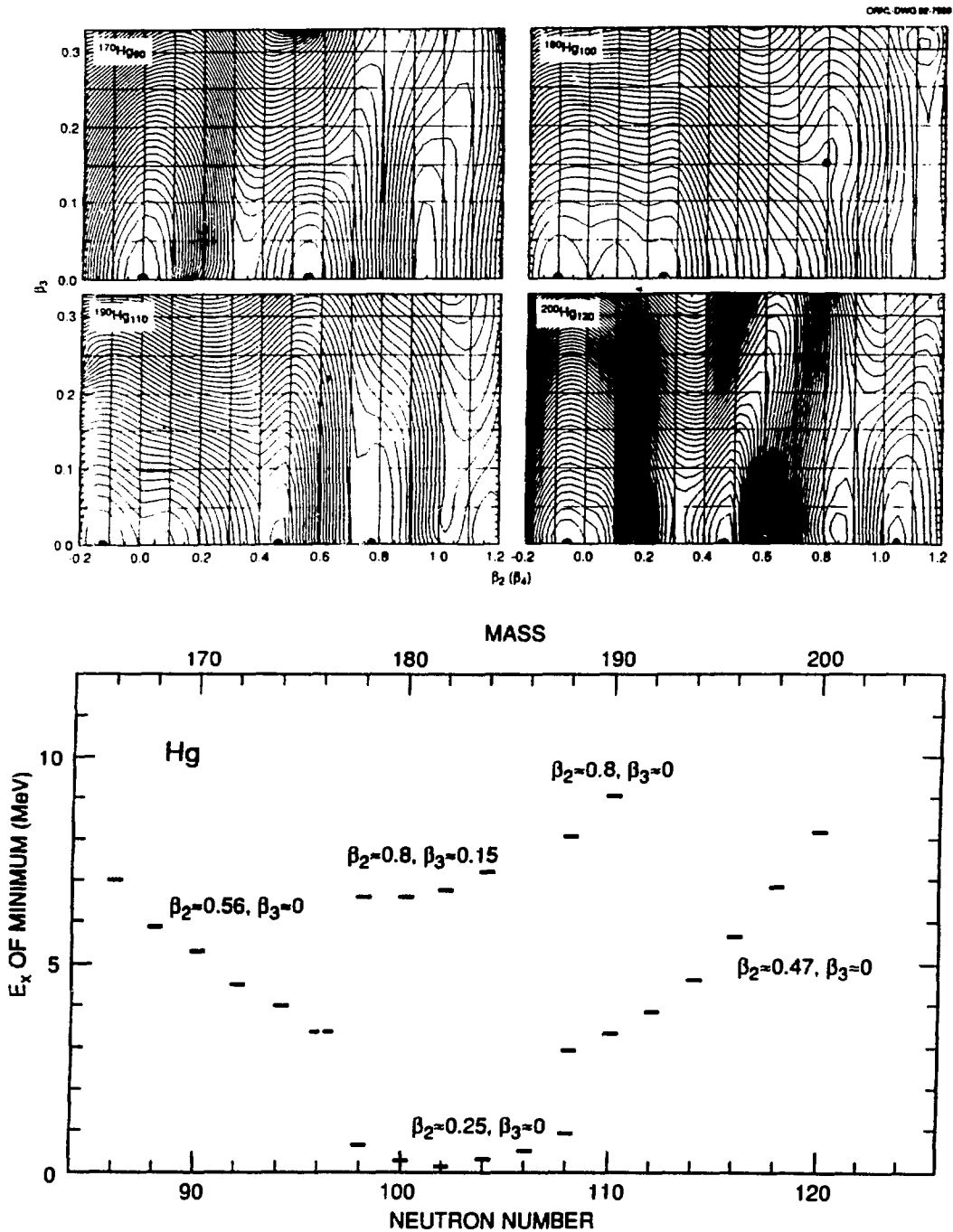


Fig. 3. (Top) Calculated zero rotational frequency potential energy surfaces³² for $^{170}\text{Hg}_{90}$, $^{180}\text{Hg}_{100}$, $^{190}\text{Hg}_{110}$, and $^{200}\text{Hg}_{120}$ as a function of the quadrupole deformation, β_2 , and the octupole deformation, β_3 . These calculations also have been minimized with respect to β_4 - β_8 deformations. The loci of the various minima are indicated by solid points. (Bottom) Calculated excitation energies of selected secondary minima in the potential energy surface for mercury isotopes. The approximate β_2 and β_3 values are given for each set of minima, and the complete potential energy surfaces for $^{170,180,190,200}\text{Hg}$ are shown in above.

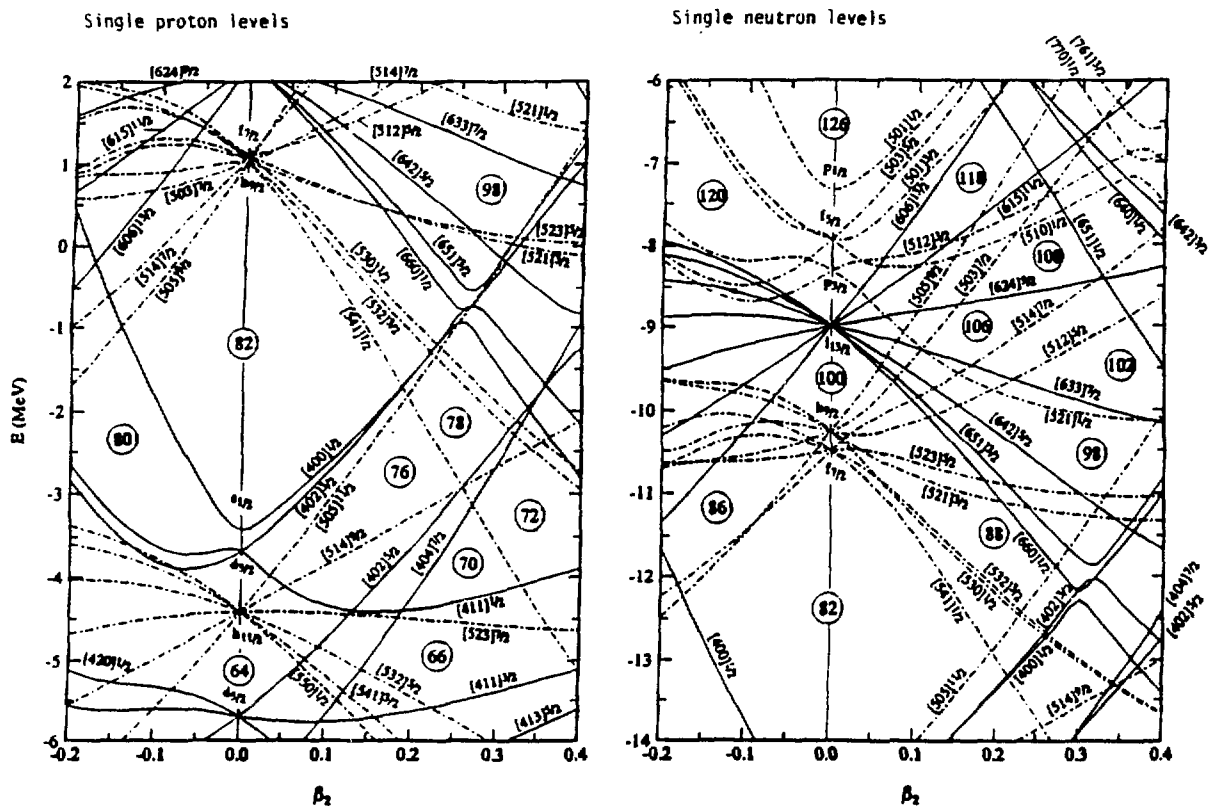


Fig. 4. Plot of predicted single-proton (left) and single-neutron (right) energies for Nilsson configurations as a function of the quadrupole deformation (β_2). These single-particle levels, calculated using Woods-Saxon potential³⁴ and assuming $\beta_4 = \gamma = 0$, are taken from ref. 35.

III. EXOTIC NUCLEAR CONFIGURATIONS ACCESSIBLE WITH RADIOACTIVE BEAMS

The major rearrangement of the spectrum of single-particle states associated both with a prolate quadrupole deformation of the mean nuclear field (most deformed nuclei are prolate) and with nuclear rotation (see Fig. 1) have a common feature. Both favor (i.e. decrease the energy of) the high- j , low- Ω configurations. (Ω , the projection of j on the nuclear symmetry axis, $= 1/2, 3/2, \dots, j$.) These are the states whose wave functions are most highly localized. The spacial localization of the high- j Nilsson states is a result of the quantization of axially-deformed nuclei (nonrotating) with respect to Ω . Plots of the probability distribution of wave functions as a function of the intrinsic angular momentum and the angles of the orbital median plane, shown in Fig. 5, help to demonstrate the concept of quantal localization. The localization is enhanced for large j , since a large number of states (reflecting the large number of Ω values) must be accommodated in the same phase space, i.e. angular range. Likewise, for a specific j the localization is further enhanced for small values of Ω , see refs. (4,12).

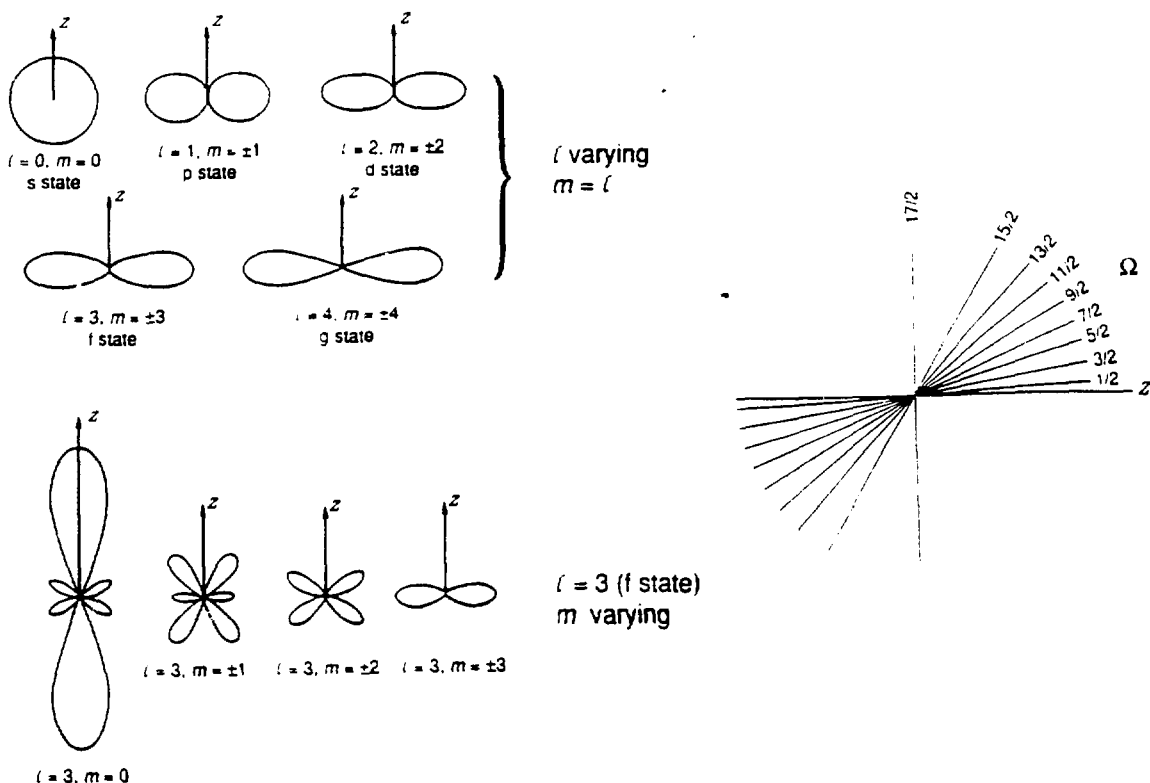


Fig. 5. Pedagogical depiction of the concept of quantal localization. (Left) Probability distributions relative to the polar axis for the $m = \ell$ component as a function of the orbital angular momentum (ℓ) and for the $\ell = 3$ (f state) as a function of the magnetic quantum number (m). Adapted from ref. 36; see also refs. 4 and 12. (Right) Semiclassical depiction of the median angles of the orbital planes for different Ω states of a $1k_{17/2}$ orbit in a deformed nucleus. Note the extreme localization for the lowest Ω , equatorial orbits. Adapted from ref. 12.

In a rotating system the Coriolis interaction destroys⁸ the nuclear pair correlations by decreasing the energy of the state corresponding to the highly-localized, low- Ω components of the wave function that orbit the nucleus in the direction of the nuclear rotation and increasing the energy of those orbiting in the opposite direction, see Fig. 6. Indeed, this interaction not only enhances the spacial localization, but in the limiting case it also restricts the valence nucleons (i.e., those of non-filled shells) to equatorial orbitals in the direction of nuclear rotation, see Fig. 7. The large overlap of such orbits is a necessary, though not sufficient, condition for new types of rotationally-induced correlations.⁴

Such quantal localization effects (which will produce, e.g. coexisting nuclear shapes, large residual interactions,^{4,10,11} and perhaps even new types of nuclear correlations⁴) become increasingly important for heavy nuclei, where many of the intrinsic (i.e. shell-model) configurations occupied have large angular momentum, j , see e.g. Figs. 1 and 4. Even larger enhancements are expected in regions where the low- Ω states of the nuclear wave functions and/or the same neutron and proton configurations are selectively

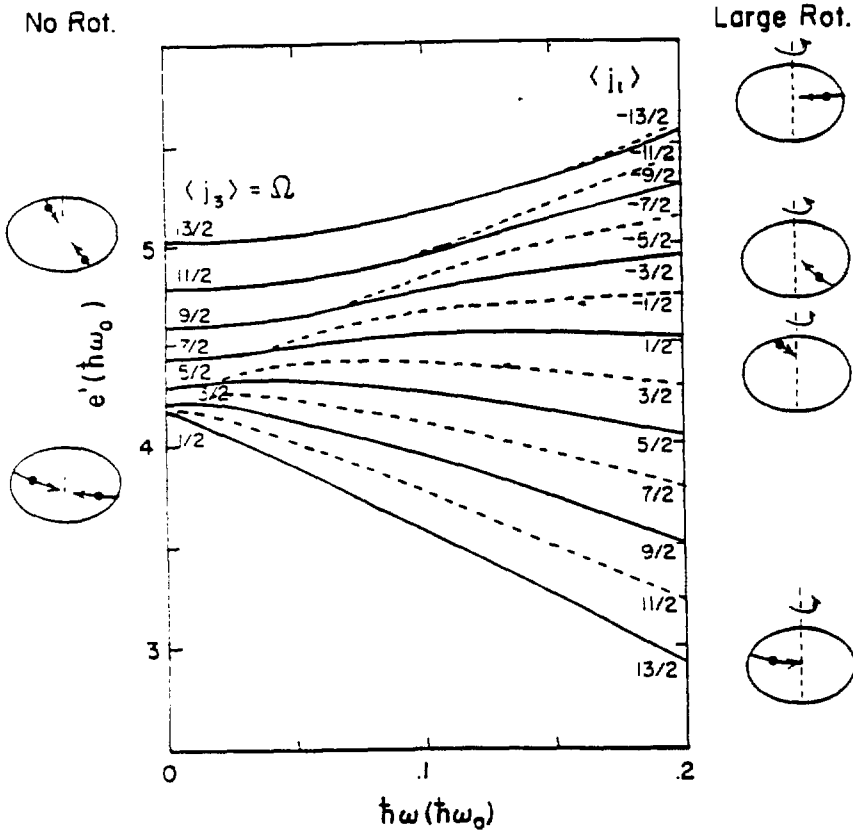


Fig. 6. Calculated spectrum of single-particle states for a single- j shell in a rotating axially-deformed nucleus ($\epsilon_2 = 0.25$) in the absence of pair correlations. Solid and dashed curves correspond to states of signature (α) $1/2$ and $-1/2$, respectively. Though the details of the calculation correspond to $i_{13/2}$ neutrons, this spectrum of states is typical of any high- j shell. Values of $\langle j_3 \rangle = \Omega$ and $\langle j_1 \rangle$ are given in the limiting case of no rotation and infinite rotation, where they are conserved quantities. Schematic diagrams depicting selected nucleonic orbital with respect to the nuclear-symmetry and rotational axes also are shown in the right- and left-hand portions of the figure. Adapted from ref. 4.

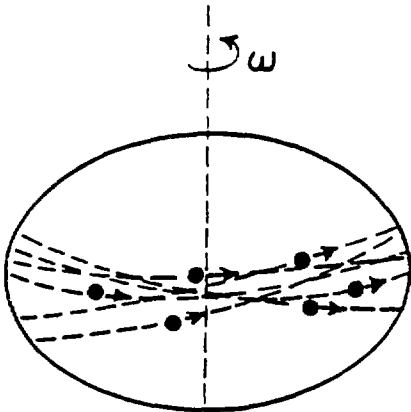


Fig. 7. Idealized figure depicting the valence nucleonic motion in a very rapidly-rotating deformed nucleus.

occupied. Indeed, all these conditions are met for the "new" proton-rich isotopes of xenon - neodymium ($Z \approx N = 54 - 60$) which can be studied with proton-rich radioactive beams, see Fig. 2 and ref. 14. Even more dramatic examples of quantal localization perhaps can be studied in the "new" very proton-rich isotopes of platinum to lead³⁵ ($Z = 78 - 82$). For the prolate deformations, associated with the occupation of down-sloping orbitals on the Nilsson diagram, the low- Ω components of the intruding $i_{13/2}$, $h_{9/2}$, and $f_{7/2}$ proton orbitals have nearly maximal overlap with each other and even more importantly with the same neutron orbitals which are occupied just above the $N = 82$ neutron shell, see Fig. 4.

IV. COMPLETE SPECTROSCOPY -- OVERLAPPING HIGH- AND LOW-SPIN DATA IN THE SAME NUCLEUS

A dream of nuclear structure physicists (see e.g. refs. 5 and 47) since the advent of heavy-ion beams -- obtaining state of the art low- and high-spin data in the same nucleus -- can be realized for the first time utilizing beams of heavy neutron-rich radioactive nuclei. The combination of the increased neutron excess in heavy stable nuclei (to counteract the repulsive Coulomb interaction), the emission of neutrons from highly-excited nearly-stable and neutron-rich compound nuclei, and the necessity of heavy ions for imparting angular momentum to the system (in heavy-ion induced fusion evaporation reactions), has confined previous high spin studies to proton-rich nuclei. For example, nuclei such as $^{168}\text{Er}_{100}$, which are studied^{37,38} in great detail at low spin in (n,γ) studies and transfer reactions, cannot be populated at high spin using heavy-ion induced fusion, evaporation reactions with stable beams and targets.

Figure 8 illustrates the population of erbium isotopes by a variety of reactions utilizing both stable and radioactive ions. All of the stable erbium isotopes can be studied at high, as well as at low, spin using the neutron-rich radioactive beams planned for the IsoSpin Laboratory¹² (and other similar second-generation ISOL facilities). Likewise, the predicted³⁹ best cases for hyperdeformation, $^{166-168}\text{Er}$, also can be populated at high spin with these radioactive beams. Indeed, for complete spectroscopic studies a variety of light-, intermediate-, and heavy-ion induced fusion-evaporation reactions are needed to populate the nucleus with the complete range of angular momenta. Of course, this nucleus also must be studied at low spin with (n,γ) , (e,e') , and transfer reactions and Coulomb excited.

Besides the obvious use of "complete spectroscopic studies" for more stringent tests of nuclear models, several other interesting nuclear structure questions can be addressed by "complete" data. For example, statistical analyses of level spacing distributions, often associated with the order or chaos of the nuclear system,^{40,41} can be studied as a function of angular momentum.⁴² Likewise, detailed studies of strongly-correlated nuclear states, such as the γ -, β -, and octupole-vibrations, at larger angular momentum, will be greatly facilitated by the use of radioactive ion beams to overlap low- and high-spin studies in the same nucleus. Such data can provide information on the spin dependence of the microscopic basis of such correlations, see e.g. refs. 43 and 47.

Population of Erbium Nuclei

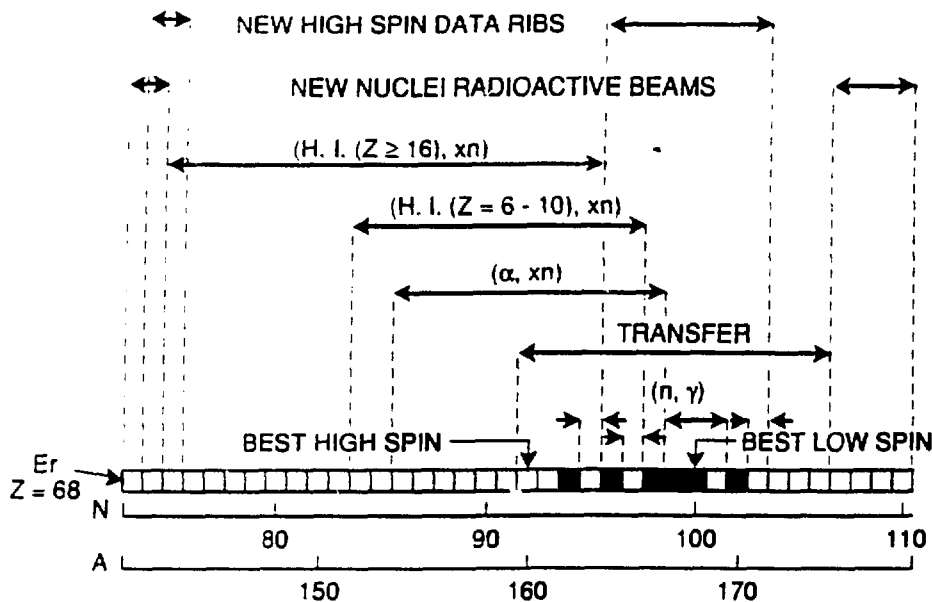


Fig. 8. Comparison of the population of erbium isotopes by a variety of nuclear reactions using both stable and radioactive ion beams (RIBS). The stable erbium isotopes are shown by filled squares, and the best studied cases at both low and high spin are indicated.

VI. CONCLUSION

The purpose of this report is to present a flavor of the rich and exciting nuclear structure studies of rapidly-rotating exotic nuclei afforded by radioactive ion beams. It is not intended to be a comprehensive review of this frontier field. Indeed, the possibilities for studies on the limits of both angular momentum and isospin are just starting to be seriously considered. Additional information is contained in refs. 12-14, 35, and 44.

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