

CN9201772

CNIC-00592

IAE-0100

中国核科技报告

CHINA NUCLEAR SCIENCE & TECHNOLOGY REPORT

直线加速器中空间电荷束团的非线性效应

NONLINEAR SPACE CHARGE EFFECT
OF BUNCHED BEAM IN LINAC



原子能出版社

中国核情报中心

China Nuclear Information Centre



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摘 要

提出了在强流直线加速器中,由空间电荷束团的密度分布所引起的非线性效应。针对直线加速器中常用的盘电荷模型和有限圆柱电荷模型下的 $K-V$ 分布、水袋分布、抛物线分布和高斯分布等密度分布形态,推导出了相应的非线性空间电荷场和力的表达式。

NONLINEAR SPACE CHARGE EFFECT OF BUNCHED BEAM IN LINAC

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ABSTRACT

The nonlinear space charge effect due to the nonuniform particle density distribution in bunched beam of a linac is discussed. The formulae of nonlinear space charge effect and nonlinear focusing forces were derived for the bunched beam with Kapchinskij – Vladimirskij (K – V) distribution, waterbag (WB) distribution, parabolic (PA) distribution, and Gauss (GA) distribution in both of the space charge disk model and space charge cylinder model in the waveguide of a linac.

INTRODUCTION

In high-current beam for Free Electron Laser (FEL) and linear accelerator for high-energy physics, induction linac for heavy ion fusion, microwave devices and other applications, the space charge force is no longer small compared with the externally applied focusing forces. And the space charge effect is assumed to be one of the fundamental factors governing the beam dynamics. With regard to the current in surrounding structures, many articles have been published^[1-7]. However, most of them studied the beam bunch with uniform density distribution except the Ref. [7] in which the general formulae for calculating the space charge effect in a waveguide have been derived.

Furthermore, the nonlinear effect of the space charge is one of the important reason that induces the emittance growth because of the conversion of field energy to kinetic energy^[8-10]. Theoretical study and numerical simulation showed that the nonuniform particle distributions have more electrostatic field energy per unit length than that of the equivalent uniform beam with the same current I , RMS radius, and RMS emittance. Therefore, additional field energy is converted into particle kinetic and potential energy (and hence emittance growth) as the distribution tends to become more homogeneous. This concept has been already accepted by some further studies^[11,12]. However, we should point it out that above results concerning the calculation of the space charge field energy is based on the continue beam in free space. Therefore, it is necessary to derive the formulae of nonlinear space charge effect and nonlinear focusing forces for the beam with some common space charge distributions such as Kapchinskij – Vladimirskij (K – V), waterbag (WB), parabolic (PA), and Gaussian (GA) distributions in both of the space charge disk model and space charge cylinder model in the waveguide of a linac.

1 GENERAL FORMULAE FOR CALCULATING THE SPACE CHARGE EFFECT IN WAVEGUIDE

For the convenience of understanding and application, here we review the main point of Ref. [7] in which the general formulae for calculating the space charge effect in a waveguide are obtained.

Assuming the space charge bunch model is central symmetry, we have the po

potential induced by the space charge with uniform density distribution ρ in a cylindrical coordinate system as follows:

$$\varphi_0 = \rho f_0(r, z; b, L/2) \quad (1)$$

where b and $L/2$ are the edges of the model in r and z directions, respectively, f_0 is the potential induced by the unit space charge density and the subscript 0 stands for the uniform density distribution.

Using Eq. (1), we get the potential induced by the same space charge bunch model, but with nonuniform charge density distribution $\rho(r)$ as

$$\varphi = \int_0^b \rho(\xi) \frac{\partial f_0(r, z; \xi, L/2)}{\partial \xi} d\xi \quad (2)$$

Analogously, for the same space charge bunch model with the nonuniform charge distribution $\rho(z)$ one finds the potential as:

$$\varphi = \int_0^{L/2} \rho(\zeta) \frac{\partial f_0(r, z; b, \zeta)}{\partial \zeta} d\zeta \quad (3)$$

And, hence, for the same space charge bunch model with assuming nonuniform charge density distribution $\rho(r, z) = \rho(r)\rho(z)$, we get the potential as follows:

$$\varphi = \int_0^{L/2} \int_0^b \rho(\xi, \zeta) \frac{\partial^2 f_0(r, z; \xi, \zeta)}{\partial \zeta \partial \xi} d\xi d\zeta \quad (4)$$

It should be pointed out that the assumption of $\rho(r, z) = \rho(r)\rho(z)$ is always satisfied for any function because that can be expressed as a summation of many disconnecting functions.

Therefore, according to Eqs. (2)~(4), the potential induced by the space charge bunch with nonuniform charge distribution can be obtained if the potential induced by the same space charge bunch model but with uniform charge density distribution is known as Eq. (1).

2 CALCULATION OF SPACE CHARGE EFFECT

Now Eqs. (2)~(4) are applied to some common space charge models with different charge density distributions in a linac.

2.1 Disk Model of Space Charge

We have the potential of this disk space charge as ^[1]

$$\varphi_0 = \frac{\rho b}{\epsilon_0 a^2} \sum_{l=1}^{\infty} \frac{J_1(k_l b) J_0(k_l r)}{k_l^2 J_1^2(k_l a)} e^{-k_l |z|} \quad (5)$$

where a is the waveguide radius and b is the disk radius, $J_l(k_l x)$ is Bessel function, l ,

and k_i satisfies the equation: $J_0(k_i a) = 0$. We discuss the space charge density distribution in the disk as follows:

(1) Kapchinskij – Vladimirskij (K – V) distribution

According to Ref. [8], the charge density in real space with the K – V distribution in phase space is homogeneous, i. e.

$$\rho = \rho_0 = \frac{q}{\pi b^2} \quad (6)$$

where q is the total charge in the disk. Then, the potential induced by the K – V distribution is just as the expression (5), or rewritten as:

$$\varphi_0 = \frac{q}{\epsilon_0 \pi b a^2} \sum_{i=1}^{\infty} \frac{J_1(k_i b) J_0(k_i r)}{k_i^2 J_1^2(k_i a)} e^{-k_i |z|} \quad (7)$$

(2) Waterbag (WB) distribution

The charge density in real space with a WB distribution in phase space is parabolic^[9]:

$$\rho = \rho_{WB} \left(1 - \frac{r^2}{b^2}\right) \quad (8)$$

where ρ_{WB} can be expressed with the total charge q :

$$\rho_{WB} = \frac{2q}{\pi b^2} \quad (9)$$

According to Eq. (2), the potential induced by the WB distribution is as follows:

$$\varphi = \int_0^b \rho_{WB} \left(1 - \frac{\xi^2}{b^2}\right) \sum_{i=1}^{\infty} \frac{\xi J_1(k_i \xi) J_0(k_i r)}{\epsilon_0 a^2 k_i^2 J_1^2(k_i a)} e^{-k_i |z|} d\xi$$

Using the expression:

$$\frac{d}{d\xi} \xi J_1(k_i \xi) = k_i \xi J_0(k_i \xi)$$

one gets

$$\varphi = \int_0^b \rho_{WB} \left(1 - \frac{\xi^2}{b^2}\right) \sum_{i=1}^{\infty} \frac{\xi J_0(k_i \xi) J_0(k_i r)}{\epsilon_0 a^2 k_i J_1^2(k_i a)} e^{-k_i |z|} d\xi$$

Substituting the integral of Bessel function^[13]

$$\int_0^x x^3 J_0(x) dx = x^3 J_1(x) - 2x^2 J_2(x)$$

the final result is:

$$\varphi = 2\rho_{WB} \sum_{i=1}^{\infty} \frac{J_2(k_i b) J_0(k_i r)}{\epsilon_0 a^2 k_i^3 J_1^2(k_i a)} e^{-k_i |z|} = \frac{4 \cdot a}{\epsilon_0 \pi b^2} \sum_{i=1}^{\infty} \frac{J_2(k_i b) J_0(k_i r)}{(k_i a)^3 J_1^2(k_i a)} e^{-k_i |z|} \quad (10)$$

The disk charge with the WB distribution becomes to a point charge when the disk radius $b \rightarrow 0$, and hence, Eq. (10) becomes to

$$\varphi = \sum_{i=1}^{\infty} \frac{q J_0(k_i r)}{2\pi \epsilon_0 a b^2 (k_i a) J_1^2(k_i a)} e^{-k_i |z|} \quad (11)$$

which agrees the potential of a point charge in Ref. [1].

(3) Parabolic (PA) distribution

The charge density in real space with a PA distribution in phase space presents the following form^[8]:

$$\rho = \rho_{PA} \left(1 - \frac{r^2}{b^2}\right)^2 \quad (12)$$

where ρ_{PA} can be expressed with the total charge q :

$$\rho_{PA} = \frac{3q}{\pi b^2} \quad (13)$$

According to Eq. (2), we obtain the potential induced by the PA distribution as follows :

$$\begin{aligned} \varphi &= \int_0^b \rho_{PA} \left(1 - \frac{\xi^2}{b^2}\right) \sum_{i=1}^{\infty} \frac{\xi J_1(k_i \xi) J_0(k_i r)}{\epsilon_0 a^2 k_i J_1^2(k_i a)} e^{-k_i |z|} d\xi \\ &= \int_0^b \rho_{PA} \left(1 - \frac{2\xi^2}{b^2} + \frac{\xi^4}{b^4}\right) \sum_{i=1}^{\infty} \frac{\xi J_0(k_i \xi) J_0(k_i r)}{\epsilon_0 a^2 k_i J_1^2(k_i a)} e^{-k_i |z|} d\xi \end{aligned}$$

Again, using the integral of Bessel function^[13]

$$\int_0^2 x^3 J_0(x) dx = x^3 J_1(x) - 4x^4 J_2(x) + 8x^3 J_3(x)$$

we can write :

$$\varphi = 8\rho_{PA} \sum_{i=1}^{\infty} \frac{J_3(k_i b) J_0(k_i r)}{\epsilon_0 a^2 k_i^4 J_1^2(k_i a)} e^{-k_i |z|} = \frac{24qa^2}{\epsilon_0 \pi b^3} \sum_{i=1}^{\infty} \frac{J_3(k_i b) J_0(k_i r)}{(k_i a)^4 J_1^2(k_i a)} e^{-k_i |z|} \quad (14)$$

The disk charge with the PA distribution becomes to a point charge when the disk radius $b \rightarrow 0$, and hence, Eq. (14) also becomes to Eq. (11) of the potential for a point charge.

(4) Gauss (GA) distribution

The charge density in real space with a GA distribution in phase space can be expressed as follows^[8]:

$$\rho = \rho_{GA} e^{-\frac{r^2}{2\alpha^2}} \quad (15)$$

where $\alpha^2 = \langle x^2 \rangle$, and ρ_{GA} can be expressed with the total charge q :

$$\rho_{GA} = \frac{q}{2\pi\alpha^2} \quad (16)$$

According to Eq. (2), we get the potential induced by the GA distribution as follows:

$$\begin{aligned}\varphi &= \int_0^b \rho_{GA} e^{-\frac{\xi^2}{2\alpha^2}} \sum_{i=1}^{\infty} \frac{\xi J_1(k_i \xi) J_0(k_i r)}{\varepsilon_0 a^2 k_i J_1^2(k_i a)} e^{-k_i r} d\xi \\ &= \int_0^b \rho_{GA} e^{-\frac{\xi^2}{2\alpha^2}} \sum_{i=1}^{\infty} \frac{\xi J_0(k_i \xi) J_0(k_i r)}{\varepsilon_0 a^2 k_i J_1^2(k_i a)} e^{-k_i r} d\xi\end{aligned}$$

Using the following formulae ^[14]

$$\begin{aligned}\int_0^{\infty} x^{\nu} e^{-\alpha x^2} J_{\nu}(\beta x) dx &= \frac{\beta^{\nu} \Gamma(\frac{1}{2}\nu + \frac{1}{2}\mu + \frac{1}{2})}{2^{\nu+1} \alpha^{\frac{1}{2}(\nu+\mu+1)} \Gamma(\nu+1)} {}_1F_1\left(\frac{\nu+\mu+1}{2}; \nu+1; -\frac{\beta^2}{4\alpha}\right) \\ \int_0^{\infty} \xi e^{-\frac{\xi^2}{2\alpha^2}} J_0(k_i \xi) d\xi &= \alpha^2 {}_1F_1\left(1; 1; -\frac{k_i^2 \alpha^2}{2}\right)\end{aligned}$$

where ${}_1F_1(a; \gamma; z)$ is the Kummer function, which has the series expansion ^[15]

$${}_1F_1(a; \gamma; z) = \frac{\Gamma(\gamma)}{\Gamma(a)} \sum_{n=0}^{\infty} \frac{\Gamma(n+a) z^n}{\Gamma(n+\gamma) n!}$$

And hence, in our case:

$${}_1F_1(1; 1; -\frac{k_i^2 \alpha^2}{2}) = \sum_{n=0}^{\infty} \frac{(-k_i^2 \alpha^2 / 2)^n}{n!} = e^{-\frac{k_i^2 \alpha^2}{2}}$$

Substituting into the potential expression, we obtain

$$\begin{aligned}\varphi &= \rho_{GA} \sum_{i=1}^{\infty} \frac{\alpha^2 J_0(k_i r)}{\varepsilon_0 a^2 k_i J_1^2(k_i a)} e^{-\frac{k_i^2 \alpha^2}{2}} e^{-k_i r} \\ &= \frac{q}{2\pi \varepsilon_0 a} \sum_{i=1}^{\infty} \frac{J_0(k_i r)}{\varepsilon_0 a^2 k_i J_1^2(k_i a)} e^{-\frac{k_i^2 \alpha^2}{2}} e^{-k_i r}\end{aligned}\tag{17}$$

The disk charge with the GA distribution becomes to a point charge when the disk radius $b \rightarrow 0$, (and hence $\alpha \rightarrow 0$, too), and, then we have the expected point of the charge potential from Eq. (17).

It should be pointed out that, we expanded the upper limit b of the integral in to ∞ in the above procedure. This is reasonable due to the character of Gauss distribution of the beam.

2.2 Cylinder Model of Space Charge

We have the potential of the cylinder space charge as^[1]

$$\left. \begin{aligned} \varphi_{1,2} &= \frac{2\rho b}{\epsilon_0 a^2} \sum_{i=1}^{\infty} \frac{J_1(k_i b) J_0(k_i r)}{k_i^3 J_1^2(k_i a)} \operatorname{sh} \frac{k_i L}{2} e^{-k_i |z|}, & (|z| > \frac{L}{2}) \\ \varphi_3 &= \frac{2\rho b}{\epsilon_0 a^2} \sum_{i=1}^{\infty} \frac{J_1(k_i b) J_0(k_i r)}{k_i^3 J_1^2(k_i a)} \left(1 - e^{-\frac{k_i L}{2} \operatorname{ch} k_i z} \right), & (|z| < \frac{L}{2}) \end{aligned} \right\} \quad (18)$$

where b and L are the radius and length of the cylinder, respectively.

By the procedure analogous to that of deducing the above disk model of space charge and taking notice of the relationship between the charge density ρ and the total charge q in the cylinder model of space charge, we obtain the potentials of the space charge cylinder with K - V, WB, PA, GA distributions in the following:

(1) K - V distribution

The potential induced by the K - V charge density distribution is just as Eq. (18), or rewritten as:

$$\left. \begin{aligned} \varphi_{1,2} &= \frac{2qa}{\pi \epsilon_0 b L} \sum_{i=1}^{\infty} \frac{J_1(k_i b) J_0(k_i r)}{(k_i a)^3 J_1^2(k_i a)} \operatorname{sh} \frac{k_i L}{2} e^{-k_i |z|}, & (|z| > \frac{L}{2}) \\ \varphi_3 &= \frac{2qa}{\pi \epsilon_0 b L} \sum_{i=1}^{\infty} \frac{J_1(k_i b) J_0(k_i r)}{(k_i a)^3 J_1^2(k_i a)} \left(1 - e^{-\frac{k_i L}{2} \operatorname{ch} k_i z} \right), & (|z| < \frac{L}{2}) \end{aligned} \right\} \quad (19)$$

(2) WB distribution

The potential induced by the WB charge density distribution is

$$\left. \begin{aligned} \varphi_{1,2} &= \frac{4\rho_{WB}}{\epsilon_0 a^2} \sum_{i=1}^{\infty} \frac{J_2(k_i b) J_0(k_i r)}{k_i^4 J_1^2(k_i a)} \operatorname{sh} \frac{k_i L}{2} e^{-k_i |z|}, & (|z| > \frac{L}{2}) \\ \varphi_3 &= \frac{4\rho_{WB}}{\epsilon_0 a^2} \sum_{i=1}^{\infty} \frac{J_2(k_i b) J_0(k_i r)}{k_i^4 J_1^2(k_i a)} \left(1 - e^{-\frac{k_i L}{2} \operatorname{ch} k_i z} \right), & (|z| < \frac{L}{2}) \end{aligned} \right\} \quad (20a)$$

or can be rewritten in the form

$$\left. \begin{aligned} \varphi_{1,2} &= \frac{8qa^2}{\pi \epsilon_0 b^2 L} \sum_{i=1}^{\infty} \frac{J_2(k_i b) J_0(k_i r)}{(k_i a)^4 J_1^2(k_i a)} \operatorname{sh} \frac{k_i L}{2} e^{-k_i |z|}, & (|z| > \frac{L}{2}) \\ \varphi_3 &= \frac{8qa^2}{\pi \epsilon_0 b^2 L} \sum_{i=1}^{\infty} \frac{J_2(k_i b) J_0(k_i r)}{(k_i a)^4 J_1^2(k_i a)} \left(1 - e^{-\frac{k_i L}{2} \operatorname{ch} k_i z} \right), & (|z| < \frac{L}{2}) \end{aligned} \right\} \quad (20b)$$

(3) PA distribution

The potential induced by the PA charge density distribution is

$$\left. \begin{aligned} \varphi_{1,2} &= \frac{16\rho_{PA}}{\epsilon_0 a^2 b} \sum_{i=1}^{\infty} \frac{J_0(k_i b) J_0(k_i r)}{k_i^3 J_1^2(k_i a)} \operatorname{sh} \frac{k_i L}{2} e^{-k_i |z|}, \quad (|z| > \frac{L}{2}) \\ \varphi_3 &= \frac{16\rho_{PA}}{\epsilon_0 a^2 b} \sum_{i=1}^{\infty} \frac{J_0(k_i b) J_0(k_i r)}{k_i^3 J_1^2(k_i a)} \left(1 - e^{-\frac{k_i L}{2} \operatorname{ch} k_i z} \right), \quad (|z| < \frac{L}{2}) \end{aligned} \right\} \quad (21a)$$

or is taken to be given by the equation

$$\left. \begin{aligned} \varphi_{1,2} &= \frac{48qa^3}{\pi\epsilon_0 b^3 L} \sum_{i=1}^{\infty} \frac{J_0(k_i b) J_0(k_i r)}{(k_i a)^3 J_1^2(k_i a)} \operatorname{sh} \frac{k_i L}{2} e^{-k_i |z|}, \quad (|z| > \frac{L}{2}) \\ \varphi_3 &= \frac{48qa^3}{\pi\epsilon_0 b^3 L} \sum_{i=1}^{\infty} \frac{J_0(k_i b) J_0(k_i r)}{(k_i a)^3 J_1^2(k_i a)} \left(1 - e^{-\frac{k_i L}{2} \operatorname{ch} k_i z} \right), \quad (|z| < \frac{L}{2}) \end{aligned} \right\} \quad (21b)$$

(4) GA distributon

The potential induced by the GA charge density distribution is

$$\left. \begin{aligned} \varphi_{1,2} &= \frac{2\rho_{GA}}{\epsilon_0 a^2} \sum_{i=1}^{\infty} \frac{a^2 J_0(k_i r)}{k_i^2 J_1^2(k_i a)} e^{-\frac{k_i^2 a^2}{2}} \operatorname{sh} \frac{k_i L}{2} e^{-k_i |z|}, \quad (|z| > \frac{L}{2}) \\ \varphi_3 &= \frac{2\rho_{GA}}{\epsilon_0 a^2} \sum_{i=1}^{\infty} \frac{a^2 J_0(k_i r)}{k_i^2 J_1^2(k_i a)} e^{-\frac{k_i^2 a^2}{2}} \left(1 - e^{-\frac{k_i L}{2} \operatorname{ch} k_i z} \right), \quad (|z| < \frac{L}{2}) \end{aligned} \right\} \quad (22a)$$

or is found to be given by the expression

$$\left. \begin{aligned} \varphi_{1,2} &= \frac{q}{\pi\epsilon_0 L} \sum_{i=1}^{\infty} \frac{J_0(k_i r)}{(k_i a)^2 J_1^2(k_i a)} e^{-\frac{k_i^2 a^2}{2}} \operatorname{sh} \frac{k_i L}{2} e^{-k_i |z|}, \quad (|z| > \frac{L}{2}) \\ \varphi_3 &= \frac{q}{\pi\epsilon_0 L} \sum_{i=1}^{\infty} \frac{J_0(k_i r)}{(k_i a)^2 J_1^2(k_i a)} e^{-\frac{k_i^2 a^2}{2}} \left(1 - e^{-\frac{k_i L}{2} \operatorname{ch} k_i z} \right), \quad (|z| < \frac{L}{2}) \end{aligned} \right\} \quad (22b)$$

Obviously, the cylinder charge becomes to a disk charge when the cylinder length $L \rightarrow 0$. And hence the above potential formulae Eqs. (19)~(22) become to the potentials of Eq. (7), Eq (10), Eq. (14), and Eq. (17) of the disk space charge density with K - V, WB, PA and GA distributions, respectively.

It should be pointed out that all the formulae are derived here in the frame of reference moving with the space charge bunch in the same velocity. However, the motion of the space charge bunch in the longitudinal direction can be relativistic in a linac. Therefore, the formulae should be transformed into that of the relativistic case according to Ref. [5].

3 NONLINEAR SPACE CHARGE FORCE

The space charge forces on a point charge e can be derived by means of the derivatives of the potentials obtained for different models. In the following, we first give the force expressions in the frame of reference moving with the charge, and then transform them into that of the relativistic case.

3.1 Disk Model of Space Charge

(1) K - V distribution

From Eq. (7), we get the force expressions as:

$$\left. \begin{aligned} F_z &= -e \frac{\partial \varphi}{\partial z} = \frac{eq}{\epsilon_0 \pi b a^2} \sum_{i=1}^{\infty} \frac{J_1(k_i b) J_0(k_i r)}{k_i J_1^2(k_i a)} e^{-k_i z} & (z > 0) \\ F_z &= -e \frac{\partial \varphi}{\partial z} = -\frac{eq}{\epsilon_0 \pi b a^2} \sum_{i=1}^{\infty} \frac{J_1(k_i b) J_0(k_i r)}{k_i J_1^2(k_i a)} e^{k_i z} & (z < 0) \\ F_r &= -e \frac{\partial \varphi}{\partial r} = \frac{eq}{\epsilon_0 \pi b a^2} \sum_{i=1}^{\infty} \frac{J_1(k_i b) J_1(k_i r)}{k_i J_1^2(k_i a)} e^{-k_i |z|} \end{aligned} \right\} \quad (23)$$

And the force expressions in laboratory system can be obtained according to the transformation of Ref. [5]:

$$\left. \begin{aligned} F_z &= \frac{eq}{\epsilon_0 \pi b a^2} \sum_{i=1}^{\infty} \frac{J_1(k_i b) J_0(k_i r)}{k_i J_1^2(k_i a)} e^{-k_i \gamma z} & (z > 0) \\ F_z &= -\frac{eq}{\epsilon_0 \pi b a^2} \sum_{i=1}^{\infty} \frac{J_1(k_i b) J_0(k_i r)}{k_i J_1^2(k_i a)} e^{k_i \gamma z} & (z < 0) \\ F_r &= \frac{eq}{\epsilon_0 \pi b a^2 \gamma} \sum_{i=1}^{\infty} \frac{J_1(k_i b) J_1(k_i r)}{k_i J_1^2(k_i a)} e^{-k_i \gamma |z|} \end{aligned} \right\} \quad (24)$$

where $\gamma = (1 - \beta^2)^{-1/2}$, $\beta = v/c$ is the ratio of the particle velocity to the speed of light.

(2) WB distribution

From Eq. (10), we get the force expressions as:

$$\left. \begin{aligned} F_z &= \frac{4eq}{\epsilon_0 \pi b^2 a^2} \sum_{i=1}^{\infty} \frac{J_2(k_i b) J_0(k_i r)}{k_i^2 J_1^2(k_i a)} e^{-k_i z} & (z > 0) \\ F_z &= -\frac{4eq}{\epsilon_0 \pi b^2 a^2} \sum_{i=1}^{\infty} \frac{J_2(k_i b) J_0(k_i r)}{k_i^2 J_1^2(k_i a)} e^{k_i z} & (z < 0) \\ F_r &= \frac{4eq}{\epsilon_0 \pi b^2 a^2} \sum_{i=1}^{\infty} \frac{J_2(k_i b) J_1(k_i r)}{k_i^2 J_1^2(k_i a)} e^{-k_i |z|} \end{aligned} \right\} \quad (25)$$

And the force expressions in laboratory system are

$$\left. \begin{aligned} F_z &= \frac{4eq}{\epsilon_0 \pi b^2 a^2} \sum_{i=1}^{\infty} \frac{J_2(k_i b) J_0(k_i r)}{k_i^2 J_1^2(k_i a)} e^{-k_i z} & (z > 0) \\ F_z &= -\frac{4ea}{\epsilon_0 \pi b^2 a^2} \sum_{i=1}^{\infty} \frac{J_2(k_i b) J_0(k_i r)}{k_i^2 J_1^2(k_i a)} e^{k_i z} & (z < 0) \\ F_r &= \frac{4eq}{\epsilon_0 \pi b^2 a^2} \sum_{i=1}^{\infty} \frac{J_2(k_i b) J_1(k_i r)}{k_i^2 J_1^2(k_i a)} e^{-k_i |z|} \end{aligned} \right\} \quad (26)$$

(3) PA distribution

From Eq. (14), we get the force expressions as:

$$\left. \begin{aligned} F_z &= \frac{24eq}{\epsilon_0 \pi b^3 a^2} \sum_{i=1}^{\infty} \frac{J_3(k_i b) J_0(k_i r)}{k_i^3 J_1^2(k_i a)} e^{-k_i z} & (z > 0) \\ F_z &= -\frac{24eq}{\epsilon_0 \pi b^3 a^2} \sum_{i=1}^{\infty} \frac{J_3(k_i b) J_0(k_i r)}{k_i^3 J_1^2(k_i a)} e^{k_i z} & (z < 0) \\ F_r &= \frac{24eq}{\epsilon_0 \pi b^3 a^2} \sum_{i=1}^{\infty} \frac{J_3(k_i b) J_1(k_i r)}{k_i^3 J_1^2(k_i a)} e^{-k_i |z|} \end{aligned} \right\} \quad (27)$$

And the force expressions in laboratory system are

$$\left. \begin{aligned} F_z &= \frac{24eq}{\epsilon_0 \pi b^3 a^2} \sum_{i=1}^{\infty} \frac{J_3(k_i b) J_0(k_i r)}{k_i^3 J_1^2(k_i a)} e^{-k_i z} & (z > 0) \\ F_z &= -\frac{24eq}{\epsilon_0 \pi b^3 a^2} \sum_{i=1}^{\infty} \frac{J_3(k_i b) J_0(k_i r)}{k_i^3 J_1^2(k_i a)} e^{k_i z} & (z < 0) \\ F_r &= \frac{24eq}{\epsilon_0 \pi b^3 a^2} \sum_{i=1}^{\infty} \frac{J_3(k_i b) J_1(k_i r)}{k_i^3 J_1^2(k_i a)} e^{-k_i |z|} \end{aligned} \right\} \quad (28)$$

(4) GA distribution

From Eq. (17), we get the force expressions as:

$$\left. \begin{aligned} F_z &= \frac{eq}{2\pi\epsilon_0 a^2} \sum_{i=1}^{\infty} \frac{J_0(k_i r)}{(k_i a) J_1^2(k_i a)} e^{-\frac{k_i^2 a^2}{2}} e^{-k_i z} & (z > 0) \\ F_z &= -\frac{eq}{2\pi\epsilon_0 a^2} \sum_{i=1}^{\infty} \frac{J_0(k_i r)}{(k_i a) J_1^2(k_i a)} e^{-\frac{k_i^2 a^2}{2}} e^{k_i z} & (z < 0) \\ F_r &= \frac{eq}{2\pi\epsilon_0 a^2} \sum_{i=1}^{\infty} \frac{J_1(k_i r)}{(k_i a) J_1^2(k_i a)} e^{-\frac{k_i^2 a^2}{2}} e^{-k_i |z|} \end{aligned} \right\} \quad (29)$$

And the force expressions in laboratory system are

$$\left. \begin{aligned} F_z &= \frac{eq}{2\pi\epsilon_0 a^2} \sum_{i=1}^{\infty} \frac{J_0(k_i r)}{(k_i a) J_1^2(k_i a)} e^{-\frac{k_i^2 a^2}{2}} e^{-k_i z} & (z > 0) \\ F_z &= -\frac{eq}{2\pi\epsilon_0 a^2} \sum_{i=1}^{\infty} \frac{J_0(k_i r)}{(k_i a) J_1^2(k_i a)} e^{-\frac{k_i^2 a^2}{2}} e^{k_i z} & (z < 0) \\ F_r &= \frac{eq}{2\pi\epsilon_0 a^2 \gamma} \sum_{i=1}^{\infty} \frac{J_1(k_i r)}{(k_i a) J_1^2(k_i a)} e^{-\frac{k_i^2 a^2}{2}} e^{-k_i \gamma z} \end{aligned} \right\} \quad (30)$$

3.2 Cylinder model of space charge.

(1) K - V distribution

From Eq. (19), one gets the force expressions as:

$$\left. \begin{aligned} F_z &= \frac{2eq}{\pi\epsilon_0 bL} \sum_{i=1}^{\infty} \frac{J_1(k_i b) J_0(k_i r)}{(k_i a)^2 J_1^2(k_i a)} \operatorname{sh} \frac{k_i L}{2} e^{-k_i z} & (z > \frac{L}{2}) \\ F_z &= -\frac{2eq}{\pi\epsilon_0 bL} \sum_{i=1}^{\infty} \frac{J_1(k_i b) J_0(k_i r)}{(k_i a)^2 J_1^2(k_i a)} \operatorname{sh} \frac{k_i L}{2} e^{k_i z} & (z < -\frac{L}{2}) \end{aligned} \right\} \quad (31a)$$

$$\left. \begin{aligned} F_r &= \frac{2eq}{\pi\epsilon_0 bL} \sum_{i=1}^{\infty} \frac{J_1(k_i b) J_0(k_i r)}{(k_i a)^2 J_1^2(k_i a)} \left(1 - e^{-\frac{k_i L}{2} \operatorname{sh} k_i z} \right) & (|z| < \frac{L}{2}) \\ F_r &= \frac{2eq}{\pi\epsilon_0 bL} \sum_{i=1}^{\infty} \frac{J_1(k_i b) J_1(k_i r)}{(k_i a)^2 J_1^2(k_i a)} \operatorname{sh} \frac{k_i L}{2} e^{-k_i |z|} & (|z| > \frac{L}{2}) \end{aligned} \right\} \quad (31b)$$

$$F_r = \frac{2eq}{\pi\epsilon_0 bL} \sum_{i=1}^{\infty} \frac{J_1(k_i b) J_1(k_i r)}{(k_i a)^2 J_1^2(k_i a)} \left(1 - e^{-\frac{k_i L}{2} \operatorname{ch} k_i z} \right) \quad (|z| < \frac{L}{2})$$

And the force expressions in laboratory system are

$$\left. \begin{aligned} F_z &= \frac{2eq}{\pi\epsilon_0 bL\gamma} \sum_{i=1}^{\infty} \frac{J_1(k_i b) J_0(k_i r)}{(k_i a)^2 J_1^2(k_i a)} \operatorname{sh} \frac{k_i \gamma L}{2} e^{-k_i z} & (z > \frac{L}{2}) \\ F_z &= -\frac{2eq}{\pi\epsilon_0 bL\gamma} \sum_{i=1}^{\infty} \frac{J_1(k_i b) J_0(k_i r)}{(k_i a)^2 J_1^2(k_i a)} \operatorname{sh} \frac{k_i \gamma L}{2} e^{k_i z} & (z < -\frac{L}{2}) \end{aligned} \right\} \quad (32a)$$

$$\left. \begin{aligned} F_r &= \frac{2eq}{\pi\epsilon_0 bL\gamma} \sum_{i=1}^{\infty} \frac{J_1(k_i b) J_0(k_i r)}{(k_i a)^2 J_1^2(k_i a)} \left(1 - e^{-\frac{k_i \gamma L}{2} \operatorname{sh} k_i \gamma z} \right) & (|z| < \frac{L}{2}) \\ F_r &= \frac{2eq}{\pi\epsilon_0 bL\gamma^2} \sum_{i=1}^{\infty} \frac{J_1(k_i b) J_1(k_i r)}{(k_i a)^2 J_1^2(k_i a)} \operatorname{sh} \frac{k_i \gamma L}{2} e^{-k_i \gamma |z|} & (|z| > \frac{L}{2}) \end{aligned} \right\} \quad (32b)$$

$$F_r = \frac{2eq}{\pi\epsilon_0 bL\gamma^2} \sum_{i=1}^{\infty} \frac{J_1(k_i b) J_1(k_i r)}{(k_i a)^2 J_1^2(k_i a)} \left(1 - e^{-\frac{k_i \gamma L}{2} \operatorname{ch} k_i \gamma z} \right) \quad (|z| < \frac{L}{2})$$

(2) WB distribution

From Eq. (20), one gets the force expressions as:

$$\left. \begin{aligned} F_z &= \frac{8eqa}{\pi\epsilon_0 b^2 L} \sum_{i=1}^{\infty} \frac{J_2(k_i b) J_0(k_i r)}{(k_i a)^3 J_1^2(k_i a)} \operatorname{sh} \frac{k_i L}{2} e^{-k_i z} & (z > \frac{L}{2}) \\ F_z &= -\frac{8eqa}{\pi\epsilon_0 b^2 L} \sum_{i=1}^{\infty} \frac{J_2(k_i b) J_0(k_i r)}{(k_i a)^3 J_1^2(k_i a)} \operatorname{sh} \frac{k_i L}{2} e^{k_i z} & (z < -\frac{L}{2}) \end{aligned} \right\} \quad (33a)$$

$$\left. \begin{aligned} F_z &= \frac{8eqa}{\pi\epsilon_0 b^2 L} \sum_{i=1}^{\infty} \frac{J_2(k_i b) J_0(k_i r)}{(k_i a)^3 J_1^2(k_i a)} \left(1 - e^{-\frac{k_i L}{2} \operatorname{sh} k_i z} \right) & (|z| < \frac{L}{2}) \\ F_r &= \frac{8eqa}{\pi\epsilon_0 b^2 L} \sum_{i=1}^{\infty} \frac{J_2(k_i b) J_1(k_i r)}{(k_i a)^3 J_1^2(k_i a)} \operatorname{sh} \frac{k_i L}{2} e^{-k_i |z|} & (|z| > \frac{L}{2}) \\ F_r &= \frac{8eqa}{\pi\epsilon_0 b^2 L} \sum_{i=1}^{\infty} \frac{J_2(k_i b) J_1(k_i r)}{(k_i a)^3 J_1^2(k_i a)} \left(1 - e^{-\frac{k_i L}{2} \operatorname{ch} k_i z} \right) & (|z| < \frac{L}{2}) \end{aligned} \right\} \quad (33b)$$

And the force expressions in laboratory system are

$$\left. \begin{aligned} F_z &= \frac{8eqa}{\pi\epsilon_0 b^2 L \gamma} \sum_{i=1}^{\infty} \frac{J_2(k_i b) J_0(k_i r)}{(k_i a)^3 J_1^2(k_i a)} \operatorname{sh} \frac{k_i \gamma L}{2} e^{-k_i z} & (z > \frac{L}{2}) \\ F_z &= -\frac{8eqa}{\pi\epsilon_0 b^2 L \gamma} \sum_{i=1}^{\infty} \frac{J_2(k_i b) J_0(k_i r)}{(k_i a)^3 J_1^2(k_i a)} \operatorname{sh} \frac{k_i \gamma L}{2} e^{k_i z} & (z < -\frac{L}{2}) \end{aligned} \right\} \quad (34a)$$

$$\left. \begin{aligned} F_z &= \frac{8eqa}{\pi\epsilon_0 b^2 L \gamma} \sum_{i=1}^{\infty} \frac{J_2(k_i b) J_0(k_i r)}{(k_i a)^3 J_1^2(k_i a)} \left(1 - e^{-\frac{k_i \gamma L}{2} \operatorname{sh} k_i \gamma z} \right) & (|z| < \frac{L}{2}) \\ F_r &= \frac{8eqa}{\pi\epsilon_0 b^2 L \gamma^2} \sum_{i=1}^{\infty} \frac{J_2(k_i b) J_1(k_i r)}{(k_i a)^3 J_1^2(k_i a)} \operatorname{sh} \frac{k_i \gamma L}{2} e^{-k_i |z|} & (|z| > \frac{L}{2}) \\ F_r &= \frac{8eqa}{\pi\epsilon_0 b^2 L \gamma^2} \sum_{i=1}^{\infty} \frac{J_2(k_i b) J_1(k_i r)}{(k_i a)^3 J_1^2(k_i a)} \left(1 - e^{-\frac{k_i \gamma L}{2} \operatorname{ch} k_i \gamma z} \right) & (|z| < \frac{L}{2}) \end{aligned} \right\} \quad (34b)$$

(3) PA distribution

From Eq. (21), one gets the force expressions as:

$$\left. \begin{aligned} F_z &= \frac{48eqa^2}{\pi\epsilon_0 b^3 L} \sum_{i=1}^{\infty} \frac{J_3(k_i b) J_0(k_i r)}{(k_i a)^4 J_1^2(k_i a)} \operatorname{sh} \frac{k_i L}{2} e^{-k_i z} & (z > \frac{L}{2}) \\ F_z &= -\frac{48eqa^2}{\pi\epsilon_0 b^3 L} \sum_{i=1}^{\infty} \frac{J_3(k_i b) J_0(k_i r)}{(k_i a)^4 J_1^2(k_i a)} \operatorname{sh} \frac{k_i L}{2} e^{k_i z} & (z < -\frac{L}{2}) \\ F_z &= \frac{48eqa^2}{\pi\epsilon_0 b^3 L} \sum_{i=1}^{\infty} \frac{J_3(k_i b) J_0(k_i r)}{(k_i a)^4 J_1^2(k_i a)} \left(1 - e^{-\frac{k_i L}{2} \operatorname{sh} k_i z} \right) & (|z| < \frac{L}{2}) \end{aligned} \right\} \quad (35a)$$

$$\left. \begin{aligned}
 F_r &= \frac{48eqa^2}{\pi\epsilon_0 b^3 L} \sum_{l=1}^{\infty} \frac{J_3(k_l b) J_1(k_l r)}{(k_l a)^4 J_1^2(k_l a)} \operatorname{sh} \frac{k_l L}{2} e^{-k_l |z|} & (|z| > \frac{L}{2}) \\
 F_r &= \frac{48eqa^2}{\pi\epsilon_0 b^3 L} \sum_{l=1}^{\infty} \frac{J_3(k_l b) J_1(k_l r)}{(k_l a)^4 J_1^2(k_l a)} \left(1 - e^{-\frac{k_l L}{2} \operatorname{ch} k_l z}\right) & (|z| < \frac{L}{2})
 \end{aligned} \right\} \quad (35b)$$

And the force expressions in laboratory system are

$$\left. \begin{aligned}
 F_z &= \frac{48eqa^2}{\pi\epsilon_0 b^3 L \gamma} \sum_{l=1}^{\infty} \frac{J_3(k_l b) J_0(k_l r)}{(k_l a)^4 J_1^2(k_l a)} \operatorname{sh} \frac{k_l \gamma L}{2} e^{-k_l z} & (z > \frac{L}{2}) \\
 F_z &= -\frac{48eqa^2}{\pi\epsilon_0 b^3 L \gamma} \sum_{l=1}^{\infty} \frac{J_3(k_l b) J_0(k_l r)}{(k_l a)^4 J_1^2(k_l a)} \operatorname{sh} \frac{k_l \gamma L}{2} e^{k_l z} & (z < -\frac{L}{2})
 \end{aligned} \right\} \quad (36a)$$

$$\left. \begin{aligned}
 F_z &= \frac{48eqa^2}{\pi\epsilon_0 b^3 L \gamma} \sum_{l=1}^{\infty} \frac{J_3(k_l b) J_0(k_l r)}{(k_l a)^4 J_1^2(k_l a)} \left(1 - e^{-\frac{k_l \gamma L}{2} \operatorname{sh} k_l \gamma z}\right) & (|z| < \frac{L}{2}) \\
 F_r &= \frac{48eqa^2}{\pi\epsilon_0 b^3 L \gamma^2} \sum_{l=1}^{\infty} \frac{J_3(k_l b) J_1(k_l r)}{(k_l a)^4 J_1^2(k_l a)} \operatorname{sh} \frac{k_l \gamma L}{2} e^{-k_l \gamma |z|} & (|z| > \frac{L}{2}) \\
 F_r &= \frac{48eqa^2}{\pi\epsilon_0 b^3 L \gamma^2} \sum_{l=1}^{\infty} \frac{J_3(k_l b) J_1(k_l r)}{(k_l a)^4 J_1^2(k_l a)} \left(1 - e^{-\frac{k_l \gamma L}{2} \operatorname{ch} k_l \gamma z}\right) & (|z| < \frac{L}{2})
 \end{aligned} \right\} \quad (36b)$$

(4) GA distribution

From Eq. (22), one gets the force expressions as:

$$\left. \begin{aligned}
 F_z &= \frac{eq}{\pi\epsilon_0 a L} \sum_{l=1}^{\infty} \frac{J_0(k_l r)}{k_l a J_1^2(k_l a)} e^{-\frac{k_l^2 a^2}{2}} \operatorname{sh} \frac{k_l L}{2} e^{-k_l z} & (z > \frac{L}{2}) \\
 F_z &= -\frac{eq}{\pi\epsilon_0 a L} \sum_{l=1}^{\infty} \frac{J_0(k_l r)}{k_l a J_1^2(k_l a)} e^{-\frac{k_l^2 a^2}{2}} \operatorname{sh} \frac{k_l L}{2} e^{k_l z} & (z < -\frac{L}{2})
 \end{aligned} \right\} \quad (37a)$$

$$\left. \begin{aligned}
 F_z &= \frac{eq}{\pi\epsilon_0 a L} \sum_{l=1}^{\infty} \frac{J_0(k_l r)}{k_l a J_1^2(k_l a)} e^{-\frac{k_l^2 a^2}{2}} \left(1 - e^{-\frac{k_l L}{2} \operatorname{sh} k_l z}\right) & (|z| < \frac{L}{2}) \\
 F_r &= \frac{eq}{\pi\epsilon_0 a L} \sum_{l=1}^{\infty} \frac{J_1(k_l r)}{k_l a J_1^2(k_l a)} e^{-\frac{k_l^2 a^2}{2}} \operatorname{sh} \frac{k_l L}{2} e^{-k_l z} & (|z| > \frac{L}{2}) \\
 F_r &= \frac{eq}{\pi\epsilon_0 a L} \sum_{l=1}^{\infty} \frac{J_1(k_l r)}{k_l a J_1^2(k_l a)} e^{-\frac{k_l^2 a^2}{2}} \left(1 - e^{-\frac{k_l L}{2} \operatorname{ch} k_l z}\right) & (|z| < \frac{L}{2})
 \end{aligned} \right\} \quad (37b)$$

And the force expressions in laboratory system are

$$\left. \begin{aligned}
 F_z &= \frac{eq}{\pi \epsilon_0 a L \gamma} \sum_{i=1}^{\infty} \frac{J_0(k_i r)}{k_i a J_1^2(k_i a)} e^{-\frac{k_i^2 a^2}{2}} \operatorname{sh} \frac{k_i \gamma L}{2} e^{-k_i \pi} & (z > \frac{L}{2}) \\
 F_z &= -\frac{eq}{\pi \epsilon_0 a L \gamma} \sum_{i=1}^{\infty} \frac{J_0(k_i r)}{k_i a J_1^2(k_i a)} e^{-\frac{k_i^2 a^2}{2}} \operatorname{sh} \frac{k_i \gamma L}{2} e^{k_i \pi} & (z < -\frac{L}{2}) \\
 F_z &= \frac{eq}{\pi \epsilon_0 a L \gamma} \sum_{i=1}^{\infty} \frac{J_0(k_i r)}{k_i a J_1^2(k_i a)} e^{-\frac{k_i^2 a^2}{2}} \left(1 - e^{-\frac{k_i \gamma L}{2}} \operatorname{sh} k_i \gamma z\right) & (|z| < \frac{L}{2})
 \end{aligned} \right\} (38a)$$

$$\left. \begin{aligned}
 F_r &= \frac{eq}{\pi \epsilon_0 a L \gamma^2} \sum_{i=1}^{\infty} \frac{J_1(k_i r)}{k_i a J_1^2(k_i a)} e^{-\frac{k_i^2 a^2}{2}} \operatorname{sh} \frac{k_i \gamma L}{2} e^{-k_i \gamma |z|} & (|z| > \frac{L}{2}) \\
 F_r &= \frac{eq}{\pi \epsilon_0 a L \gamma^2} \sum_{i=1}^{\infty} \frac{J_1(k_i r)}{k_i a J_1^2(k_i a)} e^{-\frac{k_i^2 a^2}{2}} \left(1 - e^{-\frac{k_i \gamma L}{2}} \operatorname{ch} k_i \gamma z\right) & (|z| < \frac{L}{2})
 \end{aligned} \right\} (38b)$$

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直线加速器中空间电荷束团的非线性效应

原子能出版社出版

(北京 2108 信箱)

中国核情报中心排版

北京市海淀区三环快速印刷厂印刷

☆

开本 787×1092 1/16 · 印张 1 · 字数 20 千字

1992 年 2 月北京第一版 · 1992 年 2 月北京第一次印刷

ISBN 7-5022-0636-1

TL · 380



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ISBN 7-5022-0636-1
TL · 380

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