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ON RENORMALIZATION GROUP FLOW IN MATRIX MODEL

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ABSTRACT

The renormalization group flow recently found by Brézin and Zinn-Justin by integrating out redundant entries of the $(N+1) \times (N+1)$ Hermitian random matrix is studied. By introducing explicitly the RG flow parameter, and adding suitable counter terms to the matrix potential of the one matrix model, we deduce some interesting properties of the RG trajectories. In particular, the string equation for the general massive model interpolating between the UV and IR fixed points turns out to be a consequence of RG flow. An ambiguity in the UV region of the RG trajectory is remarked to be related to the large order behaviour of the one matrix model.

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Recently, Brézin and Zinn-Justin^[1] have suggested an approximate scheme for studying the renormalization group (RG) flow in the one matrix model. By treating the size N of the hermitian matrix as an effective cut-off in the theory, they have derived a differential equation for matrix free energy, and reproduced approximately the scaling law of the (multi-) criticality of the one matrix model. Compared to the exact result of the double scaled limit, their approach seems to be on the right track of an alternative understanding^[2] of the puzzles^[3] related to the KdV flows in the one matrix model noted earlier.

Because of the well known problem of even potential matrix integral, people hoped to be able to use the KdV flows ~~existing in the~~ model to reach a well defined pure gravity theory from the higher multicritical point. This has failed since the flow itself suffers from instability problem^[4]. Besides, we lack an intrinsic way to see how the generalized KdV equations and the general massive theory interpolating between multicritical points are inter-related.

In this short note, we try to derive something **interesting** and **exact** for the RG flows of Brézin and Zinn-Justin. The word exact will be **explained** later. Note that, even though, the values of the parameters at criticality are calculated only approximately, the RG trajectory slightly off the criticality may well survive higher order corrections.

The starting point is the potential of $N \times N$ hermitian matrix for $m = 2$ critical point (pure gravity) used by Brézin and Zinn-Justin:

$$V(\phi) = N\left(\frac{1}{2}\text{tr}\phi^2 + \frac{g}{4}\text{tr}\phi^4\right). \quad (1)$$

In [1] the following effective potential is obtained after integrating out redundant entries of the $(N+1) \times (N+1)$ matrix,

$$V'(\phi) = (N+1)\left(\frac{1}{2}\text{tr}\phi^2 + \frac{g}{4}\text{tr}\phi^4\right) + g\text{tr}\phi^2, \quad (2)$$

where, in both equations (1) and (2), ϕ denotes $N \times N$ matrix. In deriving (2), a parametrization of matrix

$$\phi_{(N+1) \times (N+1)} = \begin{pmatrix} \phi_{N \times N} & \mathbf{u} \\ \bar{\mathbf{u}} & \alpha \end{pmatrix} \quad (3)$$

is used, and the assumption is made setting $\alpha = 0$.

In the sense of a RG flow from N to $N+1$, equation (2) corresponds to adding a counter term $(1/2 + g)\text{tr}\phi^2 + g/4\text{tr}\phi^4$ to the original potential (1). We observe that the parameter of RG transformation, $t \sim \frac{1}{N}$, is implied implicitly (which might have shown up in the rescaling

$N \rightarrow N' = e^t N$). Since in the large N limit, the model is driven to an (IR) fixed point which describes pure gravity, along the trajectory of increasing t , quantities of order $\frac{1}{N}$ which were previously neglected become important in a hypothetical massive theory. The question is then what have been thrown away before? Obviously, $\alpha \sim \frac{1}{N}$ and which has been set to zero when deriving (2). Our assumption here is that a non-zero α in (3) substitutes for the role of the implicit parameter t of the RG transformation induced by integrating the margins of the $(N+1)$ -matrix. Thus we modify the RG flow equation so that it is satisfied with a small, un-integrated parameter α .

The above assumption leads, by a calculation similar to that in [1], to the effective potential

$$V''(\phi) = (N+1)\left(\frac{1}{2}\text{tr}\phi^2 + \frac{g}{4}\text{tr}\phi^4\right) + g\text{tr}\phi^2 + \gamma\text{tr}\phi, \quad (4)$$

where, $\gamma = \alpha g$, and only terms of first order in α are kept. It is easy to check that (4) does not alter the (IR) fixed point structure as long as one also adds a counter term proportional to $\text{tr}\phi^3$.

Therefore, in the same sense as introducing counter terms corresponding to (2) does not alter the fixed point structure of (1), further counter terms can be added

$$\gamma\text{tr}\phi + \lambda\text{tr}\phi^3 \quad (5)$$

without violating the original scaling law of (1), at the present level of accuracy.

Note that the condition of tadpole cancellation imposes a relation between the two coefficients in (5) whose explicit form we will not write. This implies that when α is interpreted as RG flow parameter t , $\lambda(t)$ evolves in the augmented space of coupling constants (g, λ) .

The generalization to the multi-critical points is straightforward and the result is a structure of counter terms consisting of traces of all odd powers of ϕ ,

$$\sum \lambda_{2k+1} \text{tr}\phi^{2k+1}, \quad k = 0, 1, 2, \dots \quad (6)$$

To recapitulate, we have assumed a non-zero α which plays the role of RG flow parameter, and obtained a doubling of matrix model couplings. The appearance of terms involving odd power of ϕ should be interpreted as turning on relevant perturbations.

The immediate consequence of the doubling of couplings is the possibility of extracting massive model from the RG flow of the critical point models. In fact, starting from a general matrix model potential

$$V(\phi) = \sum_{k=1} t_k \text{tr}\phi^k, \quad k = 1, 2, \dots \quad (7)$$

and defining an *abstract* linear system^[5], one is able to "derive" the following form of string equation for general massive model connecting multicritical points in one matrix model

$$\sum_{m=1} m \tilde{t}_m \frac{\partial}{\partial \tilde{t}_{m-1}} F - \frac{\partial F}{\partial \tilde{t}_0} + \frac{1}{2} \tilde{t}_0^2 = 0, \quad (8)$$

where F is the string partition function and the flow times \tilde{t}_m and the matrix couplings t_k are related by $\tilde{t}_m \sim t_{2k+1}$ upto a suitable rescaling.

A few words about the exactness of our result. Note that the most natural interpretation of the string equation (8) is in terms of a special discrete linear system^[6], of which the string equation is a consequence of compatibility condition. Equation (8) looks so familiar to us that it should survive the *continuous* limit, although this limit is not at all necessary^[6] as long as the topological nature is concerned. It is in this sense that we have deduced exact result through approximate RG transformation. At any rate, a massive theory itself is not responsible for its crude asymptotics (but the boundary conditions are).

We conclude this letter with a comment on the possible cure of the problem associated to the KdV flows. The RG flow of [1] is formally asymptotic to the IR regime. Because we have made use of the fact α small, formal procedure for studying the *UV* asymptotics is broken down. This is also to be attributed to some sort of instability problem, but this time in the *UV* regime. However, a trick^[6] which was used in studying the $\lambda\phi^4$ field theory could be of help here. That is to analytically continue to the negative coupling in such a way the *UV* regime is approachable from the negative side near the *zero* coupling point^[7]. We have carried out a study along this line and indeed found a flow function near the *UV* regime which is free of the Borel/instanton singularities^[8].

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