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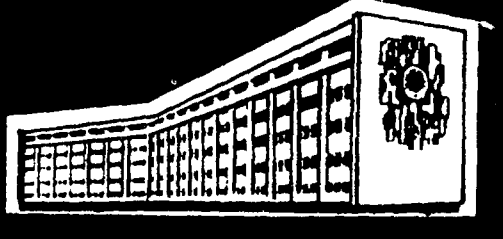

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Трьохнуклонна проблема з фазово-еквівалентними потенціалами

Вивчено вплив на трьохнуклонні параметри варіацій поведінки t-матриці поза масовою поверхнею для нелокальних фазово-еквівалентних NN потенціалів. Вказано варіації, які відповідають за зменшення або за збільшення енергії зв'язку трітія. Показано, що при деяких умовах низькоенергетичні трьохнуклонні параметри практично не залежать від поведінки від'ємних частин фаз розсіяння при високих енергіях. В методі оберненої задачі розсіяння побудовані прості потенціали S-взаємодії, які дозволили єдиним чином дати опис двохнуклонних та низькоенергетичних трьохнуклонних даних.

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Трёхнуклонная проблема с фазово-эквивалентными потенциалами

Изучено влияние на трехнуклонные параметры вариаций внемассового поведения t-матрицы для нелокальных фазово-эквивалентных NN потенциалов. Указаны вариации, ответственные за уменьшение или за увеличение энергии связи трития. Показано, что при определенных условиях низкоэнергетические трехнуклонные параметры практически не зависят от поведения отрицательных частей фаз рассеяния при высоких энергиях. В методе обратной задачи рассеяния построены простые потенциалы S-взаимодействия, позволяющие единым образом описать двухнуклонные и низкоэнергетические трехнуклонные данные.

Three-nucleon problem with phase equivalent potentials

O.M.Pushkash, D.V.Shapoval, I.V.Simenog

The effect of the t-matrix off-shell variations with nonlocal phase equivalent N-N potentials on the three-nucleon parameters is studied. The variations, which lower or increase the tritium binding energy, are revealed. We show that under certain conditions, the three-nucleon low-energy observables are almost insensitive to the high energy behavior of the negative parts of the scattering phase shifts. The inverse problem method is applied to reconstruct simple S-wave potentials which to provide a unified description of the two-nucleon and low-energy three-nucleon data.

1. INTRODUCTION

In spite of a large scale theoretical research into three-nucleon systems, involving different assumptions on the N-N interactions, a universal and simultaneous description of two- and three-nucleon data still remains to be a challenging problem. While the calculation procedure using simplified potentials is very arbitrary and provides an inadequate description already of two-nucleon characteristic, the calculation procedure involving the so-called realistic interaction potentials implies a limited number of originally determined potentials and three-nucleon parameters obtained are often at variance with experimental data.

In this paper we make an attempt to explore in a systematic way the possibilities of a complete description of the low-energy three-nucleon observables with phase equivalent potentials. For this purpose, the interaction model has to allow for the off-energy-shell variations of the t-matrix, whose on-shell behaviour holds fixed. The t-matrix off-shell variations are impossible for local central potentials that decrease fast enough at large distances. Such potentials are reconstructed unambiguously [1,2], once the two-nucleon scattering phase shifts and the deuteron binding energy are given. Although the commonly used interaction models cannot, in general, be referred to this class, some of them, at least, seem to be effectively close to it, for they show [3,4] a similarity of the off-shell t-matrices in a wide range of energies and momenta in those channels, in which the t-matrices are similar on shell.

The t-matrix off-shell variations become available when we turn, for instance, to central interaction potentials, which are nonlocal at small distances. The simplest potential is a sum of local and separable potentials. The other example is a superposition of separable potentials only. These interaction models are in common with one another both in the theoretical aspects and in the inversion procedures [5,6]. Yet, the latter model is more preferable if we are going to use the reconstructed potentials in three-particle calculations and it will be thus chosen here. Since the intent here is to examine the effects that are due to the nonlocality of the interaction, we restrict our attention to the case of the S-wave interaction only.

The plan of the paper is as follows. In Sec. 2 we solve the

inverse problem with a first rank separable potential and determine the low-energy three-nucleon parameters. A correction to the tritium binding energy, for the case of a small nonseparable perturbation, is derived in Sec. 3. Sec. 4 gives the analytical and Sec. 5 the numerical results of investigating the effects on the low-energy three-nucleon characteristics of the t-matrix off-shell variations. Also, Sec. 5 describes the effects of taking into account the negative parts of the two-nucleon scattering phase shifts. In Sec. 6 we present the inverted potentials, which simultaneously reproduce the two-nucleon and the low-energy three-nucleon data. Sec. 7 gives our concluding remarks.

2. SIMPLEST INVERSION POTENTIALS AND THREE-NUCLEON OBSERVABLES

We begin with the first rank separable potential of the two-nucleon interaction in the triplet (t) and singlet (s) states:

$$V_\mu^{(1)}(p,p') = - g_\mu v_\mu(p)\, v_\mu(p'), \quad g_\mu = \pm 1, \quad \mu = t,s. \tag{1}$$

The corresponding t-matrix is given by (the center-of-mass system; $\hbar=m=1$, m is the nucleon mass)

$$\langle p|t_\mu^{(1)}(E)|p'\rangle = - g_\mu v_\mu(p) v_\mu(p')/\det_\mu(E), \tag{2}$$

$$\det_\mu(E) = 1 - \frac{g_\mu}{2\pi^2}\int_0^\infty dq\, \frac{q^2}{q^2 - E - io}\, v_\mu^2(q), \quad \mathrm{Im}E \geq 0, \tag{3}$$

while the two-nucleon scattering phase shift $\delta_\mu(k)$ and the deuteron binding energy ε_d are determined from the relations

$$\frac{1}{k\,\cot\delta_\mu(k)} = - \frac{1}{k}\,\frac{\mathrm{Im}\,\det_\mu(k^2)}{\mathrm{Re}\,\det_\mu(k^2)}, \tag{4}$$

$$\det_\mu(-\varepsilon_d) = 0.$$

For the inverse problem method [1] to be applicable, $\delta_\mu(k)$ must be a continuous function of k, decrease, at least, in a power-like manner as $k \to \infty$, and satisfy the conditions of Levinson's theorem. Furthermore, if $\delta_\mu(k)$ crosses $n\pi$ at some $k = k_{o\mu}$, the slope of $\delta_\mu(k)$ must be negative. Under these restrictions, the interaction potential (1) can be reconstructed unambiguously. It should be mentioned that the passage of $\delta_\mu(k)$ through $n\pi$ gives rise to existence of the two-particle bound states at positive energies.

The n-p scattering phase shifts [7] once pass through zero in the lab energy range between 0 and 1 GeV, if the usual normalizations, $\delta_t(0) = \pi$ and $\delta_s(0)=0$, are used. This is observed at $E_{lab}^{0t} \approx 373$ MeV and $E_{lab}^{0s} \approx 238$ MeV, respectively. The corresponding values of the momenta $\left(k_{0\mu}= \sqrt{E_{lab}^{0\mu}/2}\right)$ are: $k_{0t} \approx 2.12$ fm^{-1} and $k_{0s} \approx 1.70$ fm^{-1}. $\delta_\mu(k)$ is positive for $k<k_{0\mu}$ and negative for $k>k_{0\mu}$. It is reasonable to consider first a simpler case, letting the scattering phase shifts $\delta_\mu(k)$ be equal to the experimental ones for $k<k_{0\mu}$ and be zero for $k \geq k_{0\mu}$. In this approach we do not run into any difficulties at positive energies. The interaction potential (1) is then attractive, $g_\mu = 1$, with the form factor expressible [1] as a functional

$$v_\mu(k) = Y\{z_\mu, \delta_\mu(k)\}, \tag{5}$$

$$Y\{z_\mu,\delta_\mu(k)\} = \left\{\frac{4\pi}{k}\,\frac{z_\mu^2+k^2}{k^2}\,\sin|\delta_\mu(k)|\right\}^{1/2} \exp\left[-\frac{1}{\pi}\int_0^\infty dq \frac{q\delta_\mu(q)}{q^2-k^2}\right], \tag{6}$$

where $z_t= \sqrt{\varepsilon_d}$, $z_s = 0$, and the integration in the neighbourhood of $q=k$ is understood in the sense of the principal value. $\delta_\mu(k)$ at arbitrary k (less than $k_{0\mu}$) were obtained by applying the spline fit interpolation procedure to the data set of ref.[7]. Also, in the region 0-1 MeV the interpolated scattering phase shifts were matched with those following from the effective range theory with $a_t= 5.424$ fm, $r_{0t}= 1.759$ fm, $a_s= -23.748$ fm, $r_{0s}= 2.75$ fm [8], where a_μ and $r_{0\mu}$ stand for the scattering lengths and the effective interaction ranges of the two nucleons, respectively. The value $\varepsilon_d= 2.2246$ MeV [8] was used for the deuteron binding energy.

The reconstructed interaction potentials (fig.1) were then used to calculate the n-d scattering parameters in the quartet state within the framework of the Faddeev integral equation method. This yields $a_4= 6.34$ fm for the scattering length and $r_4=1.76$ fm for the effective interaction range; the scattering phase shift $\delta_4(k)$ calculated for energies below the three-particle threshold is shown in fig. 2. The results obtained reproduce the ones of the nonmodel analysis [9]. This provides an additional evidence that the quartet n-d scattering low-energy parameters are insensitive to the interaction model by virtue of both the short-range character of the interaction and the crucial role of

the Pauli principle. Thus, by implying definite input data, namely, the triplet scattering phase shift and the deuteron binding energy, we can have the quartet n-d scattering amplitude as an output. The lack of precision of the experimental data [10] (points in fig. 2) prevents us from discussing any details of the scattering phase shift behavior in this energy region.

The calculations of n-d quantities in the doublet state yield $a_2^{(1)} = 0.62$ fm for the scattering length and $\varepsilon_T^{(1)} = 10.43$ MeV for the tritium binding energy. The index (1) indicates that the inversion potentials of the first rank have been used. The values obtained are found within the standard band (see, e.g. ref. [11]) in the "Phillips diagram" $\varepsilon_T(a_2)$. The net result for the doublet state is unsatisfactory: tritium turns out to be overbound as it occurs for many other S-wave interaction potentials. On the other hand, the value $\varepsilon_T^{(1)}$ is a suitable reference point, as it was obtained without introducing arbitrary parameters.

Before concluding this section, consider briefly the effect on ε_T of varying $\delta_\mu(k)$. Let the scattering phase shifts δ_μ be equal to the experimental ones for $E_{lab} \le E_{lab}^{*}$, be zero for $E_{lab} \ge E_{lab}^{*} + \Delta E$, and let them decrease in a linear manner in the intermediate region; and we also assume $\Delta E = 25$ MeV. With varying E_{lab}^{*}, ε_T and a_2 vary in the conventional correlated manner. The dependence $\varepsilon_T(E_{lab}^{*})$ is represented by the curve 1 in fig. 3. We see that the results for ε_T are already reasonable when the scattering phase shifts are reproduced up to the lab energies of about 30-40 MeV, i.e. in the region where the behaviour of δ_μ is determined by the effective range theory parameters. Also, the scale of ε_T variations decreases with increasing energy. This justifies, to a certain extent, the simplification introduced above: $\delta_\mu(k) = 0$ for $k \ge k_{0\mu}$, which is to be used in the following.

3. CORRECTION TO THE TRITIUM BINDING ENERGY

If we turn to an arbitrary nonlocal interaction potential, a wide space for the t-matrix variations becomes available. In order not to get lost in this variety of the possibilities, we consider first such interaction potentials that do not differ greatly from the separable ones. In other words, we assume that

$$\langle\vec{p}|t_\mu(E)|\vec{p}'\rangle = \langle p|t_\mu^{(1)}(E)|p'\rangle + \langle\vec{p}|\Delta t_\mu(E)|\vec{p}'\rangle, \qquad (7)$$

where $t_\mu^{(1)}$ is defined in (2) and Δt_μ is small.

The tritium binding energy ε_T is obtained by solving the equations [12,13]

$$2(k^2+\tfrac{3}{4}p^2+\varepsilon_T)\psi_\mu(\vec{p},\vec{k}) = -\sum_\nu c_{\mu\nu}\int\frac{d\vec{p}}{(2\pi)^3}\langle\vec{k}|t_\nu(-\varepsilon_T-\tfrac{3}{4}p^2)|\vec{p}'+\tfrac{\vec{p}}{2}\rangle\psi_\nu(\vec{p}',\vec{p}+\tfrac{\vec{p}}{2}), \quad (8)$$

with the t-matrix (7), $c_{\mu\mu}=1$ and $c_{\mu\nu}=3$ for $\mu\neq\nu$.

To perform the limit $\Delta t_\mu \to 0$, we introduce the corrections $\Delta\varepsilon_T=\varepsilon_T-\varepsilon_T^{(1)}$ and $\Delta\psi_\mu(\vec{p},\vec{k})=\psi_\mu(\vec{p},\vec{k})-\psi_\mu^{(1)}(p,k)$, where $\varepsilon_T^{(1)}$ and $\psi_\mu^{(1)}(p,k)$ were determined from (8) with the t-matrix (2). We further define

$$\psi_\mu^{(1)}(p,k) = \frac{1}{k^2+3/4p^2+\varepsilon_T^{(1)}}\sum_\nu c_{\mu\nu}v_\nu(k)F_\nu^{(1)}(p),$$

$$\Delta\psi_\mu(\vec{p},\vec{k}) = \frac{1}{k^2+3/4p^2+\varepsilon_T^{(1)}}\sum_\nu c_{\mu\nu}v_\nu(k)\Delta F_\nu(\vec{p},\vec{k})$$

to get the standard equations for $F_\mu^{(1)}$

$$F_\mu^{(1)}(p) = \frac{1}{2\det_\mu(-\varepsilon_T^{(1)}-3/4p^2)}\sum_\nu c_{\mu\nu}\int\frac{d\vec{p}'}{(2\pi)^3}V_{\mu\nu}(\vec{p},\vec{p}')F_\nu^{(1)}(p'), \quad (9)$$

$$V_{\mu\nu}(\vec{p},\vec{p}') = \frac{v_\mu(|\vec{p}'+\vec{p}/2|)v_\nu(|\vec{p}+\vec{p}'/2|)}{p^2+p'^2+\vec{p}\vec{p}'+\varepsilon_T^{(1)}}$$

and the equation

$$\Delta F_\mu(\vec{p},\vec{p}''+\tfrac{\vec{p}}{2}) = \frac{1}{2\det_\mu(-\varepsilon_T^{(1)}-3/4p^2)}\sum_\nu c_{\mu\nu}\int\frac{d\vec{p}'}{(2\pi)^3}V_{\mu\nu}(\vec{p},\vec{p}')\Delta F_\nu^{(1)}(\vec{p}'\vec{p}+\tfrac{\vec{p}}{2})$$

$$-\frac{1}{2}\sum_\nu c_{\mu\nu}\int\frac{d\vec{p}'}{(2\pi)^3}\frac{\langle\vec{p}'+\vec{p}/2|\Delta t_\mu(-\varepsilon_T^{(1)}-3/4p^2)|\vec{p}''+\vec{p}/2\rangle}{v_\mu(|\vec{p}'+\vec{p}/2|)v_\nu(|\vec{p}''+\vec{p}/2|)}V_{\mu\nu}(\vec{p},\vec{p}')F_\nu^{(1)}(p')$$

$$-\Delta\varepsilon_d D_\mu(\vec{p},\vec{p}'')F_\mu^{(1)}(p), \qquad (10)$$

$$D_\mu(\vec{p},\vec{p}') = \frac{1}{p^2+p'^2+\vec{p}\vec{p}'+\varepsilon_T^{(1)}} - \frac{\det'_\mu(-\varepsilon_T^{(1)}-3/4p^2)}{\det_\mu(-\varepsilon_T^{(1)}-3/4p^2)} > 0,$$

for ΔF_μ, where we have kept only the terms linear in Δt_μ. $\det'_\mu(x)$ in (10) stands for the derivative of $\det_\mu(x)$ with respect to x. To derive an expression for $\Delta\varepsilon_T$, we multiply (9) and (10) by

$$c_{\eta\mu}\Delta F_\eta(\vec{p}'',\vec{p}+\vec{p}''/2)V_{\eta\mu}(\vec{p}'',\vec{p})$$

and

$$c_{\eta\mu}F_{\eta}^{(1)}(p'')V_{\eta\mu}(\vec{p}'',\vec{p})$$

respectively, carry out the summation over η, μ and the integration over $\vec{p}$, $\vec{p}'$, and subtract the resulting equations from each other. This yields:

$$\Delta\varepsilon_{\tau} = A/B, \tag{11}$$

$$A = -\frac{1}{2}\sum_{\eta\mu\nu} c_{\eta\mu}c_{\mu\nu}\iiint \frac{d\vec{p}}{(2\pi)^3}\frac{d\vec{p}'}{(2\pi)^3}\frac{d\vec{p}''}{(2\pi)^3}F_{\eta}^{(1)}(|\vec{p}'-\vec{p}/2|)F_{\nu}^{(1)}(|\vec{p}''-\vec{p}/2|)$$

$$* \frac{v_{\eta}(|1/2\vec{p}'+3/4\vec{p}|)v_{\nu}(|1/2\vec{p}''+3/4\vec{p}|)}{(p'^2+3/4p^2+\varepsilon_{\tau}^{(1)})(p''^2+3/4p^2+\varepsilon_{\tau}^{(1)})}\langle\vec{p}'|\Delta t_{\mu}(-\varepsilon_{\tau}^{(1)}-\frac{3}{4}p^2)|\vec{p}''\rangle,$$

$$B=\sum_{\eta} c_{\eta\mu}\iint\frac{d\vec{p}}{(2\pi)^3}\frac{d\vec{p}'}{(2\pi)^3}F_{\eta}^{(1)}(p)F_{\mu}^{(1)}(p')V_{\eta\mu}(\vec{p},\vec{p}')\ D_{\mu}(\vec{p},\vec{p}').$$

Eq. (11) is the first order correction in Δt_{μ} to the tritium binding energy $\varepsilon_{\tau}^{(1)}$. As always, the correction is the perturbation operator average value over zero order functions $F_{\mu}^{(1)}(p)$ corresponding to the ground state. These functions have no nodes and are of the same sign. Hence, B is positive and the sign of $\Delta\varepsilon_{\tau}$ is determined by the quantity A which, in turn, is dependent on a specific choice of the perturbation operator Δt_{μ}. Note that Δt_{μ} is not restricted to be of the S-wave type. In the present paper, however, we shall not go beyond this approximation.

4. EFFECTS ON ε_{τ} OF THE t-MATRIX OFF-SHELL VARIATIONS

Off-shell variations become already available when another term is added to (1):

$$V_{\mu}^{(2)}(p,p')=-g_{1\mu}u_{1\mu}(p)u_{1\mu}(p')-g_{2\mu}u_{2\mu}(p)u_{2\mu}(p'),\quad g_{1\mu}=\pm 1. \tag{12}$$

To avoid duplications, we set $u_{2\mu}(0)=0$. This condition can always be satisfied by going from given functions $u_{1\mu}(k)$ and $u_{2\mu}(k)$ to their linear combination (implying also that the diagonal form of (12) is preserved). The other condition will be a definite restriction imposed on the additional term in (12). Since the model is intended to simulate variations of the nonlocal nature, we assume that the potential (12) variations are possible only for

high momenta greater than some $k_{x\mu}$, where realistic physical interaction potential might be expected to be nonlocal. The value of $k_{x\mu}$ must be taken large enough, at least, greater than the inverse value of the π-meson Compton wavelength, i.e. $k_{x\mu} \gtrsim 0.7$ fm^{-1}. Thus, $u_{2\mu}(k)$ is allowed to be much different from zero only for $k>k_{x\mu}$.

A. The off-shell variation proceedure.

The t-matrix corresponding to the interaction potential (12) is

$$\langle p|t_{\mu}^{(2)}(E)|p'\rangle = -(g_{1\mu}(1-g_{2\mu}C_{22\mu}(E))u_{1\mu}(p)u_{1\mu}(p') + g_{2\mu}(1-g_{1\mu}C_{11\mu}(E))u_{2\mu}(p)u_{2\mu}(p')+g_{1\mu}g_{2\mu}C_{12\mu}(E)(u_{1\mu}(p)u_{2\mu}(p') +u_{2\mu}(p)u_{1\mu}(p')))/\det_{\mu}(E), \quad (13)$$

where

$$\det_{\mu}(E)=(1-g_{1\mu}C_{11\mu}(E))(1-g_{2\mu}C_{22\mu}(E))-g_{1\mu}g_{2\mu}C_{12\mu}^{2}(E), \quad (14)$$

$$C_{ij\mu}(E)=\frac{1}{2\pi^2}\int_0^{\infty}dq\frac{q^2}{q^2-E-i0}u_{i\mu}(q)u_{j\mu}(q),\ i,j=1,2.$$

The scattering phase shift $\delta_{\mu}(k)$ and the deuteron binding energy ε_d are determined from the relations (4) with $\det_{\mu}(E)$ given by Eq. (14). We still assume that $\delta_{\mu}(k)$ is positive for $k<k_{0\mu}$ and is zero for $k\geq k_{0\mu}$. The interaction potential (12) can now be reconstructed within the inverse problem method. We use here the general inversion scheme which was developed in ref [6]. For the problem of interest, the general scheme of ref. [6] may be reduced to the following procedure.

Let us introduce the function $\chi_{\mu}(k)$ satisfying the conditions: (a) $\chi_{\mu}(0)=\chi_{\mu}(\infty)=0$; (b) $\chi_{\mu}(k)$ is a sign-definite and $\delta_{0\mu}(k)=\delta_{\mu}(k)-\chi_{\mu}(k)$ is a nonnegative function; (c) $|\chi_{\mu}(k)|<\pi/2$, $|\delta_{0\mu}(k)|\leq\pi$. In other respects $\chi_{\mu}(k)$ is arbitrary. The interaction potential form factors are then reconstructed as

$$u_{2\mu}(k)=Y\{0,\chi_{\mu}(k)\}, \quad (15)$$

$$u_{1\mu}(k)=\cos\chi_{\mu}(k)\cdot u_{0\mu}(k)-g_{2\mu}\tilde{C}_{\mu}(k^2)u_{2\mu}(k), \quad (16)$$

where

$$u_{0\mu}(k)=Y\{z_\mu,\delta_{0\mu}(k)\},\quad u_{2\mu}(k)=\frac{4\pi}{k}\,\frac{\sin|\chi_\mu(k)|}{u^2_{2\mu}(k)}\,u_{0\mu}(k),$$
$$\tilde{C}_\mu(k^2)=\frac{1}{2\pi^2}\int_0^\infty dq\frac{q^2}{q^2-k^2}\,u_{2\mu}(q)u_{2\mu}(q). \qquad (17)$$

The constant $g_{1\mu}$ in (12) is positive and $g_{2\mu}$ is of the same sign as the function $\chi_\mu(k)$.

By varying the arbitrary function $\chi_\mu(k)$, a set of different form factors $u_{1\mu}(k)$ and $u_{2\mu}(k)$ can be derived. Thus, variations in $\chi_\mu(k)$ lead to off-shell variations in the t-matrix, whose on-shell ($\sqrt{E}=p=p'=k$) behaviour holds fixed. The conditions (a)-(c) involve two conditions which we have imposed additionally. The first condition, $\chi_\mu(k)=0$, is equivalent to the condition $u_{2\mu}(0)=0$ that was introduced earlier. The other one, $|\chi_\mu(k)|<\pi/2$ (the general requirement is $|\chi_\mu(k)|\leq\pi$), prevents Eqs. (16), (17) from becoming more complicated. This condition is not too restrictive; yet, it may be ruled out if necessary. Except for this condition, the above procedure, in principle, enables one to derive all possible phase equivalent separable interaction potentials of the second rank.

B. Small orthogonal variations.

Let us consider the effect on ε_T of going from one inversion potential, $V^{(1)}_\mu$, to another, $V^{(2)}_\mu$, for the simplest case, when both terms of $V^{(2)}_\mu$ are attractive and act in nonintersecting regions. We thus assume that $u_{1\mu}(k)$ is nonzero for $0\leq k<k_{x\mu}$ and $u_{2\mu}(k)$ is nonzero for $k_{x\mu}<k<k_{0\mu}$, where $k_{x\mu}$ is arbitrary, but less than $k_{0\mu}$. These form factors will be referred to as orthogonal ones, since the overlapping integral $C_{12\mu}(E)$ is zero. This approach is equivalent to the choice: $\chi_\mu(k)=0$ for $0\leq k<k_{x\mu}$ and $\chi_\mu(k)=\delta_\mu(k)$ for $k_{x\mu}\leq k\leq k_{0\mu}$. Hence, $u_{2\mu}(k)$ is determined from Eq.(15) and $u_{1\mu}(k)$ from a similar equation, $u_{1\mu}(k)=Y\{z_\mu,\delta_{0\mu}(k)\}$, which immediately follows from (16). Assume also that $u_{2\mu}(k)$ is small. This is accomplished easily by making $k_{x\mu}$ tend to $k_{0\mu}$, but it is more convenient here to fix $\delta_\mu(k)$ in the region $0\leq k<k_{x\mu}$ and set $\delta_\mu(k)=\varepsilon f_\mu(k)$ for $k_{x\mu}\leq k\leq k_{0\mu}$, where $f_\mu(k)$ is a given function and ε tends to zero. We now calculate the correction (11) for the perturbation operator $\Delta t_\mu=t^{(2)}_\mu-t^{(1)}_\mu$. Since the t-matrices $t^{(2)}$ and

$t^{(1)}$ correspond to phase equivalent interaction potentials, their denominators of Eqs. (14) and (3) are identical: implying definite $\delta_\mu(k)$ and ε_d, the analytical in the upper half-plane function $det_\mu(E)$ is uniquely determined by Eq. (4) and the condition $det_\mu(\infty)=1$. To evaluate the difference between the t-matrices numerators, we divide the range of the variables of the matrix element $\langle p'|\Delta t_\mu(-\varepsilon_T^{(1)}-3/4p^2)|p''\rangle$ into three subregions labeled as I-III, where I is $0\le p',p''<k_{x\mu}$, II is $k_{x\mu}\le p',p''\le k_{0\mu}$, III is $0\le p'<k_{x\mu}$, $k_{x\mu}\le p''\le k_{0\mu}$ and $0\le p''<k_{x\mu}$, $k_{x\mu}\le p'\le k_{0\mu}$. In the limit $\varepsilon\to 0$, the form factors involved in Eqs. (2) and (13) behave as: $v_\mu(k)=u_{1\mu}(k)+0(\varepsilon)$ for $0\le k<k_{x\mu}$, $v_\mu(k)=0(\sqrt{\varepsilon})$ and $u_{2\mu}(k)=0(\sqrt{\varepsilon})$ for $k_{x\mu}\le k\le k_{0\mu}$; the function $u_{1\mu}(k)$ is independent of ε. Finally, we observe that in subregion III the matrix element $\langle p'|\Delta t_\mu(-\varepsilon_T^{(1)}-3/4p^2)|p''\rangle=v_\mu(p')v_\mu(p'')/det_\mu(-\varepsilon_T^{(1)}-3/4p^2)$ is positive and is of the order of $0(\sqrt{\varepsilon})$ as $\varepsilon\to 0$, while in subregions I and II it is of the order of $0(\varepsilon)$. Consequently, the correction (11) to the tritium binding energy is negative: $\Delta\varepsilon_T<0$.

Two remarks must be added here. First of all, in subregion I the matrix element of Δt_μ tends to zero as $\varepsilon\to 0$ nonuniformly with respect to p',p''. This is due to the singularity: $u_{1\mu}(k)\sim(k_{x\mu}-k)^{-\delta_\mu(k_{x\mu})/2\pi}$ as $k\to k_{x\mu}$. A detailled analysis however, shows that the integral of Δt_μ over subregion I is still of the order of $0(\varepsilon)$. The other comment is that the scattering phase shift $\delta_\mu(k)$, used here, is discontinuous at $k=k_{x\mu}$, by definition. This defect can be removed by minor adjustments in $\delta_\mu(k)$ to the left from $k_{x\mu}$, which do not alter the estimates for Δt_μ.

The opposite case, when the second term in (12), satisfying the conditiors of smallness and orthogonality, is repulsive, is realized by the choice: $\chi_\mu(k)=0$ for $0\le k\le k_{0\mu}$ and $\chi_\mu(k)=-\varepsilon f_\mu(k)$ for $k_{0\mu}<k\le k_{x\mu}$ where $f_\mu(k)>0$ and $\varepsilon\to 0$ (here $u_{2\mu}(k)$ and $v_\mu(k)$ are orthogonal). Then the arguments which are similar to those of the previous case reveal that $\Delta\varepsilon_T<0$.

To summarize: the tritium binding energy decreases if an attractive term is added (under the conditions of phase equivalence) to the interaction potential and increases in the case of a repulsive addition. The origin of this effect is clarified below.

C. Qualitative treatment.

Suppose again that both terms in (12) are attractive and the form factors are orthogonal and examine the difference between the two phase equivalent interaction potentials $\Delta V_\mu(p,p') = V^{(2)}_\mu(p,p') - V^{(1)}(p,p')$. From the definitions of $u_{1\mu}(k)$, $u_{2\mu}(k)$ and $v_\mu(k)$ we find that $u_{1\mu}(k) > v_\mu(k)$ for $0 \le k < k_{x\mu}$ and $u_{2\mu}(k) < v_\mu(k)$ for $k_{x\mu} \le k < k_{0\mu}$. It is readily seen that $\Delta V_\mu < 0$ in subregion I and $\Delta V_\mu > 0$ in subregions II and III. In other words, as compared to $V^{(1)}_\mu$, the potential $V^{(2)}_\mu$ exhibits an increased attraction at small momenta and a decreased attraction at high momenta. With short-ranged interaction potentials ($V^{(2)}_\mu$ is of this kind, provided that $k_{x\mu}$ is large enough), the region of high momenta is more important for low-energy three-nucleon quantities, rather than for those of a two-nucleon system, since $\varepsilon_T \gg \varepsilon_d$. Consequently, the value of ε_T calculated for the potential $V^{(2)}_\mu$ must be smaller as compared with that for $V^{(1)}_\mu$.

Note that the consideration provides a (qualitative) extension for the previous findings, since the second term in (12) was not required to be small. In the next example we introduce the form factors which are nonorthogonal, showing thus that the restriction of orthogonality is not crucial also. Let $u_{2\mu}(k)=\rho\varepsilon^2/\pi(\varepsilon^2+(k-k_{x\mu})^2)$ and let $g_{2\mu}$ in (12) be of an arbitrary sign. As $\varepsilon \to 0$, $u_{2\mu}(k)$ tends to the δ-function peaked at $k_{x\mu}$, but with a decreasing intensity. $k_{x\mu}$ is arbitrary, but large enough. For $u_{2\mu}(k)$ to be small at $k=k_{x\mu}$, we assume that $\rho \ll v_\mu(k_{x\mu})$. The simplest way to derive now an expression for $u_{1\mu}(k)$, is to use the identity of the t-matrix denominators of Eqs. (3) and (14). This yields: $u_{1\mu}(k)=v_\mu(k)+g_{2\mu}\varepsilon\rho^2 k^2_{x\mu} v_\mu(k_{x\mu})/(2\pi)^3(k^2_{x\mu}-k^2)+O(\varepsilon^2)$ as $\varepsilon \to 0$. Then, the correction ΔV_μ is of the same sign as $g_{2\mu}$ at high momenta and of the opposite sign at small momenta. Therefore, $\Delta\varepsilon_T<0$ for $g_{2\mu}=1$ and $\Delta\varepsilon_T>0$ for $g_{2\mu}=-1$.

5. NUMERICAL RESULTS

A. General regularities.

The procedure of Sect.4A has been used to derive several

families of phase equivalent interaction potentials. As $\chi_\mu(k)$ (μ=t,s) we took different functions involving 5-6 parameters. The parameters were varied under the requirements (a) - (c) of Sect.4A and the condition that $\chi_\mu(k)$ must be small for $k \le k_{x\mu}$. The recalculated $\delta_\mu(k)$ for $k<k_{0\mu}$ were found to reproduce the initial scattering phase shifts [7] within 0.5°. The inverted potentials were then used in the Faddev calculations of n-d observables. It was found that $\varepsilon_T < \varepsilon_T^{(1)}$ for positive $\chi_\mu(k)$ and $\varepsilon_T > \varepsilon_T^{(1)}$ for negative $\chi_\mu(k)$, where $\varepsilon_T^{(1)}$ is for $\chi_\mu(k)=0$, corresponding to the first rank potential. This agrees with the analytical results. The value of $|\varepsilon_T - \varepsilon_T^{(1)}|$ increases with increasing $|\chi_\mu(k)|$ and especially with decreasing $k_{x\mu}$. We present the results, obtained with one family of the interaction potentials. Let the functions $\delta_{0\mu}(k)$, involved in (17), be equal to the truncated scattering phase shifts that were introduced in the last paragraph of Sect.2 and are associated with curve 1 in fig.3. Then $u_{2\mu}(k)=0$ for $k \le k_x = \sqrt{E_{lab}^x/2}$ and $u_{1\mu}(k)$ are close to zero for $k > \sqrt{(E_{lab}^x + \Delta E)/2}$. The dependence of ε_T on E_{lab}^x is given by curve 2 in fig.3. The figure reveals that off- and on-shell variations in the t-matrix behavior produce the same, by the order of magnitude, changes in ε_T. We see that if the t-matrix varies in the region of momenta higher than 0.7 fm^{-1}, the variations in ε_T are of about 2 Mev. Note also that conventional correlations between low-energy three-nucleon parameters hold. In particular, the values of ε_T and a_2, determined from our phase equivalent potentials, are found within the standard band in the "Phillips diagram", which is formed (see, e.g., ref.[11]) by plotting the results of numerous calculations with different interaction models.

Note that one cannot be assured that the interaction radius R_μ of the nonlocal potential is small if the potential parameters are fitted only to the two-nucleon observables $\delta_\mu(k)$ and ε_d. Quite the contrary, the restriction that $R_\mu - r_{0\mu} \ll a_\mu$ must be imposed additionally, yielding

$$\langle \vec{p}|t_\mu(E)|\vec{p}'\rangle \propto \frac{4\pi}{1/a_\mu + i\sqrt{E+i0}}, \quad p,p',|E|^{1/2} \ll 1/r_{0\mu}. \qquad (18)$$

The opposite case of long-ranged potentials arises when $\chi_\mu(k)$ is allowed to vary in the region $k \ll 1/r_{0\mu}$ (and hence (18) is violated off-shell). This leads to enormous changes in low-energy three-nucleon quantities and, moreover, destroys the correlations.

For example, the band in the "Phillips diagram" becomes an order wider and thus smears into a shapeless region, although we are still concerned with the potentials fitted to the two-nucleon scattering phase shifts [7] and the deuteron binding energy [8]. This is one more evidence that the correlations between the low-energy three-nucleon parameters are due to the short-range character of the two-nucleon interaction [14, 15].

B. Taking into account the negative parts of $\delta_\mu(k)$.

Let $\delta_\mu(k)$ reproduce the scattering phase shifts of ref.[7] that are given up to 1 Gev. In the region of higher energies $\delta_\mu(k)$ are continued in such a way as to meet the requirements $-\pi \le \delta_\mu(k) \le 0$ and $\delta_\mu(\infty)=0$. Hence $\delta_\mu(k)<0$ for $k>k_{o\mu}$. With such $\delta_\mu(k)$, the inversion procedure described in Sect.4A is still valid if additional constraints are imposed, namely, $\chi_\mu(k)\le 0$ and $g_{2\mu}=-1$. To elucidate the effects of taking into account the negative parts of $\delta_\mu(k)$, we exclude the independent off-shell variations by fixing one of the terms in (12). The another term is then strongly correlated with $\delta_\mu(k)$. In this approach, the dependence of ε_T on $\delta_\mu(k)$ is found to be quite natural: the value of ε_T decreases as $\delta_\mu(k)$ become negative for $k>k_{o\mu}$. But the scale of ε_T variations is very sensitive to the model assumptions, as is seen from the following two examples.

We first fix the attractive term in (12) by assuming $\chi_\mu(k)=0$ for $k\le k_{o\mu}$ and $\chi_\mu(k)=\delta_\mu(k)$ for $k>k_{o\mu}$. This leads to orthogonal form factors: $u_{1\mu}(k)=v_\mu(k)\neq 0$ for $k<k_{o\mu}$ and $u_{2\mu}(k)\neq 0$ for $k>k_{o\mu}$. Then the t-matrix of Eq.(13) reproduces the first rank t-matrix $t_\mu^{(1)}$ of Eq.(2) for $p,p'<k_{o\mu}$ and is different from $t_\mu^{(1)}$ only in the region of high momenta. With these assumptions, taking into account the negative part of $\delta_\mu(k)$ yields extremely small changes in ε_T: $|\Delta\varepsilon_T| \lesssim 0.03$ MeV, where $\Delta\varepsilon_T=\varepsilon_T-\varepsilon_T^{(1)}$. The values obtained are almost independent of the choice of the asymptotic behaviour of $\delta_\mu(k)$. For those $\delta_\mu(k)$ that with growing k approach zero fast enough, $\Delta\varepsilon_T$ was recalculated using the perturbation theory in $u_{2\mu}(k)$. The relevant expression for $\Delta\varepsilon_T$ was derived by substituting $\langle p'|\Delta t_\mu(-\varepsilon_T^{(1)}-3/4p^2)|p''\rangle \propto -u_{2\mu}(p')u_{2\mu}(p'')$ into (11). The perturbation calculations confirmed the results of the direct calculations. The same results were obtained for infinite

repulsion which was introduced by taking, as a limiting case, $\delta_\mu(k) \to -\pi$ (the value proposed in ref. [16]) as $k \to \infty$. These findings are not surprising, since in the present model, the repulsion is simply added in the region of high momenta, while the probability of finding the nucleons here is small.

We now fix the repulsive term in (12) by fixing a definite $\chi_\mu(k)$. $\chi_\mu(k)$ must be still small for $k<k_{x\mu}$, but at higher momenta, $|\chi_\mu(k)|$ must be large enough as to provide some space for varying the negative part of $\delta_\mu(k)$ under the restriction of $\delta_{o\mu}(k)=\delta_\mu(k)-\chi_\mu(k)\geq 0$ of Sect.4A. If $k_{x\mu}$ is taken to be much smaller than $k_{o\mu}$, the form factors $u_{1\mu}(k)$ and $u_{2\mu}(k)$ are strongly overlapped. Then, as soon as the scattering phase shift becomes negative for $k>k_{o\mu}$, appreciable changes in $u_{1\mu}(k)$ occur. The relevant t-matrix of Eq.(13) is different from $t^{(1)}_\mu$ in the whole range of momenta, including the region of small momenta. This, in tern, produces large scale decrease in ε_T which is found to be of about 2 MeV for $k_{x\mu} \gtrsim 0.7$ fm^{-1}. The value of ε_T is now much dependent on the choice of the behaviour of $\delta_\mu(k)$ at energies higher than 1 GeV. Probably, the observations quoted in literature that adding the repulsion yields a strong effect on low-energy three-nucleon parameters are relevant to the present type model, when the added repulsive term is strongly overlapped with the main attractive term, leading to significant changes in the latter. We must explain further that in the present case, the function $|\chi_\mu(k)|$ exceeds the corresponding function from the model of the orthogonal form factors due to the restriction $\delta_{o\mu}(k) \geq 0$. Since an increase in $|\chi_\mu(k)|$ is accompanied by an increase in ε_T (Sect.5A), all changes in the tritium binding energy value take place in the region $\varepsilon_T > 10.4$ MeV. By the way, this means that it is impossible to reproduce the positive and negative parts of the N-N scattering phase shifts and the correct value of the tritium binding energy, 8.48 MeV, with the S-wave interaction potentials of the second rank.

6. A UNIFIED DESCRIPTION OF TWO- AND THREE-NUCLEON DATA

Let $\delta_\mu(k)$ be equal to the experimental scattering phase shifts for $k<k_{o\mu}$ and be zero for $k \geq k_{o\mu}$. (To avoid difficulties, we remove the discontinuity in the first derivative of $\delta_\mu(k)$ with

respect to k at $k=k_{0\mu}$ by taking $\delta_\mu(k)$ changing by the low $(k_{0\mu}-k)^2$ in the left-hand vicinity of $k_{0\mu}$. We shall not take care of the higher derivatives, whose discontinuities produce additional long-ranged terms in the interaction potentials, since the amplitudes of these terms are negligible). These $\delta_\mu(k)$ may be used to reconstruct second rank attractive interaction pctentials, which prcduce the correct value for ε_T. A more difficult problem is to select among the phase equivalent potentials those ones that would correctly reproduce, in addition to ε_T, the doublet n-d scattering length a_2. For this purpose, the behaviour of $u_{1\mu}(k)$ and $u_{2\mu}(k)$ was varied in a complicated manner over the whole range of momenta, from 0 to $k_{0\mu}$. A series of coordinated variations has enabled us to reconstruct the required potentials. The relevant form factors, $u_{i\mu}(k)$, are displayed in fig.4. Also, for some values of k, their values are listed in Table 1. The interaction potentials of Eq.(12) with the form factors $u_{i\mu}(k)$, recalculated via the standard spline interpolation procedure from the data set of Table 1, reproduce the N-N scattering phase shifts [7] (for $k<k_{0\mu}$) within 1.5° and produce the correct values for the three-nucleon parameters: $\varepsilon_T=8.48$ MeV and $a_2=0.65$ fm. The calculated values of other low-energy three-nucleon quantities are also in agrement with experimental or theoretical results. As for the doublet n-d scattering phase shift, this is seen from fig.5. The value of $1/2\alpha^3a_2^2r_2=1.35$, where $\alpha=\sqrt{\varepsilon_d}$ and r_2 is the n-d doublet effective interaction range, reproduces the one from ref.[17]. Following the line of reasonings of ref.[17], we conclude that the present potentials are associated with the value $\varepsilon_v-\varepsilon_d=0.53$ MeV of the tritium virtual level energy and with the values $C^2=3.5$ and $C_V^2=0.060$ of the constants, related to the scattering amplitude residues at the poles, corresponding to the bound and virtual states of tritium, respectively. Here $C^2=-1/3\varkappa_T \lim_{k\to i\varkappa_T}[(k^2+\varkappa_T^2)f_2(k)]$, where $f_2(k)=(k\cot\delta_2-ik)^{-1}$ and $\varkappa_T=\sqrt{4/3(\varepsilon_T-\varepsilon_d)}$, and C_V^2 is defined by similar expressions. For a comparison, we present the results of other predictions: $C^2=3.3\pm0.1$ [18] (ref. [19] gives $C^2=3.39$) and $\varepsilon_v-\varepsilon_d=0.482$ Mev, $C_V^2=0.0504$ [18] (close results are given in ref.[20]). The calculated value of the threshold parameter $R=d(k\cot\delta_2)/dk^2|_{k^2=4/3\varepsilon_d}=-0.38$ fm is close to the values reported

in ref. [21]. The negative slope of $k\cot\delta_2$ at the threshold (see the inset in fig.5) is one of the consequences of the anomaly associated with the nearly degenerate state of the three-particle break-up threshold [14,21]. It should be mentioned also that the results for the quartet n-d scattering parameters are almost identical to those presented in Sect.2.

It is reasonable to consider the rms radius $\langle r^2\rangle^{1/2}$ and the elastic form factor of the deuteron $F_d(q)=\int d\vec{r}\exp(i\vec{q}\vec{r})\,|\Psi_d(r)|^2$, where $\Psi_d(r)$ is the normalized deuteron wave function. In the case of nonlocal interaction potentials, these quantities are not strongly correlated with the scattering phase shift $\delta_t(k)$. Thus, our potentials may be tested once more. Since $\delta_t(k)=0$ for $k\geq k_{ot}$, $F_d(q)$ is identically zero for $q\geq 2k_{ot}$. For smaller q, $F_d(q)$, as well as the elastic structure function $A(q)$, are depicted in fig.6. We see that in the region $q\leq 3$ fm^{-1}, where the dominant contribution comes from the deutron charge form factor, the second rank interaction potential describes the experimental data [22] in a much better way, as compared with the potential of the first rank. The value of $\langle r^2\rangle^{1/2}=1.95$ fm, determined with the potential of Eq.(12) with $u_{it}(k)$ borrowed from Table 1, is also in agreement with the experimental rms radius of the deutron, $\langle r^2\rangle^{1/2}=1.9635\pm0.0045$ fm [8].

Now, the scattering phase shifts, given in accord with ref. [7] up to 1 GeV and continued in a certain way into the region of higher energies, may be described with interaction potentials of the third rank. The simplest version of such potentials is obtained by adding the other term, $-g_{3\mu}u_{3\mu}(p)u_{3\mu}(p')$, to the second rank potentials of Eq.(12) with $u_{i\mu}(k)$, $i=1,2$ from Table 1. Here $g_{3\mu}=-1$ and $u_{3\mu}(k)$ are reproduced in terms of the negative parts of $\delta_\mu(k)$ by a relation, similar to that of Eq.(5). The functions $u_{3\mu}(k)$ are much dependent on the choice of the asymptotic behaviour of $\delta_\mu(k)$ at energies higher than 1 GeV. For this reason, their values are not given in Table 1. For a definite choice of the asymptotics, the functions $u_{3\mu}(k)$ are sketched in fig.4. The inverted potentials provide a unified description of all the available S-wave two-nucleon and low-energy three-nucleon data. This is a consequence of the fact that for each μ, the function $u_{3\mu}(k)$ is orthogonal to both functions $u_{1\mu}(k)$ and $u_{2\mu}(k)$ and, thus, insertion of the third term into (12) is of minor

importance for the low-energy three-nucleon calculations.

7. CONCLUSION

We conclude with the following statements: (i) The tritium binding energy decreases if an attractive short-ranged term is added, under the conditions of phase equivalence, to the interaction potential and increases in the case of a repulsive addition. (ii) In general, the t-matrix off- and on-shell variations produce the same, by the order of magnitude, effect on three-nucleon quantities. With the (reasonable) t-matrix off-shell variations in the region of momenta higher than 0.7 fm^{-1}, the changes in the tritium binding energy value are of about 2 MeV. (iii) The effect on the low-energy three-nucleon parameters of taking into account the negative parts of the N-N scattering phase shifts is found to be extremely small, provided that no appreciable changes occur in the t-matrix off-shell behaviour in the region of small momenta; otherwise the effect is strong enough. (iv) Using the inverse problem method has enabled us to reconstruct simple potentials of the S-wave interaction, which produce a unified description of the two-nucleon and the low-energy three-nucleon data.

The conclusions are based on the investigations with the superposition of separable interaction potentials. In the case of a sum of local and separable potentials, the behaviour of the three-nucleon parameters will be still determined by the overlapping character of the potentials involved. One should expect the above results to be reproduced, accordingly. From the phenomenological standpoint, one of the main problems is to determine the lowest values of the momenta, for which the two-nucleon interaction potential may be referred to as nonlocal. Inded, if these values are sufficiently low, say, about 0.7 fm^{-1}, then too much freedom is available for the t-matrix off-shell variations. For instance, for any reasonable, e.g. "realistic", potential, there can be found a nonlocal correction, effective in the region of higher momenta that, when being added to the given potential, would produce a correct value for the tritium binding energy, without any changes in the t-matrix on-shell behaviour.

Probably, the above problem migth be solved by means of simultaneous description of all the low- and intermediate-energy data for the two- and three-nucleon systems.

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FIGURE CAPTIONS

Fig.1: The first rank interaction potential formfactors.

Fig.2: The quartet n-d scattering phase shift. The line indicates the calculations with the interaction potentials of Eqs.(1) and (12); the experimental points are from ref.[10].

Fig.3: The dependence on E^{x}_{lab} of the tritium binding energy. The solid line is for the on-shell variations (variationsin in the NN scattering phase shifts, the dashed lineis for the off-shell variations.

Fig.4: The second rank interaction potential form factors $u_{1\mu}(k)$. The lolid lines are for i=1, the dashed lines arefor i=2. The dotted lines represent the additionalform factors $u_{3\mu}(k)$ of a third rank potential.

Fig.5: The doublet n-d scattering phase shift for the second rank interaction potential; $x=k\cot\delta_2(k)/|k\cot\delta_2(k)|_{thr}$. The experimental points are from ref.[10].

Fig.6: The elastic deuteron form factor $F^2_d(q)$. Lines 1 and 2 indicate the calculations with the first and the second rank interaction potentials, respectively. The points [22] show the elastic structure function A(q).

Table 1

The form factors of the interaction potential of Eq.(12) ($g_{1\mu}=g_{2\mu}=1$); k is in fm^{-1} and $u_{i\mu}(k)$ is in $fm^{1/2}$

k	$u_{1t}(k)$	$u_{2t}(k)$	k	$u_{1s}(k)$	$u_{2s}(k)$
0	5.922	0	0	5.306	0
0.253	5.738	0.054	0.278	5.074	0.120
0.588	5.153	0.341	0.489	4.577	0.601
0.704	4.890	0.574	0.637	3.767	1.604
0.829	4.449	0.984	0.693	2.958	2.110
0.943	3.826	1.532	0.764	1.953	2.558
1.075	2.379	2.172	0.841	1.365	2.670
1.265	1.375	2.299	0.926	1.186	2.598
1.455	1.007	1.989	1.019	1.069	2.450
1.644	0.764	1.596	1.158	0.918	2.161
1.876	0.471	1.047	1.390	0.639	1.540
2.120	0	0	1.696	0	0

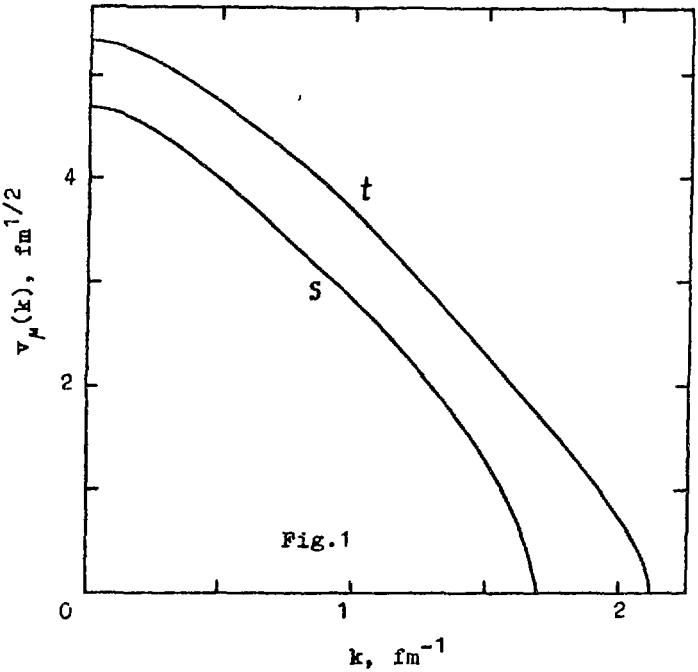
t
S
$v_\mu(k)$, $fm^{1/2}$
k, fm^{-1}
0
1
2
4

Fig.1

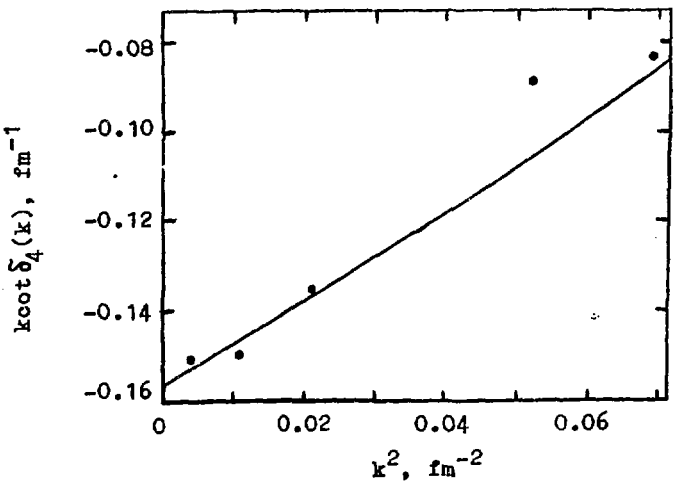
$k\cot\delta_4(k)$, fm^{-1}
k^2, fm^{-2}
-0.08
-0.10
-0.12
-0.14
-0.16
0
0.02
0.04
0.06

Fig.2

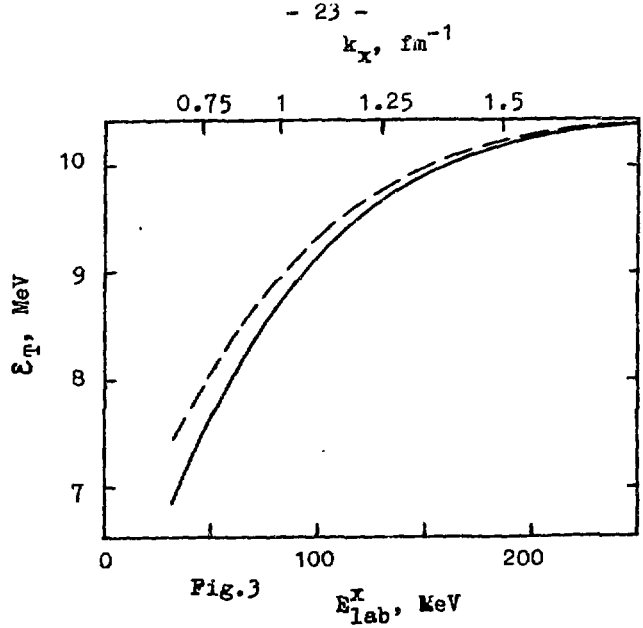
k_x, fm^{-1}
0.75
1
1.25
1.5
10
9
8
7
$\mathcal{E}_T$, MeV
0
100
200
E^x_{lab}, MeV

Fig.3

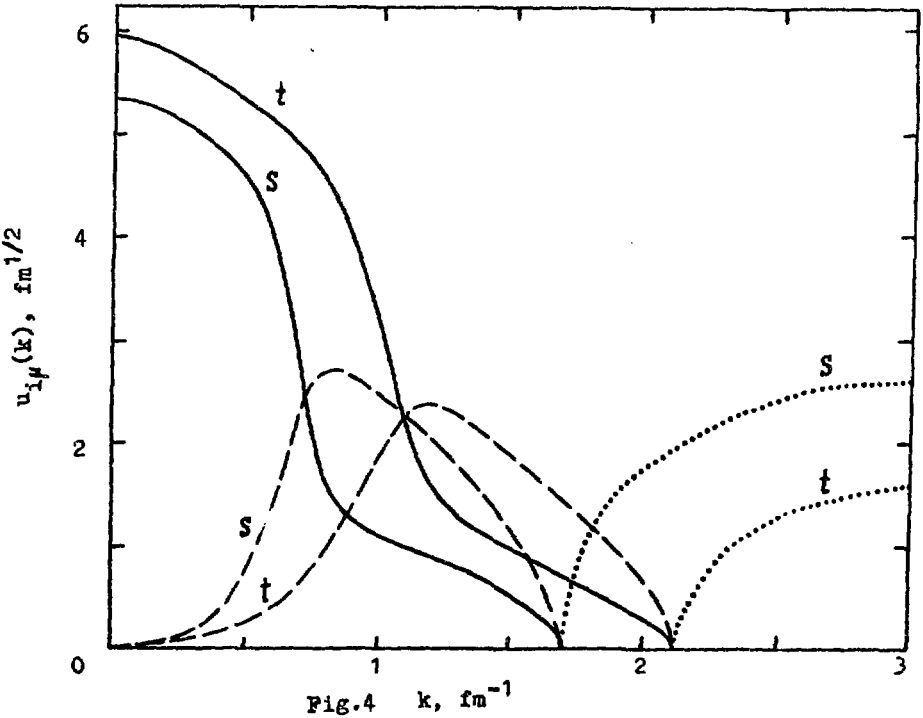
6
4
2
0
$u_{i\mu}(k)$, fm$^{1/2}$
t
S
S
t
S
t
1
2
3
k, fm^{-1}

Fig.4

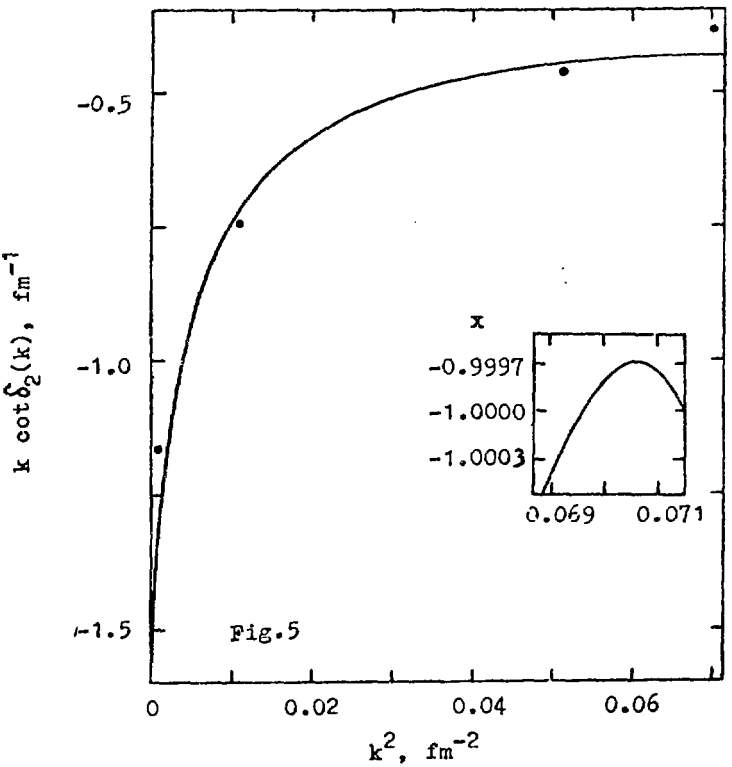
-0.5
-1.0
-1.5
k cotδ$_2$(k), fm^{-1}
0
0.02
0.04
0.06
k^2, fm^{-2}
x
-0.9997
-1.0000
-1.0003
0.069
0.071

Fig.5

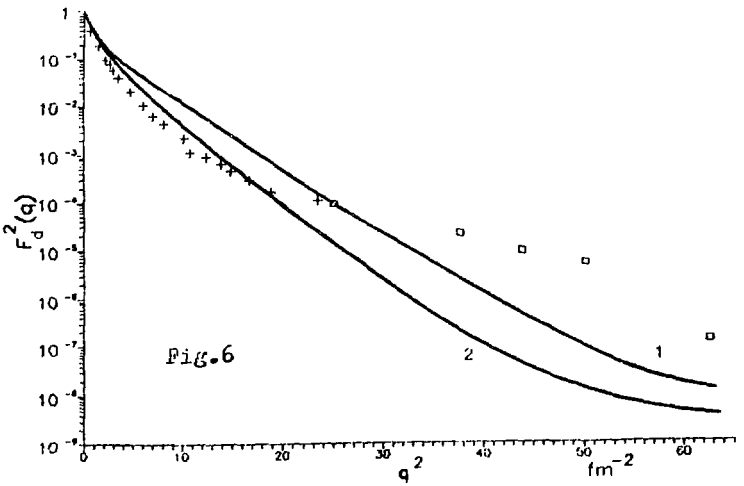
F$_d^2$(q)
q^2
fm^{-2}
1
2

Fig.6

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