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A. RÁCZ

ESTIMATION OF QUASI-CRITICAL REACTIVITY

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B U D A P E S T

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REPORT

Estimation of Quasi-Critical Reactivity

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ABSTRACT

In the present paper we critically review the bank of Kalman filter method for reactivity estimation originally suggested by D'Attellis and Cortina. It is pointed out that the procedure cannot be applied reliably in such a form as the authors proposed, due to the filter divergence. An improved method, which is free from divergence problems are presented, as well. The procedure is applied for estimation of small reactivity changes.

Rácz A.: Kvázi-kritikus reaktivitás becslése KFKI-1992-09/G

KIVONAT

Jelen munkában kritikai áttekintését adjuk D'Attellis és Cortina módszerének, melyet a szerzők reaktivitásbecslésre fejlesztettek ki. Kimutatjuk, hogy az eljárás, a szűrési folyamatban megjelenő divergencia miatt, a javasolt formában nem alkalmazható megbízhatóan. A régi eljárás helyett kidolgozunk egy javított modellt, mely mentes a divergenciaproblémáktól. A módszert alkalmazzuk kis reaktivitásváltozások meghatározására.

Introduction

In a recent communication C.E. D'Attellis and E. Cortina proposed a method to estimate reactivity as well as neutron density via a bank of Kalman filters [1]. The procedure is based on Magill's concepts of calculating unknown parameters of stochastic processes.

The adaptation of Magill's method to the case of reactivity estimation is a very attractive idea since the whole procedure seems to be quite easy and straightforward at the first sight. However, every attempt of us to reconstruct D'Attellis' and Cortina's results has failed in spite of the excellent and persuasive results of the authors. Therefore the whole theory has been carefully reviewed and hidden difficulties have been revealed. In the present paper these results are collected. Our aim is threefold. i) We would like to warn other users to be careful when using the criticized method. ii) A possible explanation why D'Attellis' and Cortina's idea has worked, is also presented. iii) A new procedure has been developed inspired by the original one. In the following the authors' model system is recalled first. Afterwards the Kalman filtering technique is overviewed with main emphasis on the filter divergence problems. Finally a new estimation technique is proposed and tested using computer simulation results.

The model system

The work in Ref.1 is based on the following assumptions:

i) The dynamic model contains the point kinetic equations, *without* a dynamic noise term:

$$\dot{n}(t) = \frac{\rho - \beta}{l} n(t) + \sum_{i=1}^{6} \lambda_i c_i, \qquad \beta = \sum_{i=1}^{6} \beta_i, \tag{1}$$

$$\dot{c}_i(t) = \frac{\beta_i}{l} n(t) - \lambda_i c_i, \qquad i = 1, \ldots 6. \tag{2}$$

where $n(t)$ is the neutron density, $c(t)$ is the precursor concentration, ρ is the reactivity, β_i is the delayed neutron fraction, l is the neutron generation time and λ_i is the decay constant of the precursors.

ii) The scalar measurement model is the noise corrupted observation of the neutron density:

$$z(t) = n(t) + v(t), \tag{3}$$

where $z(t)$ is the signal and $v(t)$ is the measurement noise. The discrete model is given in Appendix A.

iii) A discrete linear Kalman filter is applied for estimating the state vector.

iv) A bank of Kalman filters is used to determine the best approximation of the reactivity.

The authors did not give the details of the Kalman filtering technique just cited the literature. Although it is a known and well-established tool for state estimation problems,

some remarks must be made here. In general only minor attention is paid to the investigation of the possible *divergence* problems of the filtering. Here divergence means an *unbounded* estimation error. Normally divergence occurs seldom, only in extreme situations. However, the divergence does play an important role in the method in hand.

Kalman filtering and filter divergence

The necessary formulae of the Kalman filter is recapitulated in Appendix B. Here we concentrate only on the divergence. The main problems are the following. In general neither the system models [Eqs.(B.1) and (B.2)] nor the initial data $\hat{\mathbf{x}}_0$ and $\mathbf{P}_{0,0}$ are known exactly. Therefore the model used in constructing the filter can differ from the *physical system* which generates the measurable signals. However an inaccurate filter model *does* degrade the filter performance and could make the filter diverge. Inaccurate initial data $\hat{\mathbf{x}}_0$ and $\mathbf{P}_{0,0}$ can lead to similar effects, as well.

Fortunately there are *theorems* which guarantee that a *wide class* of systems can be estimated *reliably* by Kalman filters without a real risk of divergence [3]. In fact, the correction term $\mathbf{K}_k\underline{\xi}_k$ in the filter equation (B.4) does not allow the estimated state vector $\hat{\mathbf{x}}_k$ to be too far from the true value of $\mathbf{x}_k$. Obviously, the term $\mathbf{K}_k\underline{\xi}_k$ can correct the estimate unless it is zero. Unfortunately, the Kalman gain $\mathbf{K}_k$ has the following property [2,3]:

$$\mathbf{Q}_k \longrightarrow \infty \qquad \Longrightarrow \qquad \mathbf{K}_k \longrightarrow 0,$$

where $\mathbf{Q}_k$ is the variance of the dynamic noise $\mathbf{w}_k$. As a consequence, the lack of dynamic noise makes the filter unable to compensate for the fatal effect of inaccurate parameters and initial data. *Unless* every element of the system model as well as the initial data are *precise*, the estimate of the state vector *must diverge*. Schlee *et al* [4] presented a simple analytic example to show how the estimation error becomes unbounded if a non-zero valued control term is *ignored* in the noise-free dynamic model. At the same time it was proven [4] that the estimation errors can be made bounded by *adding noises* to the dynamic model. Although introduction of an increased noise can *improve* the stability of the estimation procedure it could *degrade* the performance of the filter. Therefore the optimal choice of the value of $\mathbf{Q}_k$ is usually means certain compromise. Detailed analysis of the divergence problems can be found in Ref. 2.

Divergent estimate of the neutron economy

Now let us consider Eq.(1) again. It is the noise-free dynamic model describing the neutron economy. According to the properties listed above, the Kalman filter applied to this system must be very sensitive to the parameters ρ, β, l and Λ_i's. Unless they are exactly known and a precise guess about the initial data $\hat{n}(0)$ and $\mathbf{P}_0$ can be found, the estimation procedure may become *divergent*. The values of β, l and Λ_i's can be known reliably. However, the reactivity ρ is to be determined. In order to do it a set of trial values of reactivity $\{\rho^i\}_{i=1}^N$ is chosen. Afterwards every element ρ^i of the set is substituted

into the dynamic model (1) and a Kalman filter is formed using this particular model. In each filter the applied reactivity ρ^i as well as the system model *must* differ from the real physical system. Appendix B gives the details of this parameter estimation technique called the Magill-Bogler method [5, 6].

Referring to Appendix C the main constraints of the Magill-Bogler method are the followings:

- It can estimate unknown parameters present only in the *control term*.
- No divergence is allowed during the estimation procedure.

Usually the first item is acknowledged, and the second one is ignored. The neglection of the second term could be understood from the tacit assumption, that only such an estimation is used which does not diverge. Consequently, it is usually believed, that no *inherent* divergence occurs. However, it is not trivial and should be checked.

As it will be pointed out, D'Attellis' and Cortina's method *does not* comply with either of the above constraints. In fact, the method is meant to estimate an unknown parameter of the state-transition matrix $\Phi_{k+1,k}$ and not of the control term $\mathbf{B}_k\mathbf{u}_k$, using the Magill-Bogler procedure. The procedure may be correct but it should be proven. The second problem arises from the filter divergence. The supposed noise-free dynamic model [see Eqs.(1) and (2)] makes the Kalman filter very sensitive to the uncertainties in the system parameters as well as in the initial conditions. Therefore even if the Magill-Bogler procedure could have been applied without changes for estimating the parameters of the state-transition matrix, the procedure must have been failed because of the filter divergence. Since the Magill-Bogler procedure needs a finite set of trial values, *some* best approximate must always be achieved. Therefore the reliability of the approximation must be checked otherwise. It can be shown [7] that the score statistic of the true control parameter has a χ^2_N distribution, where N is the sampling number. Therefore the value of $\mathcal{L}^I(N)$ corresponding to the best approximation ρ^I must be in the *vicinity* of N (see Appendix C). Let us apply these results to the values of Table I in Ref. 1. The minimal value of $\mathcal{L}$ is 30.30 corresponding to the best approximation of the reactivity. However, the sampling number was $N = 50$, therefore $\mathcal{L}(N)$ should have been around 50. Certainly a realization of $\mathcal{L}(N)$ might take the value of 30.30 but it has a very low probability (cf. Ref.7 for details). In general a very rough trial set could also produce such an effect. However the applied resolution was $\Delta\rho/\rho \simeq 0.06$ which was dense enough. Therefore the contradiction must be yielded by the filter divergence, probably caused by omitting the dynamic noise.

In order to illustrate the above statements some results of different computational simulation runs are presented. Here a simplified, one precursore group point kinetic model is used instead of the general one given by Eqs.(1) and (2). The parameters are taken from the literature [8] and are as follows: $\beta = 6.4 \times 10^{-3}$, $l = 9.5 \times 10^{-4}$s and $\lambda = 8 \times 10^{-2}$s^{-1}. The reactivity is set to make the reactor supercritical, namely $\rho = 0.0025$. The sampling interval is chosen to be $dt = 0.1$s.

In the first part of the simulation artifical measurement data are generated using the point kinetic model. Afterwards the Kalman filter is implemented and applied on these data in order to estimate the state vector. The estimated neutron density $\hat{n}$, the measured neutron density y_n and the innovation ξ_n are displayed. For transparency, the neutron

density n as the first component of the state vector, is also presented, keeping in mind that this quantity is *not* available in real situations.

First a typical run is presented. The initial values $\hat{n}_0$ and $\hat{c}_0$ are deliberately chosen not too close to the true values of n_0 and c_0. (Since it is a computer simulation, n_0 and c_0 are known.) The results are shown in Figure 1. It can be seen that the filter needs some time to compensate for the effect of inaccurate initial conditions. Afterwards (about 20 time steps) the estimate becomes accurate. The phenomenon is well-known and the time which is necessary to achieve convergency is called the *learning period* of the Kalman filter [2,3].

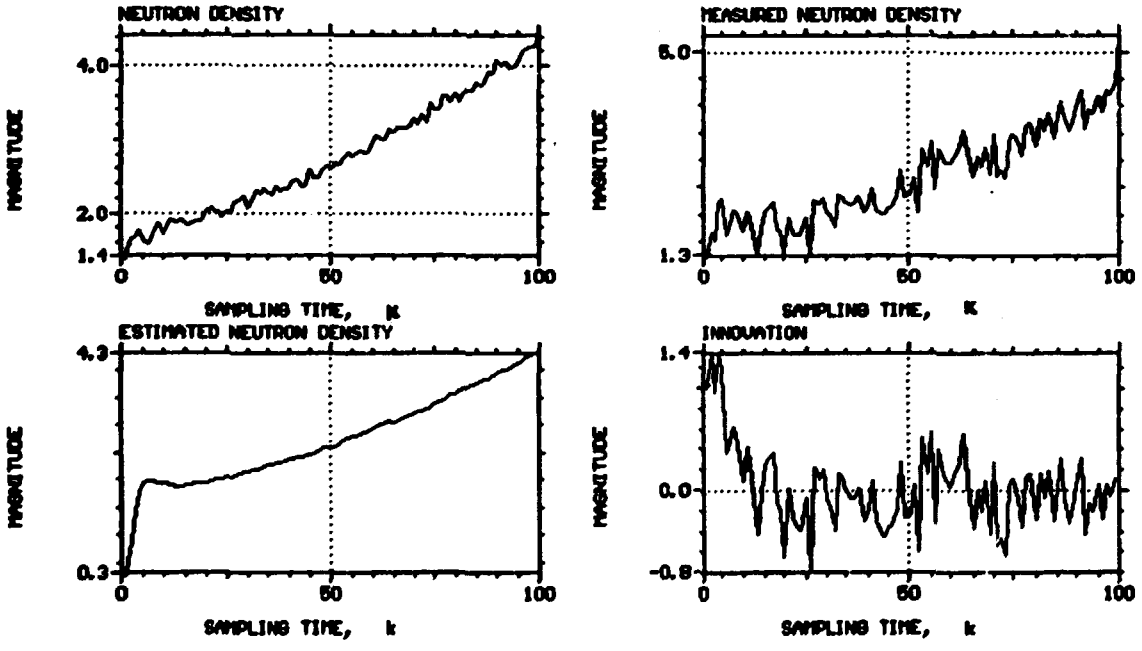

Figure 1. The neutron density n and its measured value y_n (upper curves)
The estimated neutron density $\hat{n}$ and the innovation ξ_n (lower curves)
$\rho = 0.0025$, there *is* dynamic noise

In the second run the effect of the dynamic noise is illustrated. To this effect, the measurement data are processed by *four* different Kalman filters. The filters are different in the following two aspects: i) the applied reactivity in the filter equations. ii) the presence or absence of a dynamic noise. Table I shows the possible variations.
The results are shown in Figure 2. It can be seen that a dynamic noise term does *stabilize* the Kalman filer.

Now we are in the position to give some explanation, why D'Attellis' and Cortina's results were seemingly so excellent. Since the authors did not give any technical details about their calculation, our explanation is not more than a guess; we may be totally wrong.

There are two questions to answer: i) How could the estimated reactivity be so close to the true value? ii) Why does the minimal value of the score statistics differs seriously from the theoretically prescribed one?

Table I

№	Reactivity ρ	Dynamic noise w
# 1	2.3×10^{-3}	No
# 2	2.7×10^{-3}	No
# 3	2.3×10^{-3}	Yes
# 4	2.7×10^{-3}	Yes

The different set-up values of the Kalman filters

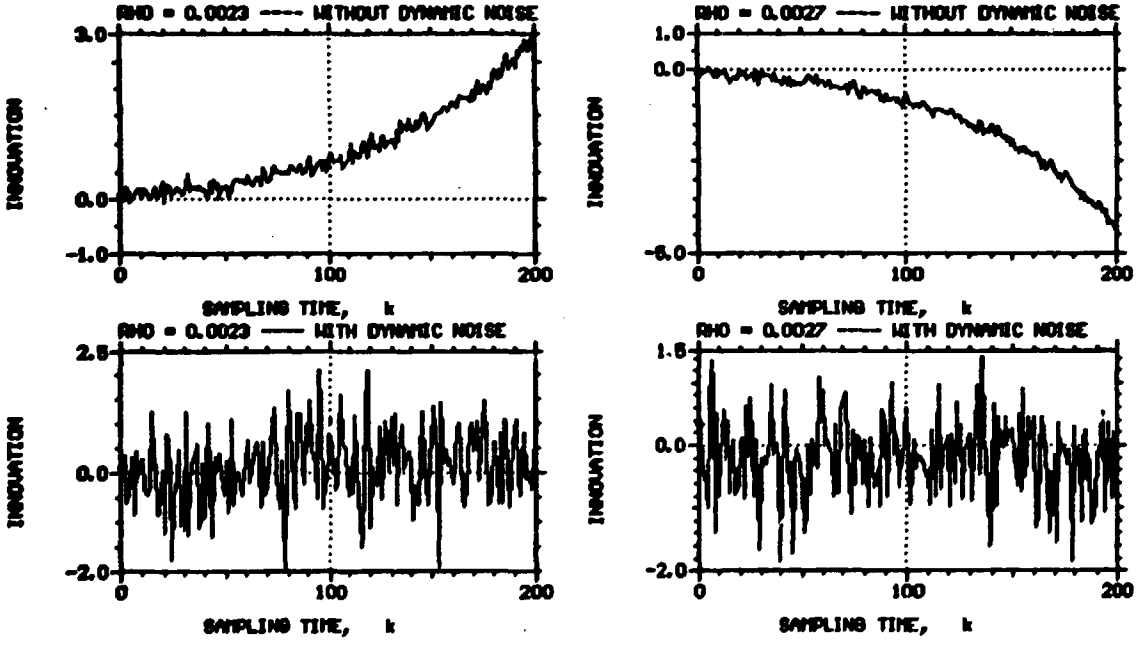

Figure 2. The innovations
$\rho = 0.0023$ and $\rho = 0.0027$ *without* dynamic noise (upper curves)
$\rho = 0.0023$ and $\rho = 0.0027$ *with* dynamic noise (lower curves)

Guess 1) The processed time-series was very short ($N = 50$) thus no serious divergence was able to develop during this time (see Figure 2). In addition, the score statistic is constructed using the *square* of the innovation thus *every* divergence does *increase* the score statistic. Therefore the smaller the divergence the smaller the score statistic. The amount of the divergence depends on the goodness of the approximation. As a consequence, the better the approximation the smaller the score statistics.

Guess 2) The authors in Ref. 1 probably used the following form $\mathcal{L}_{AC}(N)$ for the score statistics

$$\mathcal{L}_{AC}(N) = -\mathcal{L}_{MB}(N). \tag{4}$$

It has been mentioned that $\mathcal{L}(N)$ could take such a value, which is far from its expected

value, but this event has a very low probability. Therefore in such a case the whole calculation should be repeated again in order to check the reliability of the estimation. Ref. 1 has not mentioned it. Finally, if the authors applied noise corrupted dynamic model with the score statistics given by Eq.(4), the whole problem disappears. In this case Ref. 1 has only forgotten to mention this fact.
We pointed out that the application of the Magill-Bogler procedure in Ref. 1 is *improper* in that sense that the unknown parameter appears in the state transition matrix Φ instead of the control term. This invalid interpretation of the Magill-Bogler method also could be responsible for the whole anomalous effect. In the sequel a proper treatment of the problem is delineated.

Reactivity estimation

For further analysis let us consider the simplified, one precursor group model

$$\begin{aligned} \dot{n}(t) &= \frac{\rho-\beta}{l}n(t) + \lambda c, \\ \dot{c}(t) &= \frac{\beta}{l}n(t) - \lambda c, \end{aligned} \tag{5}$$

with the observation process

$$z(t) = n(t) + v(t). \tag{6}$$

In accordance with Appendix A, the above continuous-time model is discretized as

$$\begin{aligned} \mathbf{x}_{k+1} &= \Phi_{k+1,k}\mathbf{x}_k, \\ y_k &= x_k + v_k, \end{aligned} \tag{7}$$

where the state-transition matrix $\Phi_{k+1,k}$ is approximated as

$$\Phi^{\rho}_{k+1,k} \equiv \Phi_{k+1,k} = \begin{pmatrix} 1 + \frac{\rho-\beta}{l}\Delta t & \lambda\Delta t \\ \frac{\beta}{l}\Delta t & 1-\lambda\Delta t \end{pmatrix}, \tag{8}$$

supposing that the time step Δt is small enough.

The variables β, l and λ are known. The reactivity ρ is unknown and needs to be estimated. The reactivity is supposed to be a sum of a known part ρ^0 and an unknown one $\Delta\rho$:

$$\rho = \rho^0 + \Delta\rho. \tag{9}$$

Therefore the task is to determine $\Delta\rho$. After inserting Eq.(9) into Eq.(2), the state-transition matrix reads

$$\Phi^{\rho}_{k+1,k} = \Phi^{0}_{k+1,k} + \Delta\Phi_{k+1,k}, \tag{10}$$

where

$$\Phi^0_{k+1,k} = \begin{pmatrix} 1+\frac{\rho^0-\rho}{l}\Delta t & \lambda\Delta t \\ \frac{\rho}{l}\Delta t & 1-\lambda\Delta t \end{pmatrix} \quad \text{and} \quad \Delta\Phi_{k+1,k} = \begin{pmatrix} \frac{\Delta\rho}{l}\Delta t & 0 \\ 0 & 0 \end{pmatrix} \tag{11}$$

Utilizing Eq.(10), the dynamic equation (7) reads:

$$\mathbf{x}_{k+1} = (\Phi^0_{k+1,k} + \Delta\Phi_{k+1,k})\mathbf{x}_k. \tag{12}$$

Let us assume that the state vector $\mathbf{x}_k$ can be approximated as

$$\mathbf{x}_k = \mathbf{x}^0_k + \Delta\mathbf{x}_k, \tag{13}$$

where $\mathbf{x}^0_k$ satisfies the following equation:

$$\mathbf{x}^0_{k+1} = \Phi^0_{k+1,k}\mathbf{x}^0_k. \tag{14}$$

Let us substitute Eqs.(13) and (14) into Eq.(1). As long as the second order term $\Delta\Phi_{k+1,k}\Delta\mathbf{x}_k$ is small compared to the other terms the following equation is valid:

$$\Delta\mathbf{x}_{k+1} = \Phi^0_{k+1,k}\Delta\mathbf{x}_k + \Delta\Phi_{k+1,k}\mathbf{x}^0_k. \tag{15}$$

Let us consider Eq.(15). It can be regarded as a *dynamic* equation of the state vector $\Delta\mathbf{x}_k$, where the state-transition matrix is $\Phi^0_{k+1,k}$, the control matrix is $\Delta\Phi_{k+1,k}$ and the control vector is $\mathbf{x}^0_k$. In accordance with Eq.(9), the term ρ^0 is supposed to be known, thus so is the state-transition matrix $\Phi^0_{k+1,k}$. Since $\mathbf{x}^0_k$ is the *deterministic* input vector in the dynamic equation (15), it must be known. In fact, $\mathbf{x}^0_k$ is the solution of Eq.(14).

It can be seen that Eq.(15) is suitable for the Magill-Bogler procedure since the unknown parameter appears in the *control term*. Only one task has remained, namely constructing the *observation* model. Since the perturbation $\Delta\mathbf{x}_k$ is *not* measurable directly, the (discrete) measurement process must be the following:

$$\mathbf{y}_k = \mathbf{H}_k\mathbf{x}^0_k + \mathbf{H}_k\Delta\mathbf{x}_k + \mathbf{v}_k, \tag{16}$$

where Eq.(13) is utilized. The above observation model is similar to the general one in Eq.(B.2). In fact, $\mathbf{H}_k\mathbf{x}^0_k$ can be regarded as a *known* input term in the measurement process. In our case Eq.(16) reduces to the following *scalar* measurement model

$$y_k = \Delta n_k + n^0_k + v_k, \tag{17}$$

where the neutron density n is the first component of the state vector $\mathbf{x}_k$.

As a result, the point kinetic equations with unknown reactivity can be transformed into a special form, in which the reactivity is separated into a known and an unknown part. The unknown part $\Delta\rho$ of the reactivity appears in the control term of the dynamic model. Therefore the classical Magill-Bogler estimation procedure can be applied to determine $\Delta\rho$. In accordance with the arguments of the previous Subsections, the divergence of the filtering processes can be avoided by adding fictitious noises to the dynamic model. Therefore the final equations are the following:

$$\Delta\mathbf{x}_{k+1} = \Phi^0_{k+1,k}\Delta\mathbf{x}_k + \Delta\Phi_{k+1,k}\mathbf{x}^0_k + \mathbf{w}_k \,, \tag{18}$$

$$y_k = \Delta n_k + n^o_k + v_k \,. \tag{19}$$

In the following Sections the procedure is tested.

The stationary case

Regarding Eqs.(18) and (19) it can be seen that there are two principally different cases, depending on the value of ρ^0. If $\rho^0 = 0$ the reactor is critical and $\mathbf{x}^0_k$ is time-independent. However, if $\rho^0 \neq 0$ then $\mathbf{x}^0_k$ is time-dependent and can be obtained from Eq.(13). Since Eqs.(18) and (19) contain $\mathbf{x}^0_k$, its value must be known (or guessed) at every time point k. As we have seen previously the control term $\mathbf{x}^0_k$ plays a crucial role in the estimation procedure. Therefore a time-dependent, not too correct guess of $\mathbf{x}^0_k$ could degrade the estimation. Particularly it is the situation when $\rho^0 > 0$ (supercritical reactor) since in this case $\mathbf{x}^0_k$ increases exponentially. In this article only the critical case, i.e. $\rho^0 = 0$ is treated. The detailed analysis of the complicated off-critical situation is a topic of a subsequent paper.

It is supposed that after a long stationary period the reactivity suddenly changes, but the deviation from the critical value $\rho^0 = 0$ is small and constant, i.e. $\Delta\rho \neq \Delta\rho(t)$. The direction of $\Delta\rho$ can be either positive (supercritical) or negative (subcritical). The stationary $\mathbf{x}^0_k$ is supposed to be known as well. The effect of a not too good $\mathbf{x}^0_k$ value on the reactivity estimation procedure will be tested.

Computer simulation results

The method is tested via computer simulation. In the first step a $\Delta\rho$ value is chosen and the exact state transition matrix $\Phi^{\mathcal{L}}_{k,k-1} \equiv \exp\{\mathbf{A}\Delta t\}$ is determined (see Eq.(A.3) using a Laplace transformation method [9]. The subscript $\mathcal{L}$ indicates the Laplace method. The discrete dynamic model

$$\mathbf{x}_{k+1} = \Phi^{\mathcal{L}}_{k,k-1}\mathbf{x}_k + \mathbf{w}_k \tag{20}$$

gives the state vector. Afterwards the signals y_k are generated as

$$y_k = n_k + v_k, \tag{21}$$

where n_k is the first component of the state vector $\mathbf{x}_k$. The v_k and $\mathbf{w}_k$ white noises are generated by the computer.

In the second step the Magill-Bogler procedure is applied (see Appendix C) on the $\{n_1, n_2, \ldots, n_k, \ldots\}$ sequence in order to estimate the $\Delta\rho$ value.

Although the control vector $\mathbf{x}_k^0$ can be known almost exactly (in accordance with the long steady state operation), the effect of a not too correct value of $\mathbf{x}_k^0$ on the reactivity estimation is tested, as well. During the Magill-Bogler procedure a $q \cdot \mathbf{x}_k^0$ control vector is applied instead of the true one ($\mathbf{x}_k^0$) where the quantity q can vary. The $q \neq 1$ value mimics a wrong estimate of the control vector $\mathbf{x}_k^0$. Table II shows the results of some different simulation runs.

Table II

N	Δt	Δ ($\times 10^{-4}$)	q	$\Delta\rho$ ($\times 10^{-4}$)	$\Delta\hat{\rho}$ ($\times 10^{-4}$)	$\mathcal{L}^I(N)$
300	0.2	5	1.5	22	25	308.4
300	0.2	5	2.0	22	20	328.2
300	0.2	5	0.7	22	35	327.6
300	0.2	5	1.0	22	20	292.1
300	0.2	5	1.5	18	20	320.5
300	0.2	5	2.5	25	25	286.0
300	0.2	5	1.5	35	45	306.0

The different set-up values of the Kalman filters

The displayed quantities are the following: N is the sampling number, dt is the sampling time. The distance between two consecutive trial reactivity values is Δ. The n_k signals are generated using $\Delta\rho$ reactivity value. The estimated reactivity is $\Delta\hat{\rho}$. The score statistics belonging to the best estimation $\Delta\hat{\rho}$ is $\mathcal{L}^I(N)$ (see Appendix C for details). The parameter q gives the correctness of the initial guess of the control vector $\mathbf{x}_k^0$.

It can be seen that the procedure yields a good estimate of the reactivity even if the initial guess about the control vector $\mathbf{x}_k^0$ is not too precise. The reliability of the estimated reactivity could be checked using the score statistic $\mathcal{L}^I(N)$. Namely, if $\mathcal{L}^I(N) \approx N$ then the estimation is reliable [7]. Comparing the first column of the table to the last one, it can be seen that the calculated reactivity values $\Delta\hat{\rho}$ are reliable estimates of the true reactivities $\Delta\rho$. (Note that since $\Delta = 5$, the estimation can only be exact up to ± 2.5.)

Closing Remarks

In this paper D'Attellis' and Cortina's reactivity estimation method has been carefully analyzed. We pointed out that the authors used the bank of Kalman filters technique in an improper way. The appearance of an unknown (therefore inaccurate) reactivity value in the state transition matrix may cause divergence problems during the Kalman filtering procedure. The divergence is artificially amplified by omitting the dynamic noise in the dynamic, as well as in the filter equations.

We proposed another method to determine the reactivity value. Supposing small changes in the reactivity, the effect can be described by the appearance of an extra input term in the point kinetic equations. The unknown reactivity shift becomes an unknown control parameter suitable for the "bank of Kalman filters" procedure. The control vector is the solution of the homogeneous point kinetic equations using the reactivity value prior the to changes. In the present article a long critical operation is supposed before the reactivity change happens. The effect of the inaccurate guess about the critical operation conditions is investigated.

An interesting problem is to investigate the applicability of this new method for the *super- or subcritical* situation, when a sudden reactivity change occurs during a non-stationary period. This research is a topic of subsequent papers.

The present method is based on the transformation of the reactivity changes from the state transition matrix into the control term. However, this manipulation cannot be done in general. It would deserve a detailed analysis to see how the Magill-Bogler technique has to be modified in order to be able to handle unknown parameters in the state transition matrix, too.

APPENDICES

A. The discrete model of the neutron economy

The discrete dynamic model corresponding to the continuous one described by Eqs.(1) and (2) is the following:

$$\mathbf{x}_{k+1} = \Phi_{k+1,k}\mathbf{x}_k \ , \tag{A.1}$$

where the state vector $\mathbf{x}_k$ is

$$\mathbf{x}_k^T = \Big(n(t_k), c_1(t_k), \ldots, c_6(t_k)\Big) \tag{A.2}$$

and the state transition matrix $\Phi_{k+1,k}$ is

$$\Phi_{k+1,k} = \exp\{\mathbf{A}\Delta t\} \ . \tag{A.3}$$

The time dependent matrix **A** is

$$\mathbf{A} = \begin{pmatrix} \frac{\varrho-\beta}{l} & \Lambda_1 & \Lambda_2 & \dots & \Lambda_6 \\ \frac{\beta_1}{l} & -\Lambda_1 & 0 & \dots & 0 \\ \frac{\beta_2}{l} & 0 & -\Lambda_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\beta_6}{l} & 0 & 0 & \dots & -\Lambda_6 \end{pmatrix}. \qquad (A.4)$$

The discrete time t_k is defined as $t_k \stackrel{\text{def}}{=} t_0 + k\Delta t$, where t_0 is an initial time point, Δt is a fixed time step and k is an integer. Henceforth k is used as a shorthand notation of t_k.

The measurement model is the following:

$$\mathbf{y}_k = \mathbf{H}_k\mathbf{x}_k + \mathbf{v}_k. \qquad (A.5)$$

In accordance with Eq.(3) the above *general* model takes the following form:

$$y_k = n_k + v_k \quad \text{with} \quad \mathbf{H}_k^T = (1,0,\dots,0)\ , \qquad (A.6)$$

Having known the matrix **A** the discretization procedure is more or less straightforward [6, 7]. Henceforth the discrete model of the point kinetic equations is supposed to be known.

B. The Kalman filtering technique

Let us consider a discrete, linear stochastic system described by the following dynamic and measurement models, respectively:

$$\mathbf{x}_{k+1} = \Phi_{k+1,k}\mathbf{x}_k + \mathbf{B}_k\mathbf{u}_k + \mathbf{w}_k, \qquad (B.1)$$

$$\mathbf{y}_k = \mathbf{H}_k\mathbf{x}_k + \mathbf{C}_k\mathbf{d}_k + \mathbf{v}_k, \qquad (B.2)$$

where $\mathbf{x}_k$ is the state vector, $\mathbf{y}_k$ is the measurement vector, $\Phi_{k+1,k}$ is the state-transition matrix, $\mathbf{H}_k$ is the measurement matrix, $\mathbf{B}_k$ is the control matrix, $\mathbf{u}_k$ is the control vector, $\mathbf{w}_k$ and $\mathbf{v}_k$ are the plant and the measurement noise, respectively; k is the discrete time. The noise terms $\mathbf{w}_k$ and $\mathbf{v}_k$ are supposed to be zero mean white Gaussian processes, i.e. $\mathbf{w}_k \in \mathcal{N}(0,\mathbf{Q}_k)$ and $\mathbf{v}_k \in \mathcal{N}(0,\mathbf{R}_k)$. The variances $\mathbf{Q}_k$ and $\mathbf{R}_k$ are known.

Having obtained the measurement vectors $\{\mathbf{y}_i\}$ up to $i = k$, the discrete, linear Kalman filter (dkf) can provide an estimation of the state vector. Henceforth the estimated quantities are denoted by a symbol *hat*, i.e. the estimated state vector is $\hat{\mathbf{x}}_k$. The estimation error $\tilde{\mathbf{x}}_k$ is defined as

$$\tilde{\mathbf{x}}_k \stackrel{\text{def}}{=} \mathbf{x}_k - \hat{\mathbf{x}}_k. \qquad (B.3)$$

The dkf is a recursive procedure [2] with the following basic equations:

$$\hat{\mathbf{x}}_k = \Phi_{k,k-1}\hat{\mathbf{x}}_{k-1} + \mathbf{B}_k\mathbf{u}_k + \mathbf{K}_k\xi_k. \qquad (B.4)$$

Here $\Phi_{k,k-1}\hat{\mathbf{x}}_{k-1}+\mathbf{B}_k\mathbf{u}_k$ is the one-step *prediction* while the term $\mathbf{K}_k\xi_k$ gives the *correction*. The correction is formed by the innovation ξ_k (which is the new information extracted from the new measurement vector $\mathbf{y}_k$)

$$\begin{aligned}\underline{\xi}_k &\stackrel{\text{def}}{=} \mathbf{y}_k - \hat{\mathbf{y}}_k = \\ &= \mathbf{y}_k - \mathbf{H}_k(\Phi_{k,k-1}\mathbf{x}_{k-1} + \mathbf{B}_k\mathbf{u}_k) - \mathbf{C}_k\mathbf{d}_k \end{aligned} \qquad (B.5)$$

and the Kalman gain $\mathbf{K}_k$. The Kalman gain is calculated as

$$\mathbf{K}_k = \mathbf{P}_{k,k-1}\mathbf{H}_k^T \times \left[\mathbf{R}_k + \mathbf{H}_k\mathbf{P}_{k,k-1}\mathbf{H}_k^T\right]^{-1}, \qquad (B.6)$$

where $\mathbf{P}_{k+1,k}$ is the *a priori* error variance

$$\mathbf{P}_{k+1,k} = \Phi_{k+1,k}\mathbf{P}_{k,k}\Phi_{k+1,k}^T + \mathbf{Q}_k \ . \qquad (B.7)$$

The *a posteriori* error variance $\mathbf{P}_{k,k}$ is determined as

$$\mathbf{P}_{k,k} = (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k), \mathbf{P}_{k,k-1} \ , \qquad (B.8)$$

where $\mathbf{I}$ is the identity matrix. The quantities $\Phi_{k+1,k}, \mathbf{B}_k, \mathbf{H}_k, \mathbf{C}_k, \mathbf{u}_k, \mathbf{d}_k, \mathbf{Q}_k$ and $\mathbf{R}_k$ must be known. An initial guess $\hat{\mathbf{x}}_0$ and $\mathbf{P}_{0,0}$ must be used to start the recursion.

C. The basic idea of the parameter estimation

As first Magill [5] afterwards Bogler [6] pointed out, if a discrete, linear stochastic system Eq.(4) contains an unknown *control* term $\mathbf{B}_k\mathbf{u}_k$, this term can be estimated using a bank of Kalman filters. For the sake of simplicity let the control term $\mathbf{B}_k\mathbf{u}_k$ contain only one unknown, *constant* parameter b. Our aim is to determine b. It can be proven that a certain change in the value of this parameter b causes a *constant* shift in the *mean* value m of the innovation. As long as b is the *exact* control parameter denoted by b^*, the innovation $\underline{\xi}_k^*$ is a zero mean white Gaussian process $\underline{\xi}_k^* \in \mathcal{N}(0, \mathbf{V}_k^*)$ where $\mathbf{V}_k^* = \mathbf{R}_k + \mathbf{H}_k\mathbf{P}_{k,k-1}\mathbf{H}_k^T$. The Magill-Bogler procedure utilizes the fact, that the innovation $\underline{\xi}_k^+$ (determined via inaccurate parameter value $b^+ \neq b^*$) does have the same variance as $\underline{\xi}_k$, (i.e. $\mathbf{V}_k^+ = \mathbf{V}_k^*$) but has a different mean $m^+ \neq 0$. In fact, m^+ is a *linear* function of the other quantities

$(\Phi, \mathbf{H}, \ldots)$ of the system. The smaller the difference between b^* and b^+ the closer the mean value m^+ to zero. Taking advantage of the features listed above Magill and Bogler proposed the following procedure to determine the best approximation of the unknown b^*. A finite set $\{b^i\}_{i=1}^M$ is formed from the different approximating values b^i of the parameter b^*. The elements b^i must be guessed somehow. The goal is to find that particular element $\hat{b} \in \{b^i\}$, $i = 1, \ldots, M$, which is the best approximation of b^*, i.e. $|\hat{b} - b^*| < |b^i - b^*|$, $\forall b^i \neq \hat{b}$. To do it let M different Kalman filters (bank of Kalman filters) be constructed using the M different b^i elements. Let the filters operate on the *same* signals $\{\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_k, \ldots \mathbf{y}_N\}$ and produce M different innovation processes $\{\underline{\xi}_k^i\}_{i=1}^M$. Let the following *score statistic* denoted by $\mathcal{L}_{MB}^i(N)$ be determined:

$$\mathcal{L}_{MB}^i(N) \stackrel{\text{def}}{=} -\frac{1}{2} \sum_{k=1}^{N} \frac{[(\underline{\xi}_k^i)_l]^2}{(\mathbf{R}_k + \mathbf{H}_k \mathbf{P}_{k,k-1} \mathbf{H}_k^T)_{l,l}}, \qquad l \in [1, m] \ , \tag{C.1}$$

where m is the dimension of the measurement vector $\mathbf{y}_k$. The best approximation from the set $\{b^i\}_{i=1}^M$ is the particular element $b^I \in \{b^i\}_{i=1}^M$ which *maximizes* the score statistic

$$\mathcal{L}_{MB}^I(N) \stackrel{\text{def}}{=} \max_{1 \leq i \leq M} \{\mathcal{L}_{MB}^i(N)\} \ . \tag{C.2}$$

In our case a modified score statistic $\mathcal{L}^i(N)$, namely

$$\mathcal{L}^i(N) \stackrel{\text{def}}{=} -2\mathcal{L}_{MB}^i(N) \tag{C.3}$$

has proved to be useful instead of the original one. Therefore the decision rule is modified too:

$$\mathcal{L}^I(N) \stackrel{\text{def}}{=} \min_{1 \leq i \leq M} \{\mathcal{L}^i(N)\} \ . \tag{C.4}$$

Using $\mathcal{L}^i(N)$ instead of the original one $\mathcal{L}_{MB}^i(N)$ makes performing some statistical test more handy. In fact, the actual value of $\mathcal{L}^i(N)$ can be used to check the reliability (or goodness) of the estimated value b^i. As long as b^i is close enough to the true value b^*, the minimal value $\mathcal{L}^i(N)$ can be regarded as a *realization* of a χ_N^2 distribution. For more details and examples see Ref.7.

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