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ABSTRACT

The neutrino problem is set anew in the light of a reformulation of the Dirac field theory that provides a natural account for the effect commonly interpreted as 'P-violation', and restores P-mirror symmetry. A two-component (left-handed) neutrino field is automatically derived, whose P-mirror image does not correspond to a 'missing' particle but is the (right-handed) antineutrino field.

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1. *Introduction.*

The 'parity violation' effect, in spite of more than thirty years from its theoretical and experimental discovery [1-2], can still be treated only at a phenomenological level in QFT. According to standard views there is yet no definite answer as to the origin of such an effect. The fact is that QFT can accomodate but cannot naturally account for the appearance of 'parity violation': in order to match theory to experience, two *ad hoc* external prescriptions must be invoked. These are the two-component neutrino scheme (aimed at avoiding the 'unwanted' right-handed neutrino solution) [3-6] and the '$V - A$' scheme [7-9]. Even the 'standard model' [10-12] relies essentially upon such a phenomenological approach and can but *postulate* the 'parity violating' nature of the weak-isospin fermionic current [13]. On the other hand, there have been some contemplations to restore mirror symmetry [14]. It has also been argued [15] that the laws of Nature, i.e. the S-matrix or Hamiltonian, conserve parity, but the states are not in general eigenstates of parity, and in the case of a massless neutrino, half of the states are missing whence the problem fundamentally is about these missing states.

2. *The two mass-conjugate Dirac equations.*

Recently, a fruitful new basic way of trying to get a theoretic insight into the 'P-violation' effect was outlined [16]. The approach is based on the original idea [17] that the

Dirac free fermion and its charge-conjugate antifermion should be described by two opposite-mass (rather than equal-mass) field equations: if $\psi_f(x^\mu)$ stands for the free fermion Dirac field and $\psi_{\bar{f}}(x^\mu)$ for its charge-conjugate, then (c=ℏ=1)

$$i\gamma^\mu \partial_\mu \psi_f = +m\,\psi_f \quad , \quad i\gamma^\mu \partial_\mu \psi_{\bar{f}} = -m\,\psi_{\bar{f}} \tag{1}$$

($\mu=0,1,2,3$; $\gamma^{k\dagger}=-\gamma^k$, k = 1,2,3 ; metrics: + − − − ; $m>0$). The pair of equations (1) can actually be obtained as a direct consequence of demanding that the Dirac parity operator should covariantly have the *same* form, say $U_P=\eta\gamma^0$ ($\eta=\pm1$), in both fermion and antifermion four-spinor spaces [16]. To see this, let $u_f(\mathbf{p})$ and $u_{\bar{f}}(\mathbf{p})$ denote two equal-helicity mutually charge-conjugate free four-spinors *both with a positive energy eigenvalue* $E=(\mathbf{p}^2+m^2)^{1/2}$. Under the extended covariance requirement above, the usual *opposite-intrinsic-parity condition* for fermions and antifermions [18] then reads

$$U_P\,u_f(0)=\eta u_f(0) \quad , \quad U_P\,u_{\bar{f}}(0)=-\eta u_{\bar{f}}(0) \tag{2}$$

or, equivalently,

$$\gamma^0 m\,u_f(0)=m\,u_f(0) \quad , \quad \gamma^0(-m)\,u_{\bar{f}}(0)=m\,u_{\bar{f}}(0) \tag{3}$$

where m on the right-hand side of both eqs. (3) may be taken as the (positive) rest *energy* eigenvalue $E=m$ associated with $u_f(0)$ and $u_{\bar{f}}(0)$. So, if form invariance of the free Dirac Hamiltonian under charge conjugation is still to be preserved, the two four-spinors $u_f(\mathbf{p})$ and $u_{\bar f}(\mathbf{p})$ should just be eigenspinors of the opposite-mass Hamiltonians

$$H_f \equiv H(\mathbf{p},+m) \quad , \quad H_{\bar f} \equiv H(\mathbf{p},-m) \tag{4}$$

where $H(\mathbf{p},\pm m) = \boldsymbol{\alpha}\cdot\mathbf{p} + \beta(\pm m)$ $(\beta = \gamma^0)$. Such a description still allows real spin-$\frac{1}{2}$ fermions and antifermions to have an identical (positive) rest-energy sign, and no contradiction can therefore arise with the *'CPT'* theorem [19]: owing to the quadratic relationship $E^2 = \mathbf{p}^2 + m^2$, that theorem merely requires particle and antiparticle to have the same absolute value of mass rather than two identical masses! Note moreover that the factorization of the Klein-Gordon equation leads to the two equation types (1).

3. *Charge conjugation.*

Unlike the usual Dirac scheme, the operation of charge conjugation may now be defined not only in terms of fields, but also (and primarily) in terms of wave functions alone. If invariance of the energy-eigenspinor equation under charge conjugation C is invoked and a linear link of $u_{\bar f}(\mathbf{p})$ to $u_f(\mathbf{p})$ is assumed, then, because of (4), the *non*-trivial transformation is obtained (up to a phase factor)

$$u_{\bar f}(\mathbf{p}) \equiv U_C\, u_f(\mathbf{p}) = \gamma^5 u_f(\mathbf{p}) \tag{5}$$

$(\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3)$. Hence C should be given a *covariant* definition

$$C: \quad \psi_f(x^\mu) \longrightarrow \psi_{\bar f}(x^\mu) = \gamma^5 \psi_f(x^\mu) \tag{6}$$

that, in line with (1), identifies $\mathcal{C}$ with *proper-mass conjugation* [20-22]. By use of (6) it is easy to check that equations (1) are both derivable from the $\mathcal{C}$-invariant free Lagrangian density

$$\mathcal{L} = \tfrac{1}{2}\{ [\tfrac{1}{2}(i\,\bar{\psi}_f\,\gamma^\mu \partial_\mu \psi_f + \mathrm{H.c.}) - m\,\bar{\psi}_f\,\psi_f]$$
$$+ [\tfrac{1}{2}(i\,\bar{\psi}_{\bar{f}}\,\gamma^\mu \partial_\mu \psi_{\bar{f}} + \mathrm{H.c.}) - (-m)\,\bar{\psi}_{\bar{f}}\,\psi_{\bar{f}}]\}\,. \qquad (7)$$

The meaning of definition (6) can fully be seen in the case of electromagnetic coupling: if ψ_f stands for a fermion field of electric charge $(-e)$ in the presence of an external four-potential A_μ ,

$$i\gamma^\mu[\partial_\mu + i(-e)A_\mu]\,\psi_f = +m\,\psi_f \quad , \qquad (8)$$

then, as can be easily checked, $\psi_{\bar{f}} = \gamma^5\,\psi_f$ turns out to be, *quite symmetrically*, the field of the corresponding antifermion (of charge e) in the presence of the *charge-conjugate* four-potential $(-A_\mu)$,

$$i\gamma^\mu[\partial_\mu + ie(-A_\mu)]\,\psi_{\bar{f}} = -m\,\psi_{\bar{f}} \quad . \qquad (9)$$

In other words, unlike the usual charge conjugation operation, $\mathcal{C}$ as given by (6) acts automatically on the *whole* interacting system ($\psi_f \to \psi_{\bar{f}}$, $A_\mu \to -A_\mu$) and not on the mere fermionic part, so that symmetry under $\mathcal{C}$ is maintained. Coming back to the free-particle case, let us adopt a description covariant under the extended Lorentz group, so that the same fermion or antifermion may be assigned positive-energy as well as negative-energy states [23]. We then see that (6) strictly requires a *single* fermion-antifermion Fock space (mapped by $\mathcal{C}$ onto itself) where Dirac-fermion and

Dirac-antifermion states are *already* distinguished by the (opposite) sign of the associated proper-mass parameters. Such a Fock space should have the manifestly covariant structure

$$\mathcal{F} \equiv \mathcal{F}^{\circ} \otimes \mathcal{S}_{in} \tag{10}$$

where $\mathcal{F}^{\circ}$ is an ordinary Fock space for one *indistinct* type of positive- and negative-energy identical spin-$\frac{1}{2}$ particles (without regard to the proper-mass sign) and $\mathcal{S}_{in}$ is a two-dimensional internal space spanned by the proper-mass eigenstates $|+m\rangle$, $|-m\rangle$ thus doubling $\mathcal{F}^{\circ}$. This allows ψ_f and $\psi_{\bar{f}}$ to be *mixed* if a rotation is performed in $\mathcal{S}_{in}$. One may in particular consider the orthogonal transformation

$$\psi_f = 2^{-1/2}(\chi_f + \chi_{\bar{f}}) \quad , \quad \psi_{\bar{f}} = 2^{-1/2}(-\chi_f + \chi_{\bar{f}}) \tag{11}$$

that defines the two asymptotically left-handed and right-handed generalized chiral fields

$$\chi_f = 2^{-1/2}(1-\gamma^5)\,\psi_f \quad , \quad \chi_{\bar{f}} = 2^{-1/2}(1+\gamma^5)\,\psi_{\bar{f}} \tag{12}$$

as an *alternative field-coordinate set* in the two-dimensional (internal) space marked by ψ_f and $\psi_{\bar{f}}$. Two basic features of (11) are to be stressed: the first is that (11) enables us to introduce *massive* chiral fields on the same footing as *massive* Dirac fields; the second is that *no further* linearly independent pair of massive chiral fields can be introduced beside the pair $(\chi_f, \chi_{\bar{f}})$: the two other combinations

$$\xi_f \equiv 2^{-1/2}(1+\gamma^5)\,\psi_f \quad , \quad \xi_{\bar{f}} \equiv 2^{-1/2}(1-\gamma^5)\,\psi_{\bar{f}} \tag{13}$$

give, due to (6), the same fields again

$$\xi_f = \chi_{\bar{f}} \quad , \quad \xi_{\bar{f}} = -\chi_f \quad . \tag{14}$$

4. *Massive chiral fermions.*

In the light of the last remarks we may associate χ_f with the actual "chiral" fermion and $\chi_{\bar{f}}$ with the actual "chiral" antifermion, thereby providing a quite *natural* room for the apparent 'parity violation' effect, with no further *ad hoc* external prescriptions. In this regard, setting

$$|f\rangle \equiv |+m\rangle \qquad , \qquad |\bar{f}\rangle \equiv |-m\rangle \tag{15}$$

we may define a "chiral" basis $\left(|f^{ch}\rangle, |\bar{f}^{ch}\rangle\right)$ in S_{in},

$$|f\rangle = 2^{-1/2}\left(|f^{ch}\rangle + |\bar{f}^{ch}\rangle\right) \ , \ |\bar{f}\rangle = 2^{-1/2}\left(-|f^{ch}\rangle + |\bar{f}^{ch}\rangle\right) \ , \tag{16}$$

$|f^{ch}\rangle$ being associated with χ_f and $|\bar{f}^{ch}\rangle$ with $\chi_{\bar{f}}$. If M denotes the mass operator in S_{in}, states $|f^{ch}\rangle$ and $|\bar{f}^{ch}\rangle$ are not M-eigenstates, though they are still eigenstates of M^2. From C: $|f\rangle \rightleftarrows |\bar{f}\rangle$, it follows that

$$C|f^{ch}\rangle = -|f^{ch}\rangle \qquad , \qquad C|\bar{f}^{ch}\rangle = |\bar{f}^{ch}\rangle \quad . \tag{17}$$

5. *Two kinds of charges.*

If Q is a charge operator diagonal in $\left(|f\rangle, |\bar{f}\rangle\right)$ and Q^{ch} is a charge operator diagonal in $\left(|f^{ch}\rangle, |\bar{f}^{ch}\rangle\right)$ (both with opposite eigenvalues), it can be easily checked that Q and Q^{ch} *anticommute* and are such that on one hand,

$$\langle f | Q^{ch} | f \rangle = \langle \bar{f} | Q^{ch} | \bar{f} \rangle = 0 \tag{18}$$

while on the other,

$$\langle f^{ch} | Q | f^{ch} \rangle = \langle \bar{f}^{ch} | Q | \bar{f}^{ch} \rangle = 0 \ . \tag{19}$$

The two charge varieties Q and Q^{ch} – whose *anticommutivity* property gives rise to the "dual" fermion model sketched in Ref.[16] – differ for being (the former) *scalar* and (the latter) *pseudoscalar* under parity. To see this, let us rewrite the opposite-intrinsic-parity condition (2) in the form

$$P | f \rangle = \eta | f \rangle \quad , \quad P | \bar{f} \rangle = -\eta | \bar{f} \rangle \quad . \tag{20}$$

Choosing $\eta = 1$, we correspondingly have in the "chiral" basis:

$$P : \ | f^{ch} \rangle \rightleftarrows | \bar{f}^{ch} \rangle \ . \tag{21}$$

By use of (20) and (21), we thus obtain

$$P Q = Q P \quad , \quad P Q^{ch} = - Q^{ch} P \quad . \tag{22}$$

Let now Q and Q^{ch} be both conserved. If the same fermion or antifermion may look like a Q-eigenstate as well as a Q^{ch}-eigenstate, this cannot happen simultaneously because Q and Q^{ch} do not commute. Hence, each pair of internal states $| f \rangle$, $| \bar{f} \rangle$ and $| f^{ch} \rangle$, $| \bar{f}^{ch} \rangle$ can only *partially* describe the "dual" charge nature of the fermion and antifermion, the former pair disclosing the *scalar* aspect and the latter one the *pseudoscalar* aspect of such a nature. The correct physical interpretation of eqs. (18) and (19) is therefore the following: The signs of Q and Q^{ch} eigenvalues are in turn *hidden* freedom degrees, so that the transitions $|f\rangle \rightleftarrows |f^{ch}\rangle$ and $|\bar{f}\rangle \rightleftarrows |\bar{f}^{ch}\rangle$ should only *apparently* violate the conservation of either kind of charge.

6. *Limits to the massless case.*

We shall devote here our attention to the consequences of the above scheme in the limiting case when $m = 0$. If the new Dirac-field charge conjugation (6) is extended to this case, a key to the solution of the problem of the 'missing' (right-handed) neutrino and (left-handed) antineutrino is obtained. The Dirac field theory in its standard formulation yields at first *four* independent zero-mass chiral-field solutions, one pair of solutions being related to a left-handed neutrino and a right-handed antineutrino and the other pair vice versa to a right-handed neutrino and a left-handed antineutrino. The fundamental question is therefore left unanswered as to why *only one* (the former) chiral-field pair seems to have an actual counterpart in Nature[1]. This apparent 'mystery' is cleared in the new formulation, which *naturally* yields only the two-component neutrino scheme. Indeed, for $m = 0$, we may put $\psi_f = \psi_\nu$, $\psi_{\bar{f}} = \psi_{\bar{\nu}}$ (whence $\psi_{\bar{\nu}} = \gamma^5 \psi_\nu$) and eqs. (14) read

$$\xi_\nu = \chi_{\bar{\nu}} \quad , \quad \xi_{\bar{\nu}} = -\chi_\nu \tag{23}$$

where

$$\xi_\nu = 2^{-1/2}(1 + \gamma^5)\,\psi_\nu \quad , \quad \xi_{\bar{\nu}} = 2^{-1/2}(1 - \gamma^5)\,\psi_{\bar{\nu}} \tag{24}$$

and

$$
\begin{aligned}
\chi_{\nu} &= 2^{-1/2}(1-\gamma^{5})\,\psi_{\nu} = 2^{-1/2}(1-\gamma^{5})\,\psi \\
\chi_{\bar{\nu}} &= 2^{-1/2}(1+\gamma^{5})\,\psi_{\bar{\nu}} = 2^{-1/2}(1+\gamma^{5})\,\psi
\end{aligned}
\qquad (25)
$$

ψ standing for *the* Dirac-field solution of the *unique* equation which is obtained from eqs.(1) as $m \rightarrow 0$. Hence, on account of (6), only *two* massless chiral-field solutions are now available in all, that may clearly be identified with the *actual* solutions (25)! More precisely, if $a_{\nu}^{(R)}(\mathbf{p})$ denotes the annihilation operator of a (positive-energy) right-handed 'neutrino' and $a_{\bar{\nu}}^{(L)}(\mathbf{p})$ the one of a (positive-energy) left-handed 'antineutrino', identities (23) imply

$$
a_{\nu}^{(R)}(\mathbf{p}) = a_{\bar{\nu}}^{(R)}(\mathbf{p}) \equiv a^{(R)}(\mathbf{p}) \quad , \quad a_{\bar{\nu}}^{(L)}(\mathbf{p}) = a_{\nu}^{(L)}(\mathbf{p}) \equiv a^{(L)}(\mathbf{p}) \qquad (26)
$$

and likewise, for the corresponding creation operators,

$$
a_{\nu}^{(R)\dagger}(\mathbf{p}) = a_{\bar{\nu}}^{(R)\dagger}(\mathbf{p}) \equiv a^{(R)\dagger}(\mathbf{p}) \ , \ a_{\bar{\nu}}^{(L)\dagger}(\mathbf{p}) = a_{\nu}^{(L)\dagger}(\mathbf{p}) \equiv a^{(L)\dagger}(\mathbf{p}) \ . \qquad (27)
$$

So, only *two* (left- and right-handed) kinds of annihilation and creation operators are actually involved, the former, $a^{(L)}(\mathbf{p}), a^{(L)\dagger}(\mathbf{p})$, associated with the 'physical' neutrino and the latter, $a^{(R)}(\mathbf{p}), a^{(R)\dagger}(\mathbf{p})$, associated with the 'physical' antineutrino. The same result is reached also as follows: The negative-energy solutions of the first equation in (1) are identical to the positive-energy solutions of the second equation; the set of positive-energy solutions of both equations (1) is accordingly a *complete* one, and this will also be true for the set of positive-energy solutions of the *unique* zero-mass equation obtained from eqs.(1). If in particular using Weyl's representation, we have

$$\psi = \begin{pmatrix} \Phi^{(L)} \\ \Phi^{(R)} \end{pmatrix} \tag{28}$$

where $\Phi^{(L)}$ and $\Phi^{(R)}$ are two-component (left- and right-handed) fields such that

$$\chi_{\nu} = -\zeta^{c}_{\bar{\nu}} \propto \begin{pmatrix} \Phi^{(L)} \\ 0 \end{pmatrix} \quad , \quad \chi_{\bar{\nu}} = \zeta^{c}_{\nu} \propto \begin{pmatrix} 0 \\ \Phi^{(R)} \end{pmatrix} \ . \tag{29}$$

We are thus in a position to say, e.g., that the two 'missing' *neutrino* field components provided by ζ^{c}_{ν} are not missing at all in Nature, since they may be taken as just the actual *antineutrino* field components! The scheme under consideration gives also a theoretical account of the fact that an electrically charged fermion is never massless: the reason is that according to (11) or (16), a chiral fermion state (even massive) can now have, by its nature, only a *null* expectation value of electric charge – see (19) – and for $m=0$, the states are necessarily chiral.

7. *Parity.*

Such a key to the neutrino problem seems also to provide a deeper understanding of the neutrino symmetry properties. This can immediately be argued if noting that, under parity,

$$P: \quad a^{(L)}(\mathbf{p}) \rightleftarrows a^{(R)}(-\mathbf{p}) \quad , \quad a^{(L)\dagger}(\mathbf{p}) \rightleftarrows a^{(R)\dagger}(-\mathbf{p}) \tag{30}$$

whence *neutrino and antineutrino may now be straightforwardly interpreted as the ordinary mirror image of each other*! The same conclusion can be drawn from the more general fact that *now CP is equivalent to P if acting on either massless or massive* γ^5*-eigenfields* (C being represented by γ^5 itself!). To gain a further insight into that, let us for a moment come back to the case when $m \neq 0$. If $\eta = 1$ is substituted in (2), transformation (11) shows that we may define, in the internal space S_{in}, an *"intrinsic parity" operator* P_{in} ($P_{in}^2 = 1$) [16] such that

$$P_{in}: \qquad \chi_f \rightleftarrows \chi_{\bar{f}} \; . \tag{31}$$

This special operator — to which P is reducible only when acting on $m=0$ positive-energy eigenspinors — has the *exclusive* (internal) effect of inverting the *chirality* sign by inducing the transformations $(1-\gamma^5) \to (1+\gamma^5)$, $(1+\gamma^5) \to -(1-\gamma^5)$. Of course, in both (20) and (21) we could have written P_{in} in place of P. From (11) we can also infer that the whole parity P acts on chiral fields $\chi_f, \chi_{\bar{f}}$ as follows:

$$P: \qquad \chi_f \longrightarrow \gamma^0 \chi_f \quad , \quad \chi_{\bar{f}} \longrightarrow \gamma^0 \chi_{\bar{f}} \; . \tag{32}$$

In this way, P can be split into

$$P = P_{in} P_{ex} = P_{ex} P_{in} \tag{33}$$

where P_{ex} ($P_{ex}^2 = 1$) — we may call "external parity" — is the part of P which acts properly on the pure Fock space $\mathcal{F}^\circ$ (without affecting the chirality):

$$P_{ex}: \; (1-\gamma^5)\,\psi_f \to (1-\gamma^5)\gamma^0\,\psi_f \;\; , \;\; (1+\gamma^5)\,\psi_{\bar{f}} \to -(1+\gamma^5)\gamma^0\,\psi_{\bar{f}} \; . \tag{34}$$

Note that the action of P_{ex} upon the chiral-field current $\bar{\chi}_f \gamma^\mu \chi_f$ is the same as the ordinary action of P upon the '$V - A$' current $\bar{\psi}_f \gamma^\mu (1 - \gamma^5) \psi_f$. Let us then take the general free Lagrangian density (7) and rewrite it in terms of χ_f and $\chi_{\bar{f}}$:

$$\mathcal{L} = \tfrac{1}{2}[\tfrac{1}{2}\mathrm{i}(\bar{\chi}_f \gamma^\mu \partial_\mu \chi_f + \bar{\chi}_{\bar{f}} \gamma^\mu \partial_\mu \chi_{\bar{f}}) + \mathrm{H.c.}] - \tfrac{1}{2} m (\bar{\chi}_f \chi_{\bar{f}} + \bar{\chi}_{\bar{f}} \chi_f). \quad (35)$$

Of course, $\mathcal{L}$ is invariant under P as well as under the single operations P_{ex} , P_{in} ; and this property is kept even when $m \to 0$. On setting $m = 0$, we thus still get a P-invariant Lagrangian density, which is further both P_{ex}- and P_{in}-invariant: it reads

$$\mathcal{L} = \tfrac{1}{2}(\mathrm{i}\bar{\psi} \gamma^\mu \partial_\mu \psi + \mathrm{H.c.}) = \tfrac{1}{2}(\Lambda_\nu + \Lambda_{\bar{\nu}}) \quad (36)$$

where

$$\Lambda_\nu = \tfrac{1}{2}(\mathrm{i}\bar{\chi}_\nu \gamma^\mu \partial_\mu \chi_\nu + \mathrm{H.c.}) \quad , \quad \Lambda_{\bar{\nu}} = \tfrac{1}{2}(\mathrm{i}\bar{\chi}_{\bar{\nu}} \gamma^\mu \partial_\mu \chi_{\bar{\nu}} + \mathrm{H.c.}) . \quad (37)$$

The basic remarkable feature of (36) is that Λ_ν and $\Lambda_{\bar{\nu}}$ are just the free Lagrangian densities for the *actual* neutrino and antineutrino in spite of the P-invariance property of $\mathcal{L}$! Taking (32) into account, we see that *even* such Lagrangian densities are individually left invariant by P; this is not the case, however, if they are singly acted upon by either P_{in} or P_{ex} alone, since we have

$$P_{in} : \qquad \Lambda_\nu \rightleftarrows \Lambda_{\bar{\nu}} \quad (38)$$

and

$$P_{ex} : \qquad \Lambda_\nu \rightleftarrows \Lambda_{\bar{\nu}} \quad . \quad (39)$$

Transformation (38) follows directly from (31), while transformation (39) is equivalent, e.g., to the ordinary one (in the '$V-A$' interpretation)

$$P: \quad \mathcal{L}_{\nu}(\chi_{\nu}) \quad \rightleftarrows \quad \mathcal{L}_{\nu}(\xi_{\nu}) \tag{40}$$

$\mathcal{L}_{\nu}(\xi_{\nu})$ $\left(=\mathcal{L}_{\bar{\nu}}(\chi_{\bar{\nu}})\right)$ being the free Lagrangian density for the 'missing' (right-handed) neutrino.

8. *The P-mirror image of β-decay.*

As already remarked, CP is now reducible to P even when acting on *massive* chiral fields (χ_f and $\chi_{\bar{f}}$ being C-eigenfields). The usual 'CP-mirror' symmetry of the weak interaction should therefore *quite generally* be re-interpretable as a pure P-mirror one. But, how can P-symmetry be actually resurrected, say, in the β-decay $n \to p + e + \bar{\nu}$? According to the new scheme, both the quarks and the electron involved in the decay are described by fields like χ_f, thus being in "chiral" internal states like $|f^{ch}\rangle$. In other words, recalling (17), (19), (21) and (22), we may say that the weak interaction "sees" them as *pseudoscalar*-charge (rather than scalar-charge) particles! This is physically meaningful by virtue of the *anticommutivity* property between scalar charge Q and pseudoscalar charge Q^{ch}. The same argument applies to the antineutron β-decay, in which "chiral" states like $|\bar{f}^{ch}\rangle$ (associated with fields like $\chi_{\bar{f}}$) are involved. The result is that *now the P-mirror image of the actual* *process* $n \rightarrow p + e + \bar{\nu}$ *should just be identified with the actual antiprocess* $\bar{n} \rightarrow \bar{p} + \bar{e} + \nu$.

9. *Conclusion*.

A reformulation of the Dirac field theory based on the two opposite-mass charge-conjugate Dirac equations (1) automatically leads, as $m \rightarrow 0$, to a two-component neutrino-antineutrino scheme and further rigorously explains why no right-handed neutrino and left-handed antineutrino have ever been found. Such a reformulation includes the massless chiral fields (25) as a limiting case of the *massive* ones (12) and could equally well apply to a neutrino-antineutrino pair with a small mass[2]. The new scheme, unlike the usual, can naturally describe the appearance of 'P-violating' phenomenology (with no recourse to *ad hoc* external prescriptions): it is able to predict "chiral", besides "Dirac", massive fermions and antifermions, and to restore P-symmetry itself to the apparently 'P-violating' processes (without any change in the meaning of P!) by reducing the CP-mirror image of them to a pure P-mirror image. The theoretical reason for such a "dual" (either "Dirac" or "chiral") behaviour of massive spin-$\frac{1}{2}$ particles should be the *anticommutivity* property of the scalar and pseudoscalar charges carried by them.

FOOTNOTES

(1) - Even if admitting the existence of a positive-helicity neutrino and a negative-helicity antineutrino, it is unexplained why such a further neutrino-antineutrino pair is quite ignored by the weak interaction.

(2) - We add a remark concerning the conjecture of the neutrino magnetic moment, which plays an important role in many recent studies. In principle we can insert in the Dirac equation, even if $m=0$, a Pauli term proportional to

$$\sigma_{\mu\lambda} F^{\mu\lambda} \psi$$

($\sigma_{\mu\lambda} \propto [\gamma_\mu , \gamma_\lambda]$) $F^{\mu\lambda}$ being the electromagnetic field. The associated magnetic current is automatically conserved. However, in the *strict* $m=0$ case, the corresponding Hamiltonian density would be now of the type

$$\mathcal{H}_{\rm int} \propto F^{\mu\lambda} \bar{\psi} \sigma_{\mu\lambda} \psi = \frac{1}{2} F^{\mu\lambda} (\bar{\chi}_{\bar{\nu}} \sigma_{\mu\lambda} \chi_{\nu} + \bar{\chi}_{\nu} \sigma_{\mu\lambda} \chi_{\bar{\nu}})$$

thus inducing the transitions $\nu \rightleftarrows \bar{\nu}$.

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