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ON THE ALGEBRAIC STRUCTURE OF COVARIANT ANOMALIES AND COVARIANT SCHWINGER TERMS

GERALD KELNHOFER*

Institut für Theoretische Physik, Universität Wien
A-1090 Wien, Boltzmanngasse 5
Austria

ABSTRACT. A cohomological characterization of covariant anomalies and covariant Schwinger terms in an anomalous Yang-Mills theory is formulated and will be geometrically interpreted. The BRS and anti-BRS transformations are defined as purely differential geometric objects. Finally the covariant descent equations are formulated within this context.

1. Introduction

The phenomenon of anomalies in a quantum field theory has been extensively studied in the last decade. Anomalies in the divergence of the fermionic current and Schwinger terms in the equal time commutator of the gauge group generators were found to be different manifestations of these anomalies [1,2]. It has been recognized by Bardeen and Zumino [3] that anomalies occur in two different forms, known as consistent and covariant anomalies. Besides perturbative calculations, the former have been investigated by algebraic [4-6] and topological means [7]. Consistent anomalies as well as consistent Schwinger terms are characterized by an algebraic condition, which is of cohomological nature.

On the other hand, covariant anomalies and covariant Schwinger terms were studied under various viewpoints in the last years [8-10]. However, there have been only few attempts to investigate the general mathematical structure underlying the phenomenon of covariant anomalies. In [11] it was shown how the covariant anomaly can be understood in terms of presymplectic geometry But it is not clear how to interpret also the covariant Schwinger term in this context. Furthermore, it was realized that both, covariant anomalies and covariant Schwinger terms can be derived by enlarging the usual BRS algebra to include an antighost and an anti-BRS operator [12,13]. Homotopy operators have to be introduced which lead to descent equations for the covariant anomaly and the covariant Schwinger term.

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The aim of this paper is to formulate a cohomological characterization of the notion of covariance for the integrated forms of the covariant anomaly and the covariant Schwinger term. Thereby we want to elucidate the algebraic and geometric role played by the so called anti-BRS operator.

In sect. 2 we shall use the results obtained in [10] to study the general structure of covariant anomalies and covariant Schwinger terms. In sect. 3 we shall show how BRS and anti-BRS relations (without auxiliary fields) can be interpreted as differential geometric objects. Finally the covariant descent equations of [12,13] are rederived.

2. The notion of covariance for anomalies and Schwinger terms

The present section is devoted to explain the notion of covariance in cohomological terms. To make the presentation self contained, we briefly recall the main formulas of [10].

Let $\mathcal{A}$ be the space of connections of a trivial G-bundle over space-time M and consider the principal $\mathcal{G}$-bundle $\mathcal{A}(\mathcal{M}, \pi_{\mathcal{A}}, \mathcal{G})$ of all gauge potentials over the gauge orbit space $\mathcal{M}$, where $\mathcal{G}$ is the gauge group with Lie algebra $Lie\mathcal{G}$.

The consistent anomaly arises from an anomalous continuity equation

$$i_{Y_\xi} < J > \ (A) = Anom(A,\xi) = \int_M dx \ Anom^a(x)\xi^a(x), \qquad A \in \mathcal{A}, \ \xi \in Lie\mathcal{G} \tag{2.1}$$

for the vacuum expectation value of the consistent fermionic current J in the gauge field background. $< J >$ is considered as a closed one form over $\mathcal{A}$ and is connected with the generating functional Z by $< J >= iZ^{-1} \cdot d_{\mathcal{A}} Z$. The fundamental vector field generated by an element ξ of the gauge algebra $Lie\mathcal{G}$ is denoted by Y_ξ, $i_.$ is the substitution operator, and $d_{\mathcal{A}}$ is the exterior derivative on $\mathcal{A}$. We note that in local coordinates the fundamental vector field reads

$$Y_\xi = \int dx (D_A)^{ab}_\nu \xi^b(x) \frac{\delta}{\delta A^a_\nu(x)}, \tag{2.2}$$

where D_A denotes the covariant derivative and the indices a, b refer to a basis in the Lie algebra of G. So (2.1) can be written in the more familiar form

$$(D_A)^{ab}_\nu < J >^\nu_b (x) = -Anom^a(x). \tag{2.3}$$

Finally the Ward operator can be identified with the Lie derivative L_{Y_ξ} with respect to Y_ξ. So (2.1) leads to the Wess-Zumino (WZ) consistency condition for the consistent anomaly

$$L_{Y_\xi} Anom(.,\eta) - L_{Y_\eta} Anom(.,\xi) - Anom(.,[\xi,\eta]) = 0. \tag{2.4}$$

The covariant anomaly $\widetilde{Anom}$ arises from an analogous equation for the covariant current $< \tilde{J} >$,

$$i_{Y_\xi} < \tilde{J} > \ (A) = \widetilde{Anom}(A,\xi) = \int_M dx \ \widetilde{Anom}^a(x)\xi^a(x). \tag{2.5}$$

However, both forms of the anomaly are related with each other by the Bardeen-Zumino polynomial Λ [3], which will here be considered as an one form on $\mathcal{A}$. This relation is given by

$$\widetilde{Anom}(A,\xi) = Anom(A,\xi) + i_{Y_\xi}\Lambda, \tag{2.6}$$

where the condition $L_{Y_\xi}\Lambda = -d_{\mathcal{A}}Anom(\xi)$ is imposed on Λ. In [10] we have interpreted Λ as a connection one form on a certain line bundle over $\mathcal{A}$ with curvature $\mathcal{F} = d_{\mathcal{A}}\Lambda$.

Furthermore we have calculated the covariant Schwinger term in the commutator of the covariant Gauss law operator in any even dimension of space-time, where it reads

$$\tilde{S}_{ab}(x,y) = -i\frac{\delta}{\delta A_0^b(y)}\widetilde{Anom}_a(x). \tag{2.7}$$

If we define the vector field $X_\xi = \int dx\xi^a(x)\frac{\delta}{\delta A_0^a(x)} \in \mathfrak{X}(\mathcal{A})$ and use the relation $i_{Y_\xi}\mathcal{F} = -d_{\mathcal{A}}\widetilde{Anom}(\xi)$, the covariant Schwinger term can now be written in a manifestly antisymmetric form

$$\tilde{S}(\xi_1,\xi_2) = \frac{i}{2}(i_{X_{\xi_2}}i_{Y_{\xi_1}}\mathcal{F} - i_{X_{\xi_1}}i_{Y_{\xi_2}}\mathcal{F}). \tag{2.8}$$

Consistent anomalies and Schwinger terms can be studied within the space $C^q(Lie\mathcal{G},\mathcal{C}(\mathcal{A}))$ of alternate q linear maps on $Lie\mathcal{G}$ with values in $\mathcal{C}(\mathcal{A})$, the space of functions on $\mathcal{A}$. (To be precise, one has to restrict to the subset of local functionals [5] on $\mathcal{A}$.) In order to formulate an algebraic condition for covariant terms, we shall view elements of $C^q(Lie\mathcal{G},\mathcal{C}(\mathcal{A}))$ as maps $\mathcal{A} \longrightarrow \Lambda^q(Lie\mathcal{G})^*$ and denote the space of all such maps by $\mathcal{C}(\mathcal{A},\Lambda^q(Lie\mathcal{G})^*)$. Here $\Lambda^q(Lie\mathcal{G})^*$ denotes the q th exterior tensor product of the dual of $Lie\mathcal{G}$.

There is a free right action $\mathcal{R}$ of $\mathcal{G}$ on the product $\mathcal{A}\times\Lambda^q(Lie\mathcal{G})^*$, given by

$$\mathcal{R}_h(A,\mu) := (A^h, ad^*(h^{-1})\mu), \qquad \mu\in\Lambda^q(Lie\mathcal{G})^*,\ h\in\mathcal{G}, \tag{2.9}$$

with the coadjoint action ad^* of $\mathcal{G}$ on $\Lambda^q(Lie\mathcal{G})^*$

$$(ad^*(h)\mu)(\xi_1,\ldots,\xi_q) := \mu(ad(h^{-1})\xi_1,\ldots,ad(h^{-1})\xi_q), \qquad h\in\mathcal{G},\ \xi_1,\ldots,\xi_q\in Lie\mathcal{G}. \tag{2.10}$$

The corresponding derived representation, denoted by $ad^{*\prime}$, is given by

$$(ad^{*\prime}(\eta)\mu)(\xi_1,\ldots,\xi_q) = -\sum_{i=1}^{q}\mu(\xi_1,\ldots,[\eta,\xi_i],\ldots,\xi_q) \qquad \eta\in Lie\mathcal{G}. \tag{2.11}$$

An element $\phi\in\mathcal{C}(\mathcal{A},\Lambda^q(Lie\mathcal{G})^*)$ is called equivariant if

$$\phi(A^h) = ad^*(h^{-1})\phi(A), \qquad h\in\mathcal{G}, \tag{2.12}$$

and the space of equivariant maps is denoted by $\mathcal{C}_{eq}(\mathcal{A},\Lambda^q(Lie\mathcal{G})^*)$.

Using (2.4), (2.6) and (2.11) it can be easily verified that

$$(L_{Y_\xi}\widetilde{Anom})(\eta) = \widetilde{Anom}([\xi,\eta]) = -(ad^{*\prime}(\xi)\widetilde{Anom})(\eta). \tag{2.13}$$

On the other hand, we find for the covariant Schwinger term (2.8), using the gauge invariance of $\mathcal{F}$, namely $L_{Y_\xi}\mathcal{F} = 0$, and the commutator

$$[X_\xi, Y_\eta] = X_{[\xi,\eta]}, \qquad \xi, \eta \in Lie\mathcal{G} \tag{2.14}$$

that

$$(L_{Y_\eta}\tilde{S})(\xi_1, \xi_2) = \tilde{S}([\eta, \xi_1], \xi_2) + \tilde{S}(\xi_1, [\eta, \xi_2]) = -(ad^{*\prime}(\eta)\tilde{S})(\xi_1, \xi_2). \tag{2.15}$$

In summary we have proven

Proposition 1.

(1) $\widetilde{Anom} \in \mathcal{C}_{\rm eq}(\mathcal{A}, (Lie\mathcal{G})^*)$
(2) $\tilde{S} \in \mathcal{C}_{\rm eq}(\mathcal{A}, \wedge^2(Lie\mathcal{G})^*)$

Let us construct the exterior coadjoint bundle $\mathcal{E}_q^*$ over the gauge orbit space by the commutative diagram

$$\begin{array}{ccc} E_q^* := \mathcal{A} \times \wedge^q(Lie\mathcal{G})^* & \longrightarrow & \mathcal{E}_q^* := \mathcal{A} \times_{\mathcal{G}} \wedge^q(Lie\mathcal{G})^* \\ \downarrow & & \downarrow \\ \mathcal{A} & \xrightarrow{\pi_{\mathcal{A}}} & \mathcal{M} \end{array} \tag{2.16}$$

Proposition 1 tells us that $\widetilde{Anom}$ and $\tilde{S}$ descend to sections of the two vector bundles $\mathcal{E}_1^*$ and $\mathcal{E}_2^*$ respectively. The space of sections $\Gamma(E_q^*)$ of the trivial bundle E_q^* becomes a $\mathcal{G}$-module with respect to the left action

$$(h \cdot \phi)(A) := ad^*(h)\phi(A^h), \qquad h \in \mathcal{G},\ \phi \in \mathcal{C}(\mathcal{A}, \wedge^q(Lie\mathcal{G})^*), \tag{2.17}$$

which in its infinitesimal form reads

$$(\theta(\xi)\phi)(A) = (L_{Y_\xi}\phi)(A) + ad^{*\prime}(\xi)\phi(A), \qquad \xi \in Lie\mathcal{G}. \tag{2.18}$$

Now we consider the double complex $K^{p,q} := K^p(Lie\mathcal{G}, \Gamma(E_q^*))$ of n cochains with values in the space of sections of E_q^* with the two coboundary operators

$$\delta\colon K^p(Lie\mathcal{G}, \Gamma(E_q^*)) \to K^p(Lie\mathcal{G}, \Gamma(E_{q+1}^*))$$

$$(\delta\Phi)(\xi_1, \dots, \xi_p)(\eta_1, \dots, \eta_{q+1}) := \sum_{i=1}^{q+1}(-1)^{i+1}(L_{Y_{\eta_i}}\Phi(\xi_1, \dots, \xi_p))(\eta_1, \dots, \hat{\eta}_i, \dots, \eta_{q+1})$$
$$+ \sum_{1 \leqq i < j \leqq q+1} (-1)^{i+j}\Phi(\xi_1, \dots, \xi_p)([\eta_i, \eta_j], \eta_1, \dots, \hat{\eta}_i, \dots, \hat{\eta}_j, \dots, \eta_{q+1})$$

$$\delta_\theta\colon K^p(Lie\mathcal{G}, \Gamma(E_q^*)) \to K^{p+1}(Lie\mathcal{G}, \Gamma(E_q^*))$$

$$(\delta_\theta\Phi)(\xi_1, \dots, \xi_{p+1}) := \sum_{i=1}^{p+1}(-1)^{i+1}\theta(\xi_i)(\Phi(\xi_1, \dots, \hat{\xi}_i, \dots, \xi_{p+1}))$$
$$+ \sum_{1 \leqq i < j \leqq p+1} (-1)^{i+j}\Phi([\xi_i, \xi_j], \xi_1, \dots, \hat{\xi}_i, \dots, \hat{\xi}_j, \dots, \xi_{p+1}). \tag{2.19}$$

Here a caret denotes omission of the corresponding element. Note that the operator δ is just the usual coboundary operator for the consistent case [4,6]. The cohomology groups with respect to δ_θ and δ are denoted by $H_{\delta_\theta}(K^{p,q})$ respectively $H_\delta(K^{p,q})$. As it is well known [6], the consistent anomaly lies in $H_\delta(K^{0,1})$ and finally the consistent Schwinger term gives a class in $H_\delta(K^{0,2})$.

Since $H_{\delta_\theta}(K^{0,q}) \cong \Gamma(\mathcal{E}_q^*)$, we can formulate the covariance condition for anomalies and Schwinger terms cohomologically by the following.

Proposition 2.

(1) $\widetilde{Anom} \in H_{\delta_\theta}(K^{0,1})$
(2) $\tilde{S} \in H_{\delta_\theta}(K^{0,2})$

Before closing this section we want to mention an interesting fact about equations (2.1) and (2.5). There exists an exact sequence

$$0 \longrightarrow \mathcal{A} \times Lie\mathcal{G} \xrightarrow{Y} T\mathcal{A} \xrightarrow{T\pi_\mathcal{A}} \pi_\mathcal{A}^* T\mathcal{M} \longrightarrow 0 \tag{2.20}$$

of vector bundles over $\mathcal{A}$. Considering the dual of (2.20) and taking the n th exterior tensor product, one obtains the following exact sequence

$$0 \longrightarrow \Lambda^n \pi_\mathcal{A}^* T^* \mathcal{M} \longrightarrow \Lambda^n T^* \mathcal{A} \longrightarrow E_n^* \longrightarrow 0 \tag{2.21}$$

of vector bundles over $\mathcal{A}$. Let us denote the complex of $\mathcal{G}$-invariant differential forms on $\mathcal{A}$ by $\Omega_\mathcal{G}(\mathcal{A})$. Since (2.21) is also an exact sequence of $\mathcal{G}$ vector bundles, one can easily prove the exactness of the following two sequences

$$\begin{gathered} 0 \longrightarrow \Gamma(\Lambda^n \pi_\mathcal{A}^* T^* \mathcal{M}) \longrightarrow \Omega^n(\mathcal{A}) \xrightarrow{\chi} \Gamma(E_n^*) \longrightarrow 0 \\ 0 \longrightarrow \Omega^n(\mathcal{M}) \xrightarrow{\pi_\mathcal{A}^*} \Omega_\mathcal{G}^n(\mathcal{A}) \xrightarrow{\chi_\mathcal{G}} \Gamma(\mathcal{E}_n^*) \longrightarrow 0, \end{gathered} \tag{2.22}$$

where $\chi(\alpha)(\xi_1, \dots, \xi_n) := i_{Y_{\xi_1}} \cdots i_{Y_{\xi_n}} \alpha$, with $\alpha \in \Omega^n(\mathcal{A})$ and $\xi_i \in Lie\mathcal{G}$. Finally $\chi_\mathcal{G}$ is the restriction of χ to $\Omega_\mathcal{G}(\mathcal{A})$.

For $n = 1$, these sequences clearly exhibit the geometrical relationship between currents and anomalies, which underlies the anomalous continuity equations (2.1) and (2.5).

3. The BRS, anti-BRS complex

In this section we shall generalize the mathematical treatment of [4] in order to include also the anti-BRS relations.

Let $\pi: P \to M$ be a principal bundle with structure group G, whose Lie algebra will be denoted by $\mathfrak{g}$ and principal right action R. The *gauge group* $\mathcal{G}$ is defined to be the group of all fibre preserving automorphisms $Aut_0(P)$ of P. Equivalently, the gauge group may be identified with the group $C^\infty_{eq}(P,G)$ of all smooth maps $\chi: P \to G$, which are equivariant with respect to the adjoint action ad of G onto itself.

Let us consider the $\mathfrak{g}$-valued de Rham complex of P, denoted by $\Omega(P,\mathfrak{g}) = \Omega(P)\otimes\mathfrak{g}$, which admits natural left actions r respectively ρ by G respectively $\mathcal{G}$,

$$\begin{aligned} r(g)\alpha &:= (R_g^* \otimes ad(g))\alpha \qquad && g\in G \\ \rho(F)\alpha &:= (F^{-1})^*\alpha && F\in Aut_0(P),\ \alpha\in\Omega(P,\mathfrak{g}) \end{aligned} \tag{3.1}$$

Note that $(\Omega(P,\mathfrak{g}), d = d\otimes 1_{\mathfrak{g}})$ becomes a graded differential algebra with the bracket $[,]$

$$[\alpha_1,\alpha_2](X_1,\dots,X_{p+q}) = \frac{1}{p!q!}\sum_{\sigma\in\Sigma_{p+q}}(-1)^\sigma[\alpha_1(X_{\sigma(1)},\dots,X_{\sigma(p)}),\alpha_2(X_{\sigma(p+1)},\dots,X_{\sigma(p+q)})], \tag{3.2}$$

where $\alpha_1\in\Omega^p(P,\mathfrak{g})$, $\alpha_2\in\Omega^q(P,\mathfrak{g})$, $X_i\in\mathfrak{X}(P)$ and Σ_{p+q} denotes the set of permutations of the first $p+q$ numbers.

The *gauge algebra* $Lie\mathcal{G}$ can be identified with $C^\infty_{eq}(P,\mathfrak{g})$, the Lie algebra of smooth ad' equivariant maps $\xi: P\to\mathfrak{g}$. Furthermore $Lie\mathcal{G}$ can be identified with the space of all G-invariant, vertical smooth vector fields $\mathfrak{X}^G_{ver}(P)$ on P.

The derived representations of $\mathfrak{g}$ respectively $Lie\mathcal{G}$ on $\Omega(P,\mathfrak{g})$ are given by (we shall denote them by the same symbols)

$$\begin{aligned} r(u)\alpha &:= L_{Z_u}\alpha + ad'(u)\alpha \qquad && u\in\mathfrak{g} \\ \rho(\xi)\alpha &:= \frac{d}{dt}|_{t=0}\rho(exp\ t\xi)\alpha && \xi\in Lie\mathcal{G}, \end{aligned} \tag{3.3}$$

where L_{Z_u} denotes the Lie derivative with respect to the vertical vector field $Z_u\in\mathfrak{X}(P)$. The BRS transformations were studied within the complex $C^q(Lie\mathcal{G},\Omega(P,\mathfrak{g})) = \Omega(P,\mathfrak{g})\otimes\Lambda^q(Lie\mathcal{G})^*$ [4,5]. Elements of $C^q(Lie\mathcal{G},\Omega(P,\mathfrak{g}))$ are considered as $\Omega(P,\mathfrak{g})$ valued, alternate q linear maps on $Lie\mathcal{G}$.

We begin our generalization by considering the triple graded space

$$C^{p,q,n} := C^p(Lie\mathcal{G}, C^q(Lie\mathcal{G},\Omega^n(P,\mathfrak{g}))) = (\Omega^n(P,\mathfrak{g})\otimes\Lambda^q(Lie\mathcal{G})^*)\otimes\Lambda^p(Lie\mathcal{G})^* \tag{3.4}$$

of all $C^q(Lie\mathcal{G},\Omega^n(P,\mathfrak{g}))$ valued, alternate p linear maps on $Lie\mathcal{G}$.

We define a representation Θ_ρ of $Lie\mathcal{G}$ on $C^q(Lie\mathcal{G},\Omega^n(P,\mathfrak{g}))$ by

$$(\Theta_\rho(\xi)\phi)(\eta_1,\dots,\eta_q) := \rho(\xi)(\phi(\eta_1,\dots,\eta_q)) + (ad^{*\prime}(\xi)\phi)(\eta_1,\dots,\eta_q). \tag{3.5}$$

$C^{*,*,*}$ can be furnished with the structure of a triple complex by considering the three coboundary operators

$$\delta_\rho : C^{p,q,n} \to C^{p,q+1,n}$$

$$(\delta_\rho \Phi)(\xi_1,\dots,\xi_p)(\eta_1,\dots,\eta_{q+1}) = \sum_{i=1}^{q+1}(-1)^{i+1}\rho(\eta_i)(\Phi(\xi_1,\dots,\xi_p))(\eta_1,\dots,\hat{\eta}_i,\dots,\eta_{q+1})$$
$$+ \sum_{1\leqq i<j\leqq q+1} \Phi(\xi_1,\dots,\xi_p)([\eta_i,\eta_j],\eta_1,\dots,\hat{\eta}_i,\dots,\hat{\eta}_j,\dots,\eta_{q+1})$$

$$\delta_{\Theta_\rho} : C^{p,q,n} \to C^{p+1,q,n}$$

$$(\delta_{\Theta_\rho}\Phi)(\xi_1,\dots,\xi_{p+1})(\eta_1,\dots,\eta_q) := \sum_{i=1}^{p+1}(-1)^{i+1}(\Theta_\rho(\xi_i)\Phi(\xi_1,\dots,\hat{\xi}_i,\dots,\xi_{p+1}))(\eta_1,\dots,\eta_q)$$
$$+ \sum_{1\leqq i<j\leqq n+1} \Phi([\xi_i,\xi_j],\xi_1,\dots,\hat{\xi}_i,\dots,\hat{\xi}_j,\dots,\xi_{p+1})(\eta_1,\dots,\eta_q) \tag{3.6}$$

and finally the exterior derivative d on $\Omega^n(P,\mathfrak{g})$. The compatibility of these three operators can be easily checked.

Now we display the BRS and anti-BRS relations in the triple complex $C^{*,*,*}$. Define the modified coboundary operators

$$s_\rho := (-1)^{n+1}\delta_\rho : C^{p,q,n} \to C^{p,q+1,n}$$
$$s_{\Theta_\rho} := (-1)^{q+n+1}\delta_{\Theta_\rho} : C^{p,q,n} \to C^{p+1,q,n} \tag{3.7}$$

and the total complex $C^a = \bigoplus_{a=p+q+n} C^{p,q,n}$ with the differential operator $\Delta = d + s_\rho + s_{\Theta_\rho}$. Since $\Omega(P,\mathfrak{g})$ is a differential graded Lie algebra and $\wedge(Lie\mathcal{G})^*$ is a graded commutative differential algebra, the complex C^* becomes a differential graded Lie algebra with the bracket $[,]_C$ between homogenous elements, namely

$$[\psi_1,\psi_2]_C(\xi_1,\dots,\xi_{p_1+p_2})$$
$$= \frac{1}{p_1!p_2!}\sum_{\sigma\in\Sigma_{p_1+p_2}} (-1)^\sigma[\psi_1(\xi_{\sigma(1)},\dots,\xi_{\sigma(p_1)}),\psi_2(\xi_{\sigma(p_1+1)},\dots,\xi_{\sigma(p_1+p_2)})], \tag{3.8}$$

where $\psi_1 \in C^{p_1,q_1,n_1}$, $\psi_2 \in C^{p_2,q_2,n_2}$ and $\xi_i \in Lie\mathcal{G}$. Here $[,]$ is the usual bracket in the BRS complex $C^q(Lie\mathcal{G},\Omega^n(P,\mathfrak{g}))$, given by (3.2) and exterior multiplication in $\wedge^q(Lie\mathcal{G})^*$.

We choose the BRS, anti-BRS multiplet $(A,c,\bar{c})$, with $A \in \mathcal{A} \subset C^{0,0,1}$, $c \in C^{0,1,0}$, with $c(\xi) = \xi$, and finally $\bar{c} \in C^{1,0,0}$, with $\bar{c}(\xi) = \xi$, where $\xi \in Lie\mathcal{G}$. Let D_A denote the covariant exterior derivative on P with respect to the connection A. Since $\rho(\xi) = -L_{Z_\xi}$ holds on the subalgebra of horizontal, ad-equivariant differential forms on P, we finally can prove

Proposition 3. *Let $(A,c,\bar{c})$ be the multiplet from above. Then the following relations hold*

$$s_\rho A = -D_A c \qquad s_{\Theta_\rho} A = -D_A \bar{c}$$
$$s_\rho c = -\frac{1}{2}[c,c]_C \qquad s_{\Theta_\rho}\bar{c} = -\frac{1}{2}[\bar{c},\bar{c}]_C$$
$$s_\rho \bar{c} = -[c,\bar{c}]_C \qquad s_{\Theta_\rho} c = 0.$$

In literature c and $\bar{c}$ are usually called the ghost and the antighost field respectively. In order to formulate the covariant descent equations in this differential geometric setting, we shall consider the "universal" connection $\beta = A + c + \bar{c} \in C^1$, associate with it a covariant derivative $\Delta^\beta = \Delta + [\beta, .]_C$ and finally define the algebraic curvature $f^\beta = \Delta\beta + \frac{1}{2}[\beta, \beta]_C$. In consequence of the BRS and anti-BRS relations (Prop.3), an analog of the well known "Russian" formula holds, namely $f^\beta = F = dA + \frac{1}{2}[A, A]$.

Let I be an *ad* equivariant polynomial of order m on $\mathfrak{g}$. Then we obtain

$$I(F, \ldots, F) = m\Delta \int_0^1 dt\; I(\beta, f^{t\beta}, \ldots, f^{t\beta}), \tag{3.9}$$

where $f^{t\beta} = t\Delta\beta + \frac{1}{2}t^2[\beta, \beta]_C$. The form

$$Q_{2m-1} := m \int_0^1 dt\; I(\beta, f^{t\beta}, \ldots, f^{t\beta}) \in C^{2m-1} \tag{3.10}$$

of total degree $2m - 1$ can be decomposed into a sum of elements, homogenous in ghost, antighost and form degree

$$Q_{2m-1} = \sum_{p+q+n=2m-1} Q_n^{p,q}. \tag{3.11}$$

Eq. (3.9) implies the following system of descent equations

$$\begin{aligned} dQ^{0,0}_{2m-1} &= I(F, \ldots, F) \\ s_{\Theta_\rho} Q^{j-1,k-j+1}_{2m-k-1} + s_\rho Q^{j,k-j}_{2m-k-1} + dQ^{j,k-j+1}_{2m-k-2} &= 0 \qquad 0 \leqq j \leqq k+1,\ 0 \leqq k \leqq 2m-1, \end{aligned} \tag{3.12}$$

which becomes separated for $j = 0$ and $j = k + 1$, where it reads

$$\begin{aligned} s_\rho Q^{0,k}_{2m-k-1} + dQ^{0,k+1}_{2m-k-2} &= 0 \\ s_{\Theta_\rho} Q^{k,0}_{2m-k-1} + dQ^{2k+1,0}_{2m-k-2} &= 0 \qquad 0 \leqq k \leqq 2m-1. \end{aligned} \tag{3.13}$$

So we have recovered the descent equations, first obtained in [12,13], within our differential geometric setup.

References

1. R. Jackiw, Current Algebras and Anomalies (S.B. Treiman, R. Jackiw, B. Zumino, and E. Witten, eds.), World Scientific, Singapore, 1985.
2. L. Faddeev, Phys. Lett. **145B** (1984), 81.
3. W.A. Bardeen and B. Zumino, Nucl. Phys. **B244** (1984), 421.
4. D. Kastler and R. Stora, J. Geom. Phys. **3** (1986), 437.
5. M. Dubois-Violette, M. Talon and C.M. Viallet, Commun. Math. Phys. **102** (1985), 105.
6. L. Bonora and P. Cotta-Ramusino, Commun. Math. Phys. **87** (1983), 589.
7. M.F. Atiyah and I.M. Singer, Proc. Natl. Acad. Sci. **81** (1984), 2597.
8. T. Nishikawa and I. Tsutsui, Nucl. Phys. **B308** (1988), 544.
9. M. Abud, J.P Ader and F. Gieres, Nucl. Phys. **B339** (1990), 687.
10. G. Kelnhofer, Z. Phys. **C52** (1991), 89.
11. D. Bao and V.P. Nair, Commun. Math. Phys. **101** (1985), 437.
12. Y.Z. Zhang, Phys. Lett. **219B** (1989), 439.
13. I. Tsutsui, Phys. Lett. **229B** (1989), 51.