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Scaling of Gain with Energy Spread and Energy in the PEP FEL

The Sag Harbor paper on the PEP FEL[1] discusses the scaling of various FEL parameters with energy spread σ_ϵ. I will repeat some of this material here and then examine the benefit of increasing the energy spread. How much energy spread can be achieved with damping wigglers is the next topic. Finally, I consider the dependence of gain and saturation length on beam energy and undulator field.

This note was originally written during a visit to SSRL in August 1991, and has been slightly modified since then.

1. Review of the Scaling Rules

The fractional rms energy spread σ_ϵ in a storage ring, determined by synchrotron-radiation losses in the bending magnets, is proportional to beam energy and so favors low energy for the FEL. With low-emittance optics in PEP (but without wigglers),[2]

$$\sigma_{\epsilon 0} = 6.6 \cdot 10^{-5} \cdot E\,[\mathrm{GeV}]\,, \tag{1}$$

giving an energy spread of 2×10^{-4} at $E = 3$ GeV.

Synchrotron radiation from a wiggler increases the beam's energy spread and changes its emittance.[3] Damping wigglers, which are placed in low or zero dispersion locations, increase energy spread but reduce emittance:

$$\frac{\sigma_{\epsilon w}^2}{\sigma_{\epsilon 0}^2} = \frac{1 + \frac{\sqrt{2}\pi^2 N_w R_0^2 K_w^3}{\lambda_w^2 \gamma^3}}{1 + \frac{\pi N_w R_0 K_w^2}{\lambda_w \gamma^2}} = \frac{1 + PQ^3}{1 + PQ^2} = R, \tag{2}$$

$$\frac{\epsilon_{xw}}{\epsilon_{x0}} = \frac{1}{1 + \frac{\pi N_w R_0 K_w^2}{\lambda_w \gamma^2}} = \frac{1}{1 + PQ^2}. \tag{3}$$

(A small term in Eq. (3) from dispersion in the wiggler has been dropped.) Here

$$K_w = \frac{eB_w\lambda_w}{2\pi m_e c} = 0.9337 B_w\,[\mathrm{T}]\,\lambda_w\,[\mathrm{cm}] \tag{4}$$

is the dimensionless wiggler parameter, expressed in terms of the wiggler period λ_w and the peak wiggler field B_w on axis; $N_w = L_w/\lambda_w$ is the number of periods in a wiggler of length L_w; γ is the ratio of the total energy to rest energy of an electron; and R_0, the beam's radius of curvature in the ring's dipoles, is 165.5 m for PEP. For later use, we have defined parameters P and Q:

$$P = \frac{L_w}{2\pi R_0} = \frac{L_w\,[\mathrm{m}]}{1040\ \mathrm{m}}. \tag{5}$$

$$Q = \frac{\sqrt{2}\pi R_0 K_w}{\gamma \lambda_w} = 35 \frac{B_w\,[\mathrm{T}]}{E\,[\mathrm{GeV}]}. \tag{6}$$

Note that the wiggler period does not enter into the scaling, and so long periods with high fields can be used. R is the square of the increase in energy spread.

For sufficiently long damping wigglers, the change in energy spread saturates at

$$\frac{\sigma_{\epsilon w}^2}{\sigma_{\epsilon 0}^2} = \frac{\sqrt{2}\pi R_0 K_w}{\lambda_w \gamma} = Q, \tag{7}$$

but the emittance will continue to decrease with N_w (until the effect saturates due to the neglected dispersive term in Eq. (3).)

If several damping wigglers are distributed around the ring, then the effect is cumulative over their total length. However, an FEL wiggler in a bypass, with the beam switched in intermittently, will not contribute to the beam's equilibrium energy spread and emittance.

A careful choice of a damping wiggler offers two benefits for the FEL. Low values of emittance are needed to lase at short wavelengths. At low energies, the ring's peak current is seriously limited; increasing the energy spread allows higher current.

The limit on the peak current $\hat{I}$ is imposed by the longitudinal microwave instability. For an estimate, we rely on the extrapolation of bunch-length measurements made on the SPEAR ring and scaled to fit PEP data.[4–5] These scaling rules are modeled by the ZAP code,[6] which incorporates the following formula:[7]

$$\hat{I} \leq \frac{2\pi\alpha\beta E\sigma_\epsilon^2}{|Z_\parallel/n|e}. \tag{8}$$

Here, α is the ring's momentum-compaction factor, the ratio of the fractional increase in the orbit path length to the fractional change in momentum; $\beta = v/c$ for an electron of energy E; e is the unit charge; $n = \omega/\omega_0$ is the mode number; and ω_0 is the revolution frequency in the ring. This can often be simplified using Eq. (1) (but not including the effect of damping wigglers (2)):

$$\hat{I}_{\mathrm{max}} \propto E\sigma_\epsilon^2 \propto E^3, \tag{9}$$

The frequency of interest ω is not critical in most cases, since the impedance is inductive, leaving $|Z_\parallel/n|$ constant. Measurements made at 4.5 GeV,[8] with rms bunch lengths σ_s of several centimeters, agree with these extrapolations. However, for frequencies above $\omega_c = c/b$, where b is the beam-pipe radius, $|Z_\parallel/n|$ decreases. SPEAR scaling gives $|Z_\parallel/n| \propto (\omega/\omega_c)^{-1.68}$, and ZAP uses $\omega = c/\sigma_s$. For extremely short bunches, ZAP does not allow the impedance to decrease below the "free-space" longitudinal impedance of the bending magnets, screened by the beam-pipe wall: $|Z_\parallel/n| = 300(b/R_{\mathrm{av}})$, where $R_{\mathrm{av}} = L_R/(2\pi)$ is the average radius of a ring of circumference L_R. In our case, with its very short bunches, ZAP uses this free-space limit; for PEP's low-emittance mode and an energy of 3 GeV, ZAP gives a maximum peak current of 17.6 A.

In the exponential gain régime of an FEL with undulator parameters K_u and λ_u, the power grows with an e-folding length

$$L_G = \frac{\lambda_u}{4\pi\sqrt{3}\rho}. \tag{10}$$

The dimensionless gain parameter ρ is given by:

$$\frac{1}{L_u^3} = \frac{\rho^3}{\lambda_u^3} = \frac{r_e}{64\pi^2 ec} \frac{{K_u}^2}{\lambda_u \sigma_x \sigma_y} \frac{\hat{I}}{\gamma^3} (J_0 - J_1)^2 . \tag{11}$$

Here L_u is the necessary undulator length for saturation:

$$L_u = \lambda_u/\rho = 4\pi\sqrt{3} L_G = 21.8 L_G. \tag{12}$$

J_0 and J_1 are Bessel functions with the argument

$$\xi = \frac{{K_u}^2/4}{1 + {K_u}^2/2}. \tag{13}$$

Since the argument takes on values from 0 to 0.5, the squared difference of Bessel functions ranges from 1 to 0.485.

When the energy spread σ_ϵ approaches ρ, the gain is reduced:[9]

$$\frac{\rho_{\text{eff}}}{\rho} = \frac{\exp\left[-0.136\,(\sigma_\epsilon/\rho)^2\right]}{1 + 0.64\,(\sigma_\epsilon/\rho)^2}. \tag{14}$$

The increase of σ_ϵ with energy (1) and damping wigglers (2) argue for a low energy to avoid this gain reduction. This topic will be expored in Section 4.

The beam sizes σ_x and σ_y are limited both by a restriction on focusing,

$$\beta_y \geq \beta_x \geq L_G, \tag{15}$$

and by diffraction, which is expressed in terms of the Rayleigh length of the x rays:

$$z_R = \pi w_0^2/\lambda = 4\pi\sigma_y^2/\lambda \geq L_G. \tag{16}$$

(The Rayleigh length is defined in terms of the 1/e radius w_0 of the electric field. Here we equate σ_y, the smaller of the two rms electron-beam radii, not to w_0 but to the rms radius of the x-ray power.) This diffractive limit proves to be quite severe and prevents our focusing to a σ_y smaller than σ_x. Hence we set

$$\sigma_x = \sigma_y. \tag{17}$$

2. Optimization of the Energy Spread

We see from Eq. (9) that a higher energy spread will increase the peak current, and hence ρ, but will lead to a greater gain reduction from Eq. (14). Where is the optimum? For a given energy, we have

$$\rho^3 \propto \hat{I} \propto {\sigma_\epsilon}^2. \tag{18}$$

Use this scaling to define parameters S, T, and U that are independent of σ_ϵ and ρ:

$$\frac{{\sigma_\epsilon}^2}{\rho^2} = S{\sigma_\epsilon}^{2/3}, \tag{19}$$

$$\rho = TS{\sigma_\epsilon}^{2/3}, \tag{20}$$

$$\hat{I} = U{\sigma_\epsilon}^2. \tag{21}$$

Then for ρ_{eff} we have:

$$\rho_{\text{eff}} = T\left(S{\sigma_\epsilon}^{2/3}\right)\frac{\exp\left[-0.136\left(S{\sigma_\epsilon}^{2/3}\right)\right]}{1+0.64\left(S{\sigma_\epsilon}^{2/3}\right)}. \tag{22}$$

Eq. (22) has a peak, but to find it we need values for S, T, and U for a particular choice of beam and undulator parameters. Start with the values on line 2 of the Sag Harbor[1] or PAC91[10] table (the case with a 3.5-GeV energy and a hybrid magnet with a 1-cm gap):

$$\sigma_\epsilon = 7.2\times10^{-4}, \tag{23}$$

$$\rho = 9.1\times10^{-4}, \tag{24}$$

$$\hat{I} = 219\text{A}. \tag{25}$$

These give

$$S = 78, \tag{26}$$

$$T = 1.45\times10^{-3}, \tag{27}$$

$$U = 4.22\times10^{8}\ \text{A}. \tag{28}$$

The consequences of these scaling rules are shown in Table 1. Neglect for the moment the unrealistic values near the bottom of the table. Because of the strong scaling of peak current, the peak of ρ_{eff} is 1.0×10^{-3}, for a σ_ϵ of 6×10^{-3}. In comparison, the Sag Harbor result, which is also repeated in this table, uses the more cautious energy spread of 7.2×10^{-4} to get a ρ_{eff} of 6.0×10^{-4}.

Various effects will limit the range of these scaling rules. A damping wiggler is limited by the saturation value (7) in how much energy spread it can introduce; this limit will be considered in the next section. A higher beam energy raises the energy spread achievable with a damping wiggler and raises the peak current; this is discussed in Section 4. PEP's rf acceptance will restrict σ_ϵ to perhaps 2×10^{-3}, but, even at this value, a large increase in gain can be achieved. I am not certain which effect will limit $\hat{I}$, but it could perhaps go to 1 or 2 kA. (The limit on average current is considered in the next section.) Any increase obtained in ρ_{eff} corresponds to a proportionate decrease in L_G and L_w. In addition, the shorter gain length allows for tighter focusing (a shorter β_x and a shorter Rayleigh length) and the possibility of a further increase in gain.

σ_ϵ (10^{-4})	$\hat{I}$ (A)	ρ (10^{-4})	ρ_{eff} (10^{-4})
2	17	3.9	3.2
4	68	6.1	4.6
6	152	8.1	5.5
7.2	219	9.1	6.0
8	270	9.8	6.2
9.2	361	10.7	6.6
10	422	11.3	6.8
12	608	12.8	7.3
17	1220	16.1	8.1
20	1690	18.0	8.5
30	3800	23.6	9.3
40	6760	28.5	9.7
50	10600	33.1	9.9
60	15200	37.4	10.0
80	27000	45.3	9.9
100	42200	52.6	9.7
200	169000	83.5	8.2

Table 1: Effect of energy spread on gain, using Eq. (22).

3. Optimization of the Damping Wiggler

Table 1 suggests that the highest possible energy spread is desirable. A damping wiggler is limited by Eq. (7) in the increase in $\sigma_{\epsilon w}$ it can provide. Since this limit Q depends only on the wiggler field (for the 3.5-GeV energy considered here), we choose the largest field conveniently achievable. For a room-temperature magnet:

$$B_w = 2\ \text{T}, \tag{29}$$

$$Q = 20.0. \tag{30}$$

Next, we choose a large value of the energy-spread increase factor R (limited by Q), and solve Eq. (2) for P (the wiggler length):

$$P = \frac{R-1}{Q^2(Q-R)}. \tag{31}$$

Since $P > 0$, we see that $Q > R > 1$, and R cannot get too close to Q or the wiggler length will diverge. A reasonable choice might be:

$$R = 16, \tag{32}$$

$$P = 9.38 \times 10^{-3}, \tag{33}$$

$$\sigma_{\epsilon w} = 9.2 \times 10^{-4}, \tag{34}$$

$$L_w = 9.7\ \text{m}. \tag{35}$$

The wiggler period is not specified, and can be chosen for convenience in achieving the 2-T field. The current and ρ values corresponding to this choice are listed in Table 1. We see that this choice of damping wiggler raises ρ_{eff} from 6.0×10^{-4} to 6.6×10^{-4}.

A higher damping-wiggler field would give a greater energy spread and a higher peak current. For a superconducting wiggler:

$$B_w = 6 \text{ T}, \tag{36}$$

$$Q = 60.0. \tag{37}$$

Again, we choose a value of R near Q and solve for P:

$$R = 56, \tag{38}$$

$$P = 3.82 \times 10^{-3}, \tag{39}$$

$$\sigma_{\epsilon w} = 1.7 \times 10^{-3}, \tag{40}$$

$$L_w = 4.0 \text{ m}. \tag{41}$$

Table 1 shows that ρ_{eff} for this choice is 8.1×10^{-4}.

In reality, this energy spread may be a bit too large for PEP's rf acceptance. Also, the single-bunch current limit may be exceeded in this case. To check this, we need to find the average current. The bunch length in the Sag Harbor table is 4.2 mm, and this length is proportional to energy spread. We get:

$$\sigma_z = 9.9 \text{ mm}, \tag{42}$$

$$I_{\text{av}} = \sqrt{2\pi}\hat{I}\sigma_z/L_R = 13.8 \text{ mA}. \tag{43}$$

(L_R is the ring circumference.) Heinz-Dieter Nuhn (SSRL) was not certain, but he suggested that the limit is 10 mA.

4. Optimization of the Energy

Combine Eqs. (1) and (7) to find the dependence of energy spread (with a damping wiggler) on γ:

$$\sigma_{\epsilon w}^2 \propto \gamma^2 Q \propto \gamma. \tag{44}$$

(B_w is held fixed at 2 T, following Eq. (29).) Using Eq. (9), the peak current's dependence then is:

$$\hat{I} \propto \gamma^2. \tag{45}$$

We want to maintain a fixed wavelength,

$$\lambda = \lambda_u \frac{1 + K_u^2/2}{2\gamma^2}, \tag{46}$$

while maximizing the gain or minimizing the undulator length, both using Eq. (11). The beam size scaling, which comes either from Eq. (15) or Eq. (16), is the same for either limit (since a damping wiggler can provide a sufficiently low emittance to allow β_x to scale freely):

$$\sigma_x^2 = \sigma_y^2 \propto L_G \propto \lambda_u/\rho. \tag{47}$$

(a) High-Field Undulators

The premise in designing the high-field FELs was minimizing the undulator saturation length L_u in Eq. (11). Substitute for $\hat{I}$ from Eq. (45), for λ_u from Eq. (46) (with constant wavelength), and for $\sigma_x \sigma_y$ from Eq. (47):

$$\frac{1}{L_u^2} = \frac{\rho^2}{\lambda_u^2} \propto \frac{K_u^2 \left(1 + K_u^2/2\right)}{\gamma^3}. \tag{48}$$

We need a large K_u and a small γ.

First we regard the magnetic field as an independent parameter, because, for example, a variety of undulator designs are being compared, or because any one design need not be pushed to its high-field limit. To maintain synchronism, λ_u must decrease even though K_u is increasing. For large K_u and constant wavelength, Eq. (46) gives:

$$\lambda_u \propto \left(\frac{\gamma}{B_u}\right)^{2/3}, \tag{49}$$

and the saturation length scaling becomes:

$$L_u \propto \gamma^{1/6} / B_u^{2/3}. \tag{50}$$

As a result, we choose a low energy and the largest practical field, limited by the properties of materials such as samarium cobalt. Then, as λ_u gets smaller, the gap must be reduced to maintain the field on axis, until the limit of an unacceptable gap is reached.

Now consider operation at the high-field limit of any one design, with the minimum gap acceptable for injection and beam lifetime (especially if the undulator is in the main ring rather than a bypass). The magnetic field now is linked to the undulator period and, through Eq. (46), to the beam energy. For example, the field of a hybrid, planar wiggler using neodymium-iron permanent magnets is given by

$$B_w\,[\mathrm{T}] = 3.44 \exp\left[-\left(g/\lambda_w\right)\left(5.08 - 1.54 g/\lambda_w\right)\right]. \tag{51}$$

Such expressions do not lead to any simple scaling, but it may be seen that there is an optimum energy and period giving the shortest saturation length. This value must be found iteratively: at each of a range of energies, find λ_u and the corresponding maximum B_u that give the desired wavelength, and calculate the saturation length.

(b) Cusp-Field Undulators

Roman Tatchyn's premise in this case[10] is maximizing ρ while holding λ fixed. The overall undulator length is not as important here, because the cusp-field undulator is simply and sparsely constructed, and can have excellent field accuracy over long lengths. Making the same substitutions before, we get:

$$\rho^2 \propto \left(\frac{K_u^2}{1 + K_u^2/2}\right) \gamma. \tag{52}$$

Again, we need a large K_u, although this time the improvement saturates for K_u values much above 3. In contrast to Eq. (50), the energy should be large, while λ_u can increase to maintain synchronism following Eq. (49). The demand on field strength is not as great. The limit to this procedure arrives when the undulator length fills PEP's straight section ($\approx$90 m, to allow for matching the short beta function of the FEL).

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