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**FEIGENBAUM ATTRACTOR  
AND INTERMITTENCY  
IN PARTICLE COLLISIONS**

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**Abstract**

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The hypothesis is proposed that the Feigenbaum attractor arising as a limit set in an infinite pitchfork bifurcation sequence for unimodal one-dimensional maps underlies the intermittency phenomena in particle collisions.

**Аннотация**

Багунин А.В. Аттрактор Фейгенбаума и перемежаемость в столкновениях частиц. Препринт ИФВЭ 92-5. - Протвино, 1992. - 14 с., библиогр.: 23.

Выдвигается гипотеза, что аттрактор Фейгенбаума, возникающий как предельное множество в бесконечной последовательности бифуркаций удвоения периода соответствующих фазовых траекторий есть причина перемежаемости в наблюдаемых распределениях по множественности заряженных частиц.

## INTRODUCTION

In recent years, the interpretation of the intermittency in particle collisions has inspired an extensive theoretical study [1]. However, the majority of the models proposed – the known  $\alpha$ -model [2], the spin chain model [3], the branching geometric model [4], the linked pair approximation [5] and others – only reproduce the required power behavior of the scaled factorial multiplicity moments without asking about dynamical origin of the intermittency phenomena.

There is also another class of phenomenological models [6] a priori suggesting non-trivial fluctuations in hadronic matter density during particle collisions. To simulate “spikes” in multiplicity distributions, an analogy with the theory of one-dimensional maps with a **tangent** bifurcation is used.

In this paper, we also use such an analogy. However, in our model the hadron generation in high-energy collisions is like the appearance and development of vortices in turbulent liquid through a sequence of **period-doubling** bifurcation (PDB) when the governing parameter is changed. Such analogy is justified since, first, the relevant factorial moments, as will be shown below do manifest the power-like variation with bin size. and, second, we have solid experimental evidence that PDB is common in many physical systems [7]. Because of its one-dimensionality, the PDB theory describes dynamical systems for which the dissipation has effectively damped out all but one of the degrees of freedom. Thus, for a dissipative system phase space volumes shrink at different rates in different directions. The direction of the slowest convergence defines a line which contains the attractor, i.e., the region of the phase space to which the phase space

trajectory (PST) is confined at asymptotic times. As we will demonstrate below, in hadron physics such an attractor turns out to be the Feigenbaum attractor [7] while such direction - the rapidity axis.

## 1. BIFURCATION MECHANISM OF HADRONIZATION

Three facts underlie our model. The first one is the Polyakov's similarity hypothesis in strong interactions [8] suggesting scale invariance of correlations functions at large momenta and far from the mass shell. Predicted in [8] the power-law dependence of the mean multiplicity  $\langle n_{ch}(s) \rangle$  on the energy of collisions does fit well experimental data on  $p\bar{p}$  interactions up to  $\sqrt{s} = 900 \text{ GeV}$  [9]

$$\langle n_{ch}(s) \rangle \sim \sqrt{s^\Delta}, \quad \Delta = 0.449 \pm 0.018, \quad (1)$$

and in  $e^+e^-$  annihilation owing to the substitution [10]

$$\langle n_{ch}(s) \rangle_{e^+e^-} = \langle n_{ch}(k^2s) \rangle_{pp} - n_0, \quad (2)$$

where  $k = 3.00 \pm 0.32$  and  $n_0 = 2.57 \pm 0.72$ .

Note that Polyakov theory does not predict the value of  $\Delta$  imposing the only restriction  $0 < \Delta < 1$  which follows from the energy conservation.

A very simple space-time picture of the hadronization process follows immediately from the similarity hypothesis: real hadrons are produced as a result of sequential fission of a heavy virtual particle into lighter fragments. The only restriction is that a small number of fragments in each fission process should be produced and the masses of these fragments should be comparable with the mass of the antecedent fragment. Obviously, the usual cascade  $1 \rightarrow 2$  satisfies this condition. Moreover, as is known it yields the negative binomial distribution which succeeds in fitting a large number of experimental multiplicity distributions [11].

The second fact is the recent observation [12] of the striking quantitative coincidence between the value of  $\Delta$  from Eq.(1) and the corresponding exponent  $\Delta_F$  from the law of the rise of the number  $n_m$  of the stable limit  $2^m$ -cycles elements ( $n_m \approx 2^m$ )

$$n_m \sim |\lambda_m - \lambda_\infty|^{-\Delta_F}, \quad \Delta_F = 0.449806\dots \quad (3)$$

arising in the iteration succession

$$x_{n+l} = f_\lambda(x_n) \quad (4)$$

of the points  $x_n \in [0, 1]$  by one-dimensional unimodal map  $f_\lambda(\cdot)$  with unique quadratic maximum on the unit interval (we call them  $U2$ -maps) when the governing parameter  $\lambda$  is changed. In a certain interval of the  $\lambda$ -range, the limit cycles undergo an infinite sequence of period-doubling bifurcations  $2^m \rightarrow 2^{m+1}$ ,  $m = 0, 1, 2, \dots, \infty$ ,  $m$  means the number of bifurcation. The value  $\lambda_\infty$  corresponds to infinitely large period for  $x_n$ -values which is indistinguishable from chaos.

The maps like (4) are known as Poincare maps and widely used at the study of the behaviour of nonlinear dynamical systems. They appear at the intersection of PST with some given surface in the phase space. The bifurcations of maps correspond to the splittings of PST, see Fig.1. The conditions of unimodality and the existence of only one quadratic maximum of a relevant Poincare map turned out to be sufficiently general [7] and to be satisfied in very different experiments. For example, the development of turbulence goes through a sequence of period-doublings of oscillations of liquid.

The third fact is the power-like behaviour of the scaled factorial moments for the multiplicity of cycle elements as the bin size is decreased, see Fig.2. The notations will be explained in Sect.4 with a detailed discussion.

We can therefore assume that hadronization also obeys some nonlinear equation (like Yang-Mills equations suggesting the existence of some order parameter [13]), and its solution is associated with some PST, whose number of splittings is proportional to the number of observed  $\pi$ -mesons. Really, we do not know that equation however we have an opportunity to investigate properties of its solution. The energy of collision is the only possible governing parameter whose change might cause PST splittings. One can easily see that in Eq.(1), the inverse energy of collision  $1/\sqrt{s}$  plays the role of  $\lambda$  in Eq.(3), whereas the quantity  $\lambda_\infty$  corresponds to the infinite energy ( $1/\sqrt{s} = 0$ ). So, a finite number of hadrons are produced at any finite energy. We call this mechanism of particle production the bifurcation mechanism (BM).

BM slightly modifies the Polyakov's picture of hadronization: all final hadrons in the  $1 \rightarrow 2$  cascade should have the same "age", i.e., the same number of the preceding decays of virtual fragments.

The visual contradiction between the number of  $\pi$ -mesons observed in a given event ( which can be any natural number available at a given energy) and the number  $2^m$  can be easily eliminated if one take into account that BM can generate no observed  $\pi$ -mesons but some intermediate particles (or "fireballs") which in turn decay into secondaries. It is sufficient for the law (1) to be satisfied that the intermediate particles decay into the same (at least, in average over all events) number of secondaries independently of energy. From the other side, we can suppose as well the existence of another source of particles in the spirit of a two-component model.[14,15]. When the energy increases, the relative contribution of another ("coherent" source) becomes negligible, while BM will yield the major contribution.

BM provides a qualitative explanation of the observed degeneracy of the momentum phase space in hadron collisions: in practice, instead of  $3n - 4$ , only  $n - 2$  (longitudinal) dimensions "work", where  $n$  denotes the number of secondaries. Indeed, if particles belong to the same PST, they cannot be considered as independent particles, which results in the reduction of the dimension of the relevant phase space. The exclusion of the direction  $p_{||}$  (suppression of  $\langle p_T \rangle$ ) might be related with the exclusion in the space of the direction along the collision axis.

It would then be natural to expect that characteristics of the trajectories in the momentum phase space will resemble most the properties of a one-dimensional Poincare maps at the projection onto this direction (or, equally on the (pseudo)rapidity axis). So we briefly quote these properties.

## 2. FEIGENBAUM UNIVERSALITY

The  $U2$ -maps possess an universality, (i.e., an independency of a particular map) which was discovered by Feigenbaum in the late seventies [16]. First, he has found that the convergence of the governing parameter is universal:

$$|\lambda_m - \lambda_\infty| \sim \delta^{-m} \text{ as } m \rightarrow \infty, \quad (5)$$

where  $\delta = 4.669201\dots$  is the first Feigenbaum constant. From Eq.(5) and the law of doubling of the number  $n_m$ , Eq.(3) follows immediately, with  $\Delta_F = \ln 2 / \ln \delta$ .

Second, the relative scale of the successive splittings of trajectories is also universal, i.e.,

$$\lim_{m \rightarrow \infty} \frac{|x_m - x_c|}{|x_{m+1} - x_c|} = \alpha, \quad (6)$$

where  $\alpha = 2.502907\dots$  is the second Feigenbaum constant,  $x_c$  is the critical point of the map,  $x_m$  is the nearest to  $x_c$  element of the superstable  $2^m$ -cycle (see Fig. 1b, where  $x_c = 0.5$ ). The  $2^\infty$ -cycle elements forms the Feigenbaum attractor (FA) mentioned in the title of our paper. FA is the fractal object with the Hausdorff dimension  $D_H \simeq 0.538$ , analogous to the classical Cantor set [7].

In the framework of BM, FA corresponds to the projections of  $\pi$ -meson momenta onto the rapidity axis (chosen as a one-dimensional phase space cross section) in the limit  $\sqrt{s} \rightarrow \infty$ . At finite energies, there is a finite number of  $\pi$ -mesons, whose rapidities correspond to the location within the interval  $[0, 1]$  of the elements  $x_n$  ( $n = 1, 2, 3, \dots, 2^m$ ) of the limit  $2^m$ -cycles for  $U/2$ -maps. It would be interesting to compare  $D_H$  with the dimension of a set formed by the coordinates of observed particles on the normalized rapidity axis.

From the other side,  $D_H$  is the limit dimension as  $\sqrt{s} \rightarrow \infty$  of a self-similar cascade considered by Sarcevic and Satz [17] which yields a new scaling for multiplicity moments as a function of the relative rapidity.

Let us give now a brief formulation of our hypothesis.

### 3. MAIN ASSUMPTION

BM gives the major contribution to the particle multiproduction. BM-produced particles belong to the same trajectory in phase space, which corresponds to the solution of some hadronization equation at the given energy of collision  $\sqrt{s}$ . The number of splittings of PST is proportional (due to possible subsequent decays into  $\pi$ -mesons) to the number of observed particles. The interval  $[-1, 1]$  of the normalized (pseudo)rapidity axis  $\hat{y}$  corresponds to the interval  $[0, 1]$  on the  $x$ -axis, where the one-dimensional  $U/2$  map  $x_{n+1} = f(x_n)$  is defined, i.e.,

$$\hat{y} \equiv y/y_{max} \in [-1, 1] \iff x \in [0, 1], \quad (7)$$

where  $y_{max} = \ln[(\sqrt{s} - 2m_N)/m_\pi]$ , with the point  $y = 0$  corresponds to  $x = 0.5$ . The coordinates of the BM-produced particles on the  $y$ -axis

correspond to the  $x_n$ -values in superstable  $2^m$ -cycles of the map  $f(x_n)$ , and  $2^m < n_{ch} < 2^{m+1}$ ,  $m = 1, 2, \dots, \infty$ .

Corollary 1.

$$|\bar{y}| \in [0, 1] \iff \xi = 2|x - 0.5| \in [0, 1]. \quad (8)$$

Corollary 2.

To compare the experimental ( $|\bar{y}$ -axis) and theoretical ( $\xi$ -axis) particle densities, the corresponding intervals  $[0, 1]$  should be divided into the *same* number of bins.

At the successive dividing of the  $\xi$ -interval into increasing number of bins (so that the size of bin  $\delta\xi \rightarrow \infty$ ) the corresponding scaled factorial moments [1,2] for the multiplicity of cycle elements

$$F_q = \frac{\langle n(n-1)\dots(n-q+1) \rangle}{\langle n \rangle^q} \quad (9)$$

depend on  $\delta\xi$  as power showing the presence of intermittency. see Fig.2.

Corollary 3.

With  $\sqrt{s}$  growing, the bin fixed on the (pseudo)rapidity axis will correspond to the decreasing bin on the  $\xi$  axis.

## 4. COMPARISON WITH EXPERIMENT

We proceed now to comparing experimental data on hadron collisions and the calculable (by means of computer iterations) corresponding characteristics of the position of elements in the limit superstable  $2^m$ -cycles of the particular  $U2$ -map. Remember that these characteristics are independent of the particular form of  $U2$ -map due to the Feigenbaum universality.

On Fig.3 we plot the experimental points [14] corresponding to the exponent  $\Delta(y_c)$  of the rise (in the  $\sqrt{s} = 5 - 900$  GeV range) of the mean multiplicity  $\langle n_{ch}(s, y_c) \rangle$  in  $p\bar{p}$ -collisions for a fixed symmetric rapidity interval  $|y| \leq y_c$ .

$$\langle n_{ch}(s, y_c) \rangle \sim \sqrt{s^{\Delta(y_c)}}. \quad (10)$$

For convenience, we lay off the quantity  $|\bar{y}|$  as the abscissa. The theoretical step curve on Fig.3 displays the  $\xi$ -dependence of the exponent  $\Delta(\xi)$

of the rise of the number of limit superstable  $2^m$ -cycle elements ( $m$  grows from 2 to 5) falling within the interval of the length  $\xi$  with the centre at the critical point  $x_c (= 0.5)$  for a logistic map (belonging to  $U^2$ -maps)

$$x_{n+1} = \lambda x_n (1 - x_n). \quad (11)$$

Map (11) satisfies the conditions of unimodality and uniqueness of quadratic maximum on the unit interval at  $0 < \lambda \leq 4$ . Superstable cycles are chosen for convenience since they do not need an additional shift of the middle cycle element to the point  $x = 0.5$ .

The curve has been calculated without any normalization and satisfactory fits the points. We expect experimental points will reach the theoretical curve when  $\langle n_{ch}(s) \rangle_{exp}$  will exceed essentially the value  $2^5 = 32$ . In the frame of BM, any infinitesimal interval with the centre at the point  $y = 0$  will reproduce the fractal structure of the whole rapidity interval as  $s \rightarrow \infty$ .

Fig.4 shows the theoretical  $(dN/d\xi)$  and experimental  $(dN/d\bar{\eta})$ , where  $\bar{\eta} = |\eta|/y_{max}$ ,  $\eta$  is pseudorapidity), particle densities in the bin versus the position of the bin. The experimental histogram (Fig. 3d) was constructed for  $\simeq 33000$  events on  $pp$ -collisions at  $360 \text{ GeV}/c$ , with the bin dimension  $\delta\bar{\eta} = 0.1$  [19]. At  $\bar{\eta} \simeq 0.15$ , a statistically significant narrow peak is clearly seen. (We put here  $y_{max} = 2$ , because the interval  $|\eta| \leq 2$  covers more than 80% of all the events). The known Lund model is not able to describe this peak [19] whereas it is seen perfectly well in our theoretical histogram (see Fig. 3c). The latter corresponds to the 8-cycle (since in this experiment,  $\langle n_{ch} \rangle \simeq 8.3$ ).

A softer form of the experimental spectrum as compared with the theoretical one, might be due to three facts:

- the contribution of the 4-cycle with softer spectrum;
- the decay of directly produced BM-particles into observable  $\pi$ -mesons;
- two-step data selection procedure when the events at first were grouped and only then the centers of groups were plotted on the histogram.

On Fig.5, as an example, the theoretical histogram for the 32-cycle is shown. As one can see, the second peak, hardly noticeable in the 8-cycle, appears at  $\xi \in [0.6; 0.8]$ . In the 32-cycle, its value achieves 160 particles per unit of  $\xi$ , which is four times greater than that for the first peak in the 8-cycle (with the same bin size  $\xi = 0.05$ ).

Further, Fig. 6 demonstrates the average maximum particle density within an event as a function of  $n_{ch}$ . The bins given were of two sizes:  $\Delta\eta = 0.5$  and  $\Delta y = 0.1$  [20]. In fact, the observable growth of the maximum density as  $n_{ch}$  increases is well reproduced by the theoretical curve. The recipe for calculations is simple (see Main Assumption and Corollary 2): for a given  $n_{ch}$  one takes the relevant  $2^m$ -cycle (the largest integer  $m$  from the condition  $2^m \text{ less } < n_{ch} >$ ) and finds the maximum number of the cycle elements  $\Delta N_{max}$ , falling within the bin of the size  $\Delta\xi$ . Then the value  $\Delta N_{max}/\Delta y$  (or  $\Delta\eta$ ) is plotted on the graph.

The independence of the maximum particle density on energy at a fixed  $n_{ch}$ , noticed in the experiment, is quite natural for BM. In its framework, it does not matter at which  $\sqrt{s}$  the given multiplicity is achieved, since the true governing parameter is  $\hat{s}$ .

Fig.7 shows the experimental dependence of  $\langle p_T \rangle$  on  $n_{ch}$  in  $p\bar{p}$ -collisions at  $\sqrt{s} = 1.8$  TeV for the pseudorapidity interval  $|\eta| < 3.25$  [21]. The step structure of this dependence is clearly seen. This phenomenon was early interpreted as the manifestation of a high-order phase transition [22]. In the framework of BM the abrupt increases of  $\langle p_T \rangle$  can be explained as follows. Let us consider a given particle in its rest frame ( $p_T = 0, p_{||} = 0$ ). As it was mentioned above, the Feigenbaum universality reveals along the  $p_{||}$ . Then after the bifurcation, in accordance with the structure of FA, two new particles are produced with the almost same  $p_{||}$ , which are slightly different from that of the parent particle (see Fig.1). Since we consider the rest frame of the parent particle, then both:  $p_{||} \simeq 0$  and the whole missing mass reveals along the  $p_T$  direction. i.e.  $\langle p_T \rangle \neq 0$  (see Fig.8).

After the bifurcation the number of particles is doubled, so the lengths of the plateaus between the subsequent abrupt increases of  $\langle p_T \rangle$  are doubled as well, that is shown by the arrows on Fig.7. To the left from the plateau of the length  $L$  the picture will be inverse: the lengths of the plateaus will decrease by a factor two and the plateaus will become practically indistinguishable.

The recent observation of the step increase of  $\langle p_T \rangle$  in  $e^+e^-$  annihilation at  $\sqrt{s} = 91$  GeV by DELPHI Collaboration [23] confirms the mechanism considered.

## 5. CONCLUSIONS

We have proposed the new interpretation of the intermittency phenomena in particle collisions as the manifestation of self-similar structure of the Feigenbaum attractor on the rapidity axis as  $s \rightarrow \infty$ . This fractal object with the dimension  $D_H = 0.538$  arises as a limit set in an infinite sequence of pitchfork bifurcations, which correspond to the splittings of the relevant PST when the energy of collisions is changed. PST corresponds to a solution of some hadronization master equation, essentially nonlinear, whereas the energy plays the role of the governing parameter.

Such a picture follows from Polyakov's cascade mechanism of hadronization with the only modification: all final hadrons have the same "age" (the same number of the preceding decays of virtual fragments in the cascade) and only  $1 \rightarrow 2$  decays are allowed.

Our hypothesis seems to be only one which enable physicists to calculate the characteristic of intermittency and explain in the general way the particle density fluctuations, the power-law rise of the mean multiplicity and step increases of  $\langle p_T \rangle$  as  $n_{ch}$  increases. It may be just the situation where we have the opportunity to apply the powerfull apparatus of the one-dimensional unimodal map theory to the particle physics.

I would like to thank Profs. S.S. Gershtein, H. Satz, S.N. Storchak and R. Vilela Mendes for discussions and CERN Theory Division for its hospitality while this work was completed.

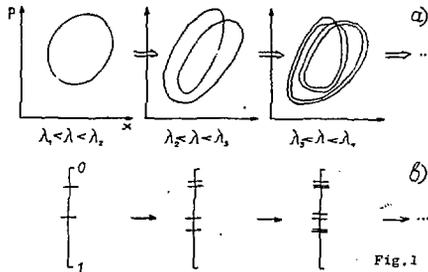


Fig. 1

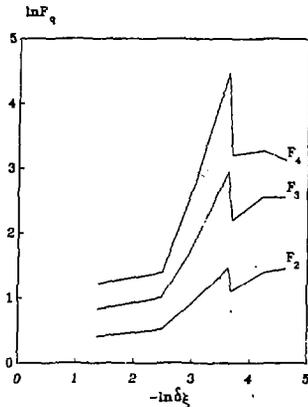


Fig. 2

Fig. 1. Splittings of PST when the governing parameter  $\lambda$  is changed (a) and corresponding doublings of the intersection points of PST and some given surface in the phase space (b).

Fig. 2. The dependence of the scaled factorial moments  $F_q$  for the multiplicity of the 32-cycle elements on the bin size  $\delta\xi$ . Two regions of a power-like rise with different slopes are clearly seen. Such a behaviour reproduces the experimental dependence [18] of  $F_q$  on the rapidity bin size  $\delta y$  in practically the same  $M$ -interval, where  $M_{exp} = 2y_{max}/\delta y$ ,  $M_{theor} = 1/\delta\xi$ . The abrupt break of  $F_q$  at  $-\ln\delta\xi > 3.61$  (or  $\delta\xi < 0.027$ ) is due to the finite number of the cycle elements. The larger the order of cycle is (64, 128, ...) the larger is the region of power-like rise of its  $F_q$ 's.

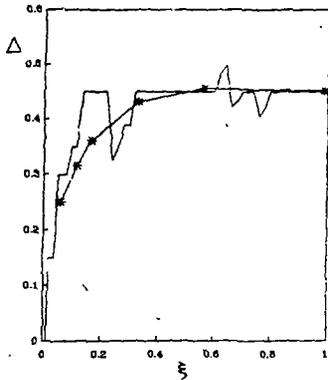


Fig. 3

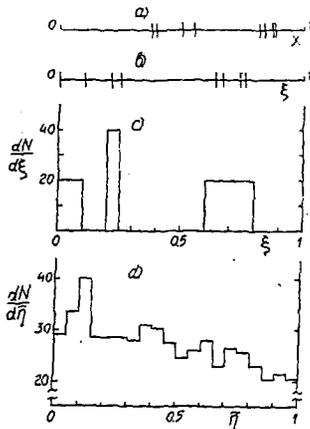


Fig. 4

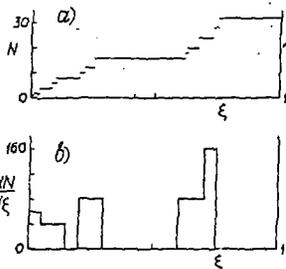


Fig. 5

**Fig. 3.** Rapidity-dependence of the exponent of  $\langle n_{ch} \rangle$ -rise in  $p\bar{p}$  collisions (experimental points [14]). The step curve corresponds to the BM predictions for  $\langle n_{ch} \rangle > 32$ ,  $\lim_{\xi \rightarrow 1} \Delta(\xi) = \ln 2 / \ln 6 = 0.449\dots$  For the details, see the text.

**Fig. 4.** Disposition of elements of the superstable 8-cycle on the  $x$ -axis (a) and  $\xi$ -axis (b). Dependence of the density of elements on the position of the bin on the  $\xi$ -axis, the bin size  $\delta\xi = 0.05$  (c). Experimental particle density as a function of the bin position on the  $\eta$ -axis [19],  $\delta\eta = 0.1$  (d).

**Fig. 5.** The multiplicity of the 32-cycle elements within a given interval of the length  $\xi$  with the centre at  $x = 0.5$  versus  $\xi$  (a). Density of the cycle elements versus the bin position on the  $\xi$ -axis,  $\delta\xi = 0.05$  (b).

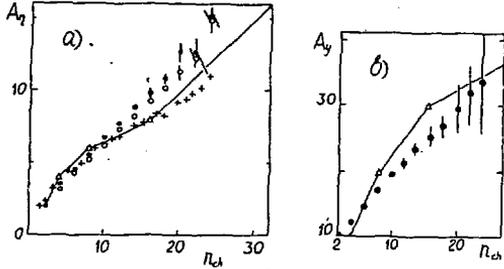


Fig. 6

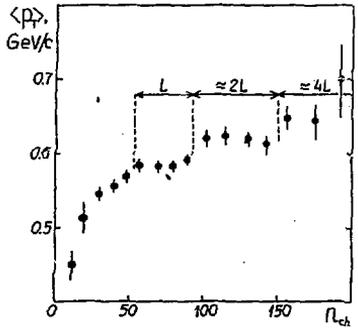


Fig. 7

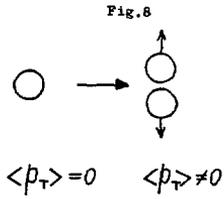


Fig. 8

- Fig. 6. The averaged maximal particle density  $A_\eta \equiv \langle (\Delta N / \delta \eta)_{max} \rangle$  within an event as a function of  $n_{ch}$  at the bin size  $\delta \eta = 0.5$ . The experimental points are taken from Ref.[20]: full dots for  $pp$ ,  $\sqrt{s} = 26$  GeV; open dots for  $\pi^+p$ ,  $\sqrt{s} = 22$  GeV, crosses for  $p\bar{p}$ ,  $\sqrt{s} = 540$  GeV (a). The analogous quantity  $A_y \equiv \langle (\Delta N / \delta y)_{max} \rangle$  at  $\delta y = 0.1$  as a function of  $n_{ch}$ . The experimental points are taken from Ref.[20] for  $pp$  collisions at  $\sqrt{s} = 26$  GeV (b). Theoretical points (triangles) are linked to guide the eye.
- Fig. 7. The dependence of  $\langle p_T \rangle$  on  $n_{ch}$  in  $p\bar{p}$  collisions at  $\sqrt{s} = 1.8$  TeV [21]. For the details, see the text.
- Fig. 8. Mechanism of the  $\langle p_T \rangle$  step increase after bifurcation.

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