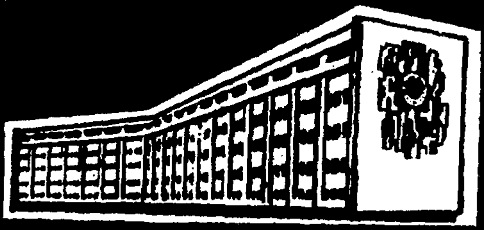
АКАДЕМІЯ НАУК УКРАЇНИ

Інститут теоретичної фізики

ITP-92-10E

S.V.Il'yin, O.A.Mogilevsky, S.A.Smolyansky, G.M.Zinov'ev

GLUON GAS VISCOSITY IN NONPERTURBATIVE REGION



Kiev

Academy of Sciences of Ukraine
Institute for Theoretical Physics

Preprint
ITP-92-10E

S.V.Illyin, O.A.Mogilevsky,
S.A.Smolyansky, G.M.Zinovjev

GLUON GAS VISCOSITY IN NONPERTURBATIVE REGION

Kiev - 1992

УДК 530.145

Г.М.Зиновьев, С.В.Ильин, О.А.Могилевский, С.А.Смолянский

Вязкость глюонного газа в непертурбативной области

Мы находим температурное поведение сдвиговой вязкости глюонного газа в области фазового перехода деконфайнмента, используя формулу типа Грина-Кубо и модель с обрезанием, опирающуюся на результаты численного Монте-Карловского изучения глюодинамики.

S.V.Illyin, O.A.Mogilevsky, S.A.Smolyansky, G.M.Zinovjev

Gluon Gas Viscosity in Nonperturbative Region

Using the Green-Kubo-type formulae and the cutoff model motivated by Monte Carlo lattice gluodynamics simulations we find the temperature behaviour of shear viscosity of gluon gas in the region of deconfinement phase transition.

During last decade a quark gluon plasma has got transformed from purely theoretical conceivable entity into an object of active experimental investigations in ultrarelativistic heavy-ion collisions. It is clear that constructing realistic model of these processes as well as the conception of the early Universe and the interior features of highly condensed stellar matter are decisively dependent on the transport properties of quark-gluon plasma since its presence is believable in such systems. Very serious activity of investigating these properties [1] is accompanied with some new suggestions on dissipative processes. Previous calculations of the kinetic coefficients (the thermal conductivity, shear and bulk viscosities) have then been added for various thermofield systems [2-8] but all of these were not much indicative in some aspects for finite nuclear systems. The recent results for quark-gluon systems obtained in the kinetic theory approach [9-12] demonstrate what physical features should be inherent to have dynamical screening in the system and give the finite answers for some important observables. However, this exciting development does not provide theoretical basis for reliable extrapolations of dissipative effect estimates in the phase transition region (but, see [9]). Here we are going to fill this gap in temperature dependence of shear viscosity estimates of gluon gas improving at the same time the preceeding results [6,8] due to the original strategy for assesing the infrared difficulty.

We calculate shear viscosity coefficient η using the Green-Kubo type formulae obtained within the relativistic statistical hydrodynamics [2,14,15]. With the estimate of η for gluon gas based on the model of real scalar field selfinteracting as $\lambda\varphi^4$ [2,3,6,8] we treat so-called "cutoff model" in lattice finite temperature gluodynamics [16-18] to simulate nonperturbative physics. Concerning a first approach it has been argued in Ref.[8] there is not only an intuitive reason to do so but, in fact, the result of η calculations in $\lambda\varphi^4$-theory is quite reliable estimate of viscosity coefficient for gluon gas. The idea of "cutoff model" is to introduce a scale K in momentum space to separate perturbative and nonperturbative regions. In Refs. [16,17] and in what follows it means to use the occupation number distribution for ideal gluon gas in the following form

$$\tilde{n}(E)=\Theta(E-K)n(E),\ n(E)=[\exp(\beta E)-1]^{-1}, \quad (1)$$

where $K(T)$ is the cutoff momentum and $\beta=1/T$ is an inverse temperature. Then the nonperturbative region which should be limited in a finite volume of momentum space does not affect the bulk features of gluon system at high enough temperature but approaching towards the critical temperature T_c the nonperturbative effects emerge though the QCD coupling constant α_s gets reduced effectively being still small enough value near T_0. It brings our estimate of at least closer to the critical region and allows to predict the violent increasing of viscosity there while the modification (1) leads to the results in agreement with ones of Refs. [10-12] in high temperature region as it is demonstrated below.

The initial point of our consideration is the well-known formula relating the shear viscosity coefficient η to equilibrium correlation functions of the energy-momentum tensor [2,14, 15]

$$\eta=\frac{1}{10}\beta\int d^3x \langle\hat{\pi}_{nm}(0)\hat{\pi}^{nm}(x)\rangle_0 .$$

Here $\hat{\pi}_{nm}$ is of shear viscous stress operator tensor which is $\hat{\pi}^{\alpha\beta}=T^{\alpha\beta}-\frac{1}{3}T_\alpha^\alpha$ $(\alpha,\beta=1,2,3)$ in the reference frame comoving with elements of a fluid, T^{nm} is an energy-momentum tensor and the statistical averaging is over the equilibrium density matrix. It can be adapted for the $\lambda\varphi^4$-model as [2,6,15]

$$\eta=\frac{2}{15}\beta\int d^3p\frac{p^4}{E(\bar{p})\Gamma(\bar{p})}n(\bar{p})[1+n(\bar{p})] \quad (2)$$

where $\Gamma(\bar{p})$ is a quasiparticle damping decrement (it is assumed that $\beta\Gamma\ll 1$)

$$\Gamma(\bar{p})=\frac{\lambda^2(2\pi)^4}{24E(\bar{p})n(\bar{p})}\int\prod_{i=1}^{3}d^3\tilde{p}_i\,\delta^{(4)}(p_1+p_2-p_3-p)(1+n_3)n_1n_2 \quad (3)$$

and $d^3\tilde{p}=[2(2\pi)^3E(\bar{p})]^{-1}d^3p$, $n_i=[\exp(\beta E(\bar{p}_i)-1]^{-1}$. According to the (1) infrared cutoff is introduced into Eqs.(2),(3) by

$$\int d^3p_i\ldots=\int d^3p_i\,\Theta[E(\bar{p}_i)-K] .$$

This innovation complicates considerably the standard technique [19] and makes the forthcoming results not so transparent.

For the sake of simplicity we deal with a massless scalar field. Then in the region of high temperature $\beta E \ll 1$ where $n(p) \simeq E(p)/\beta = 1/\beta p\ (p=|\bar p|)$ we have for the damping decrement from Eq.(3)

$$\Gamma(\bar p) = \frac{\lambda^2 T^2}{3\cdot 2^6(2\pi)^5}\int d^3 p_1\, d^3 p_2\, \frac{\theta(|\bar p+\bar p_1+\bar p_2|-K)}{p_1^2 p_2^2 (p+p_1-p_2)^2}\,\delta(p+p_1-p_2-|\bar p+\bar p_1-\bar p_2|)$$

and after a few simple integrations

$$\Gamma(\bar p) = \frac{\lambda^2 T^2}{3\cdot 2^4(2\pi)^3 p}\int_\alpha^\infty \frac{dz}{z}\left[\ln\frac{z+1-\alpha}{z} + \frac{1}{z+1}\ln z\right] \quad (4)$$

with $\alpha = K/p \leq 1$. Taking into account that [20]

$$\int \frac{dx}{x}\ln(ax+b) = \begin{cases} \ln|b|(\ln|x| - \frac{1}{2}\ln|b| + \ln|a|) - \frac{\pi^2}{6} - \mathcal{L}i_2(-\frac{ax}{b}), & |ax|<|b| \\ \frac{1}{2}\ln^2|ax| + \mathcal{L}i_2(-\frac{b}{ax}), & |ax|>b, \end{cases} \quad (5)$$

where the constant providing the continuity of indefinite integral at the point $|ax|=|b|$ has been added and introduced

$$\mathcal{L}i_2(x) = \sum_{K=1}^\infty \frac{1}{K^2} x^K$$

we have the damping decrement in the final form

$$\Gamma(p) = \theta(2K-p)\Gamma_1(p) + \theta(p-2K)\Gamma_2(p) \quad (6)$$

with the following designations

$$\Gamma_1(p) = \frac{\lambda^2 T^2}{3\cdot 2^4(2\pi)^3 p}\left\{\frac{1}{2}\left[\ln^2\frac{K+p}{p} - \ln^2\frac{K}{p}\right] + \mathcal{L}i_2\left(\frac{p}{p+K}\right) + \right. \quad (7)$$

$$\left. + \mathcal{L}i_2\left(\frac{K-p}{K}\right)\right\}$$

$$\Gamma_2(p) = \frac{\lambda^2 T^2}{3\cdot 2^4(2\pi)^3 p}\left\{\frac{1}{2}\left[\ln^2\frac{p-K}{p} - \ln^2\frac{p+K}{p}\right] - \ln\frac{p-K}{p} \cdot \right. \quad (8)$$

$$\left. \cdot \ln\frac{K}{p} + \frac{\pi^2}{6} + \mathcal{L}i_2\left(\frac{K}{K-p}\right) + \mathcal{L}i_2\left(\frac{p}{K+p}\right)\right\}.$$

It is just an accurate calculation of the integrals like (5) that allows us to avoid the threshold effects in viscosity at $p=2K$, for Eqs.(7),(8) we have $\Gamma_1(p=2K) = \Gamma_2(p=2K)$. In the limit $K\to 0$ we have quasiparticle damping decrement for massless scalar thermofield theory from Eqs.(6)-(8)

$$\Gamma_{K=0}(p) = \frac{\lambda^2 T^2}{3^2 \cdot 2^7 \pi p}. \tag{9}$$

In order to estimate the scale parameter K dependence of damping we analyze $\Gamma_1(p)$ and $\Gamma_2(p)$ and find that in the interval $K \leq p < 2K$ the damping is increasing by factor two. Its minimal value equals

$$\Gamma_1(p=K) = \frac{\lambda^2 T^2 C}{3 \cdot 2^4 (2\pi)^3 K}, \quad C = \tfrac{1}{2}\ln^2 2 + Li_2\left(\tfrac{1}{2}\right). \tag{10}$$

In the interval $p \geq 2K$ expanding in K/p we find

$$\Gamma_2(p) \simeq \frac{\lambda^2 T^2}{3 \cdot 2^4 (2\pi)^3 p}\left[\frac{\pi^2}{3} - \frac{K}{p}\left(2 - \ln\frac{K}{p}\right)\right]. \tag{11}$$

Both estimates should be utilized in viscosity coefficient formula (2) which according to Eq.(6) can be represented now as

$$\begin{aligned} \eta = \eta_1 + \eta_2 = {} & \frac{\beta}{10\pi^2}\int_K^{2K} dp \, \frac{p^4}{\Gamma_1(p)} n(p)[1+n(p)] + \\ & + \frac{\beta}{10\pi^2}\int_{2K}^{\infty} dp \, \frac{p^4}{\Gamma_2(p)} n(p)[1+n(p)]. \end{aligned} \tag{12}$$

Reminding that occupation numbers are especially large in the region of minimal momentum values we use the approximations (10) and (11) for the functions $\Gamma_1(p)$ and $\Gamma_2(p)$ in the corresponding regions of integrations. Then

$$\eta_1 = \frac{2^6 \pi T^3}{5\lambda^2 C}\left(\frac{K}{T}\right)^5 \tag{13}$$

$$\eta_2 = \eta_{K=0} - \frac{3 \cdot 2^6 T^2 K}{5\pi\lambda^2}\left\{\frac{3}{\pi^2}\left[J_4 + \left(2 - \ln\frac{K}{T}\right)I_4(0)\right] - 2\left(\frac{K}{T}\right)^4\right\}. \tag{14}$$

The constant C is defined above in Eq.(9) and the integrals J_K and $I_K(y)$ in Eq.(14) are

$$\begin{aligned} J_K &= \int_0^{\infty} dx \, x^K \ln x \, n(x)[1+n(x)] \\ I_K(y) &= \int_y^{\infty} dx \, x^K n(x)[1+n(x)]. \end{aligned} \tag{15}$$

Finally $\eta_{K=0}$ is the viscosity of massless scalar thermofield theory with the damping decrement given by Eq.(9)

$$\eta_{K=0} = \frac{3\cdot 2^6}{5\pi\lambda^2} I_5(0) T^3. \tag{16}$$

It is worth mentioning at this point that T^3 dependence in the above formula in the ultrarelativistic limit has already been predicted readily with dimensional analysis [21]. Noticing $\beta p \ll 1$ leads to $\beta K \ll 1$, we have from (12) and (13) that $\eta_1 \ll \eta_2$. Thus, the final result for shear viscosity coefficient in the model under consideration at high temperature ($\beta K \ll 1$) is the following

$$\eta = \frac{3\cdot 2^6 T^3}{5\pi\lambda^2}\left\{ I_5(0) + \frac{3K}{\pi^2 T}\left[J_4 + I_4(0)\left(2 - \ln\frac{K}{T}\right)\right]\right\}, \tag{17}$$

where numerically $I_4(0) = 26,0$; $I_5(0) = 124,4$; $J_4 = 37,5$. The infrared logarithmic factor does not change the viscosity dramatically in this region whereas extrapolating this dependence to the temperature region $T \sim K$ is much more sensible for η.

In small occupation number region where the Boltzmann statistics is valid $n(p) \lesssim \exp(\beta K) \ll 1$ and $\beta K \gtrsim 1$ the damping decrement in Eq.(3) takes the form

$$\Gamma(p) = \frac{\lambda^2 T e^{-\beta K}}{3\cdot 2^5 (2\pi)^3 p^2}\left[p(K+T) - K^2\right]. \tag{18}$$

Substituting this expression into Eq.(2) leads to the following behaviour of shear viscosity

$$\eta = \frac{2^7 \pi T^3 (K/T)^{12}}{5\lambda^2 (1+K/T)^7}\left\{ \sum_{\ell=0}^{5} \sum_{n=0}^{\ell} \frac{\ell!}{n!}\left(\frac{K}{T}\right)^{n-2\ell-2}\cdot\left(1+\frac{K}{T}\right)^{\ell+1} - \exp\left(\frac{K/T}{1+K/T}\right) Ei\left(-\frac{K/T}{1+K/T}\right)\right\} \tag{19}$$

and in asymptotic region of $K/T \gg 1$ it gets

$$\eta = \frac{2^7 \pi T^3}{5\lambda^2}\left(\frac{K}{T}\right)^5\left[\frac{5T}{K} - Ei(-1)\right]. \tag{20}$$

It is necessary to keep in mind that the limit $K \to 0$ no longer works in Eqs.(19) and (20) as $K/T \gtrsim 1$ here.

Now going to evolve the obtained estimates for gluon gas viscosity we would accept the standard procedure of changing [2,3,6]

$$\lambda^2 \longrightarrow \left(\frac{\pi^2}{18}\right) C' \alpha_s^2 \ln \alpha_s^{-1}, \quad C' = 20 \div 60 \tag{21}$$

with (for C' value, see also [10])

$$\alpha_s = 6\pi \left\{ \frac{11}{2} N \ln \frac{M^2}{\Lambda^2} \right\}^{-1} \tag{22}$$

where N is a number of colours, $M^2 = \frac{4}{3} \langle \rho^2 \rangle$ and $\langle \rho^2 \rangle$ is thermodynamically averaged squared momentum of gluon field [22]. Degeneracy factor $g = (d-1)(N^2-1)$ will absent in final result because the numerator of (2) contains it as well as the damping decrement in the denominator. Conventional result for M is to be proportional to temperature $M \simeq 4T$ (or chemical potential) of the system.

In "cutoff model" for Boltzmann statistics it has been shown [11] the good approximation for M^2 is

$$M^2 = \frac{4}{3} \cdot \frac{K^4 + 4K^3T + 12K^2T^2 + 24KT^3 + 24T^4}{K^2 + 2KT + 2T^2} \tag{23}$$

and for the scale parameter [18]

$$K = \rho T_c \left[T_c / (T - T_c) \right]^q, \tag{24}$$

where positive numbers ρ and q are extracted from Monte Carlo computational experiment for the concrete $SU(N)$ gluodynamics. For example, for $SU(2)$ pure gauge theory it has been found $\rho = 2,9$ and $q = 0,3$ [18]. The dependence $K(T)$ in Eq.(24) dictates the results with $K = 0$ at $T \gg T_c$, however, approaching the deconfinement critical temperature $T \to T_c$ we conclude substituting (23) and (24) into (22), that effective coupling constant is small enough being still out of nonperturbative region and this effect gets stronger when $T \sim T_c$. This striking fact makes it possible to extract phenomenologically well grounded estimate of temperature behaviour of shear viscosity coefficient in deconfinement phase transition region $T \sim T_c$.

Carrying out in Eq.(17) the above mentioned substitutions we find the following expression for shear viscosity of gluon gas in high temperature region ($T \gg T_c$)

$$\eta = \eta^{(0)} + \delta\eta, \tag{25}$$

where $\eta^{(0)}$ is shear viscosity of gluon gas without infrared cutoff

$$\eta^{(0)} = \frac{a I_5(0) T^3}{\pi^3 \alpha_s^2 \ln \alpha_s^{-1}} \quad , \quad a = 11,5 \div 34,6 \qquad (26)$$

and $\delta\eta$ is high temperature correction due to such a cutoff

$$\delta\eta = \eta^{(0)} \frac{3\rho}{\pi^2 I_5(0)} \left(\frac{T_c}{T}\right)^{q+1} \left\{ J_4 + I_4(0)\left[2 - \ln\rho\left(\frac{T_c}{T}\right)^{q+1}\right]\right\}. \qquad (27)$$

As in scalar thermofield theory this correction is small and positive at $T \gg T_c$ indicating only the tendency of slower viscosity decreasing with temperature decreasing as compared to $\eta^{(0)}$ from (26) (Fig.1).

However the temperature region $T \sim T_c$ becomes more attractive. Introducing infrared scale we not only keep α_s in perturbative region but would consider Boltzmann statistics as $\exp(-\beta K) \ll 1$ in that region. It seems justifiable to treat Eq.(20) with substitution (21) to get an estimate we are interested in. Then

$$\eta = \frac{\ell T^3}{\pi \alpha_s^2 \ln \alpha_s^{-1}} \left\{\rho\left(\frac{T_c}{T-T_c}\right)^q\right\}^5 \left\{\frac{5}{\rho}\left(\frac{T-T_c}{T_c}\right)^q - Ei(-1)\right\} \qquad (28)$$

where $\ell = 7,68 \div 23,04$ (more precise value can be extracted from (19)). Eq.(28) shows gluon gas viscosity in the critical region is drastically increasing (Fig.1). This possibility has been discussed qualitatively in Ref. [21]. There are reasons to believe [3,10,11,15,21] in the similar temperature behaviour of η, for quark subsystem of quark-gluon plasma.

It may not be out of place to notice that the temperature dependence of viscosity like Eq.(16) in nonperturbative region $T \sim T_c$ has been found in Ref. [13]. As a matter of fact that prediction was an extrapolation of monotonous dependence as in Eq.(9) for the damping decrement in the vicinity of critical temperature T_c. To our mind the difference between our and their results just comes from this point as in our approach the damping decrement takes the violent change at $T \sim T_c$.

As the final remark which, from the point of view developed here, is quite predictive we would like to indicate the possibility of much larger entropy growth under the process of quark-gluon plasma cooling and phase transition in comparison to the model of T^3 dependent viscosity of high temperature nuclear matter. We hope to report these details elsewhere.

The authors are indebted to V.S.Illyin, M.Namiki, A.V.Prozorkevich and Yu.M.Sinyukov for their comments, criticism and patience. Quick correspondence of Markus Thoma is acknowledged.

REFERENCES

1. H.T.Elze, U.Heinz, Phys.Rep. 183 (1989) 81.
2. A.Hosoya, M.Sakagami, M.Takao. Ann of Phys. 154, 229 (1984).
3. S.Gavin, Nucl.Phys. A435, 826 (1985).
4. P.Danielewicz, M.Gulassy. Phys.Rev. 31D, 53 (1985).
5. A.Hosoya, K.Kajantie, Nucl.Phys. B250, 666 (1985).
6. R.Horsley, W.Schoenmaker, Nucl.Phys. B280, 716; 735 (1987).
7. S.V.Illyin, A.D.Panferov, Yu.M.Sinyukov, S.A.Smolyansky, G.M.Zinovjev, preprint ITP-89-6E, Kiev, 1989.
8. S.V.Illyin, A.D.Panferov, Yu.M.Sinyukov, Phys.Lett. 227B, 455 (1989).
9. M.Mizutani, S.Muroya, M.Namiki, Phys.Rev., 37D, 3033 (1988).
10. G.Baym, H.Monien, C.Pethick, D.Ravenhall, Phys. Rev.Lett, 64, 1867 (1990).
11. M.Thoma, Phys.Lett. 269B, 144 (1991).
12. H.Heiselberg, G.Baym, C.Pethuk, J.Popp, Proc. of Latern. Conf. "Quark Matter '91", Gatlinburg, USA, 1991.
13. F.Karsch, H.W.Wyld, Proc. of a NATO Advanced Research Workshop on Lattice Gauge Theory '86 ed. H.Satz, J.Potvin, Plenum Press, New York, 1987, p.187.
14. D.N.Zubarev, Nonequilibrium Statistical Thermodynamics, Plenum Press, New York, 1974.
15. S.A.Smolyansky, A.D.Panferov, Introduction to Relativistic Statistical Thermodynamics of Normal Fluid, SSU Publishing, Saratov, 1988.
16. F.Karsch, Zeit.f.Phys.C, 38, 147 (1988).
    M.I.Gorenstein, O.A.Mogilevsky, Zeit.f.Phys.C, 38 161 (1988).
17. M.I.Gorenstein, O.A.Mogilevsky, Phys.Lett. 228B, 121 (1989).
    M.I.Gorenstein, O.A.Mogilevsky, S.Mrowczynsky, Phys.Lett., 246B, 200 (1990).
18. J.Engels, J.Finberg, K.Redlich, H.Satz, M.Weber, Zeit.f.Phys. C., 42, 341 (1989).

19. E.S.Fradkin. Trudy FIAN USSR, 29, 7 (1965)
V.B.Vologodsky, ibid, p.139.

20. A.P.Prudnikov, Yu.A.Brychkov, O.I.Marichev. Integrals and serieses, Nauka, Moscow, 1981.

21. Yu.S.Gangnus, A.V.Prozorkevich, S.A.Smolyansky. Pis'ma ZhETP, 26, 513 (1977); 28, 376 (1978).

22. J.Kapusta, Nucl.Phys.B., 148, 461 (1979).

Received March 3, 1992

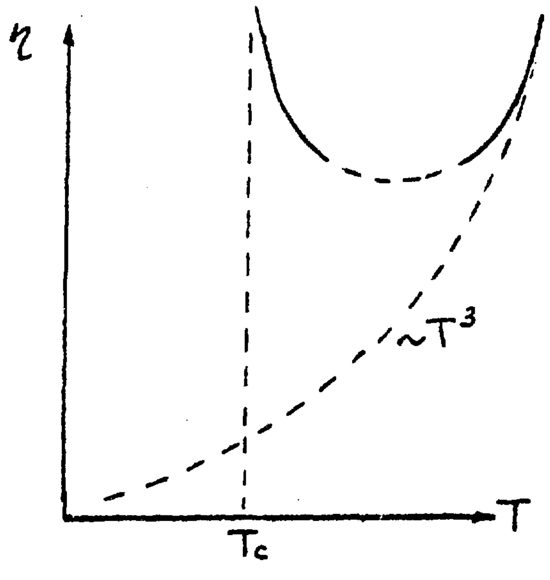


Fig.1.

NEW PUBLICATIONS

ITP-91-103E — Pavlenko O.P.

Filamentation instability of hot quark-gluon plasma with hard jet

ITP-91-104E — Kuprievich V.A.

Charge-density alternating, Peierls distortion, and charge-conjugation symmetry in correlated one-dimensional diatomic systems

ITP-91-106E — Mashkevich S.V.

Exact solutions of a many-anyon problem

ITP-92-1E — Fedutenko E.A., Lapshin V.I., Zagorodnij A.G.

Spatial helicon-wave echo in an inhomogeneous magnetized plasma

Геннадий Михайлович Зиновьев
Сергей Вадимович Ильин
Олег Арнольдович Могилевский
Станислав Александрович Смолянский

Вязкость глюонного газа в непертурбативной области

Утверждено к печати ученым советом ИТФ АН Украины

Редактор А.А.Храброва Техн.редактор Е.А.Бунькова
Зак. 92 Формат 60х84/16. Уч.-изд.л. 0,70
Подписано к печати 26.03.1992 г. Тираж 100. Цена 72 коп.
Полиграфический участок Института теоретической физики АН Украины

72 к

Препринти Інституту теоретичної фізики АН України поширюються серед наукових установ та окремих вчених шляхом взаємного обміну

252143, Київ-143, ІТФ АН України
Інформаційний відділ

The preprints of the Institute for Theoretical Physics are distributed to scientific institutions and individual scientists on the mutual exchange basis

Information Department
Institute for Theoretical Physics
252143, Kiev-143, Ukraine
Fax: (044) 2665998
*Tel.*266-91-85