BARC/1992/E/011

BARC/1992/E/011

सत्यमेव जयते

KINETIC ANALYSIS OF SUB-PROMPT-CRITICAL REACTOR ASSEMBLIES

by

S. Das
Theoretical Physics Division

1992

BARC/1992/E/011

BARC/1992/E/011

GOVERNMENT OF INDIA
ATOMIC ENERGY COMMISSION

KINETIC ANALYSIS OF SUB-PROMPT-CRITICAL REACTOR ASSEMBLIES

by

S. Das
Theoretical Physics Division

BHABHA ATOMIC RESEARCH CENTRE
BOMBAY, INDIA
1992

BARC/1992/E/011

BIBLIOGRAPHIC DESCRIPTION SHEET FOR TECHNICAL REPORT

(as per IS : 9400 - 1980)

01	Security classification :	Unclassified
02	Distribution :	External
03	Report status :	New
04	Series :	BARC External
05	Report type :	Technical Report
06	Report No. :	BARC/1992/E/011
07	Part No. or Volume No. :	
08	Contract No. :	
10	Title and subtitle :	Kinetic analysis of sub-prompt-critical reactor assemblies
11	Collation :	14 p., 4 figs., 1 tab.
13	Project No. :	
20	Personal author(s) :	S. Das
21	Affiliation of author(s) :	Theoretical Physics Division, Bhabha Atomic Research Centre, Bombay
22	Corporate author(s) :	Bhabha Atomic Research Centre, Bombay - 400 085
23	Originating unit :	Theoretical Physics Division, BARC, Bombay
24	Sponsor(s) Name :	Department of Atomic Energy
	Type :	Government
30	Date of submission :	May 1992
31	Publication/Issue date :	June 1992

Contd... (ii)

40 Publisher/Distributor : Head, Library and Information Division, Bhabha Atomic Research Centre, Bombay

42 Form of distribution : Hard Copy

50 Language of text : English

51 Language of summary : English

52 No. of references : 6 refs.

53 Gives data on :

60 Abstract : Neutronic analysis of safety-related kinetics problems in experimental neutron multiplying assemblies has been carried out using a sub-prompt-critical reactor model. The model is based on the concept of a sub-prompt-critical nuclear reactor and the concept of instantaneous neutron multiplication in a reactor system. Computations of reactor power, period and reactivity using the model show excellent agreement with results obtained from exact kinetics method. Analytic expressions for the energy released in a controlled nuclear power excursion are derived. Application of the model to a Pulsed Fast Reactor gives its sensitivity between 4 and 5.

70 Keywords/Descriptors : SUBCRITICAL ASSEMBLIES; REACTOR KINETICS; REACTOR SAFETY; REACTOR PERIOD; CRITICALITY; REACTIVITY; EXCURSIONS; POWER DISTRIBUTION; KALPAKKAM PFR REACTOR; SENSITIVITY; DELAYED NEUTRONS; MATHEMATICAL MODELS; NUCLEAR ENERGY

71 Class No. : INIS Subject Category : E3600; E3500

99 Supplementary elements :

KINETIC ANALYSIS OF SUB-PROMPT-CRITICAL REACTOR ASSEMBLIES

S. DAS

Theoretical Physics Division, Bhabha Atomic Research Centre
Trombay, Bombay-400085, India

1. INTRODUCTION

In predicting the dynamic behaviour of nuclear fission reactors, two main approaches are usually followed: the space-time kinetics approach and the point kinetics approach. While the space-time method is used for reactors which are spatially decoupled, point kinetics method is used for small-sized reactor systems (tightly coupled cores) with small perturbations [1].

This paper is not concerned with these aspects; instead, it deals with the development and applications of a simple sub-prompt-critical reactor model to the solution of a number of safety-related problems in neutron kinetics. The model, which is based on the concept of a sub-prompt-critical nuclear reactor and the concept of instantaneous neutron multiplication in a reactor assembly, provides an alternative approach to doing kinetic analysis of sub-prompt-critical reactor assemblies using point kinetics method. The applications to be discussed here include three research reactors: a fast reactor, a thermal reactor and a repetitively pulsed fast reactor.

2. THE SUB-PROMPT-CRITICAL NUCLEAR REACTOR MODEL

Consider a nuclear reactor which is sub-delayed critical by an amount $\Delta k<0$ with an extraneous neutron source emitting neutrons at the constant rate of S_{ex} neutrons per second. At steady-state equilibrium, the multiplied neutron production rate $S_{ex}/\|\Delta k\|$ equals the neutron loss rate (n/l_p) from the assembly and the balance equation in one-group, space-average approximation is:

$$n(t)/l_p=S_{ex}/\|\Delta k\|$$

where, $n(t)$ is the instantaneous average neutron population in the whole reactor, l_p is the mean prompt neutron life time and $\|\Delta k\|$ denotes the absolute value of Δk. The corresponding fissioning rate $F(t)$ in the reactor is given by:

$$F(t)=n(t)/(l_p\bar{\nu}_p)$$

where, $\bar{\nu}_p$=average number of neutrons emitted per fission. Combining the above two equations, we get:

$$F(t)=S_{ex}/(\bar{\nu}_p \|\Delta k\|) \tag{1}$$

We now extend this definition to include the super-delayed-critical region and look upon the whole reactor as a prompt sub-critical system (i.e.,a system which is sub-critical with respect to prompt neutrons), in which delayed neutrons act as an additional source $[S_d(t)]$ of neutrons in addition to the extraneous neutron source, S_{ex}. Following eq.(1), the instantaneous fission power in such a system $(k<1+\beta_{eff})$ can be written as:

$$F(t)=[S_{ex}+S_d(t)]/[\bar{\nu}_p \|\Delta k_p(t)\|] \tag{2}$$

where,

$S_d(t) = \sum_i \lambda_i c_i(t)$ is the instantaneous delayed neutron source in neutrons per second,

$\Delta k_p(t)=-[\beta_{eff}-\Delta k(t)]$ is the instantaneous degree of sub-prompt criticality,

$c_i(t)$ =the instantaneous population of the ith precursor group

β_{eff} =total effective fraction of delayed-neutrons,

λ_i =the decay constant of the ith precursor group.

Equation (2) shows that for a pre-assigned variation of $\Delta k_p(t)$, $F(t)$ and from this instantaneous reactor period $[F/(dF/dt)]^p$ and integrated fission energy $[\int F(t)dt]$ can be calculated if the time variation of delayed-neutron source $S_d(t)$ is known. Conversely, if $F(t)$ is given and $S_d(t)$ is known, $\Delta k_p(t)$ and hence $\Delta k(t)$ can be obtained from:

$$\Delta k_p(t)=[S_{ex}+S_d(t)]/[\bar{\nu}_p F(t)] \tag{3}$$

In either case, we need to know the delayed-neutron source term $S_d(t)$. This is calculated by calculating $c_i(t)$ from the equation:

$$c_i(t)=c_i(t_0)\exp(-\lambda_i t)+[\beta_{ieff}\bar{\nu}_p \int_0^t F(t)dt]\exp(-\lambda_i t) \tag{4}$$

In equation (4), the first term represents the contribution coming from the exponential decay of precursors and the second term is the production term during the time interval (0,t). The exponential factor in the second term accounts for the decay of the additional delayed-neutron precursors produced during the time interval. In the numerical computations, the expression used for calculating $c_i(t)$ was:

$$c_i(t)=c_i(t_0)\exp(-\lambda_i \Delta t)+\beta_{ieff}\bar{\nu}_p[\{F(t)+F(t_0)\}/2]\exp(-\lambda_i \Delta t) \tag{5}$$

Here,

$\Delta t = t - t_0$ is the integration time step

t_0 = the previous time point and

β_{ieff} = the effective delayed-neutron fraction of the ith precursor group

Instantaneous inverse reactor period, $\bar{\alpha}(t)$ when reactivity is added into or removed from the reactor linearly at a constant rate, a was calculated from the expression

$$\bar{\alpha}(t) \overset{Def}{\equiv} (1/F)(dF/dt)$$
$$= \pm[a/\|\Delta k_p(t)\|] + [dS_d/dt/(S_{ex}+S_d(t))] \qquad (6)$$

which was obtained by using eq.(2), its first derivative and the equation for $\Delta k_p(t) = -(\Delta k_0 - \beta_{eff} \pm a.t)$. For step reactivity perturbations, $d\|\Delta k_p(t)\|/dt=0$ and

$$\bar{\alpha}(t) = dS_d/dt/[S_{ex}+S_d(t)] \qquad (7)$$

In eqs.(6) and (7), $dS_d(t)/dt = \sum_i \lambda_i dc_i/dt$ was calculated by computing dc_i/dt from the expression:

$$dc_i/dt = -\lambda_i c_i(t_0)\exp(-\lambda_i \Delta t) + $$
$$+0.5\beta_{ieff}\bar{\nu}_p[F(t)+F(t_0)]\exp(-1.5\lambda_i \Delta t) \qquad (8)$$

Whenever computations began from equilibrium critical, initial precursor concentration at t=0 was calculated from the expression:

$$c_{i0} = (k_0 F_0 \bar{\nu}_p \beta_{ieff})/\lambda_i \qquad (9)$$

where,

$$k_0 = 1 - [S_{ex}/(F_0 \bar{\nu}_p)].$$

A six group (i=1,2,...,6) mathematical model for the delayed-neutron precursors was used in all the computations and values of λ_i's and β_i's were taken from Tuttle [2]. A conversion factor of 3.1×10^{10} was used for converting reactor power in watts to fissions per second. An equal neutron effectiveness was assumed for all the six precursor groups. It was unity for the ^{239}Pu fuelled fast system and 1.25 for the ^{233}U fuelled thermal system. The corresponding values of $\bar{\nu}_p$ used in the calculations were 3.09 and 2.48 respectively. For computing reactor power and period, eqs.(2),(5),(6),(7),(8) and (9) were used with an integration step size, Δt of either 0.01 s or 0.001 s. For reactivity computations, eqs.(3),(5),(8) and (9) were used with Δt=0.01 s. For doing the exact kinetics calculations, two point-kinetic computer programs developed by the author were used: OPTIF for the steady-flux systems and PULFR for the pulsed system. In these programs, the point kinetics equations were solved numerically using the method of Optimum Integrating Factor [3].

Before we go to the applications of the model to the solution of safety-related kinetic problems in reactors, we present in figure A two plots of the effective source strength variation calculated from the experimental power and reactivity data obtained in one (CRAC-23) of the French kinetic experiments [4]. The continuous curve is the result of point by point multiplication of the experimental P(t) and $\Delta k_p(t)$ values. The broken curve is obtained by computing the source using eq.(4) with the initial value of $c_i(t_0)$ taken at the time where power reached its minimum after the first large prompt critical burst. Thereafter, the experimental power data were used to compute $c_i(t)$ and from this the source strength [$=\sum \lambda_i c_i(t)$]. The very good agreement between the two demonstrates the validity of the basic model [Eqs.(2) and (4)]. It may be noted that the variation in the effective source strength is only about (30-40)%. But in the experiment, the oscillations in power changed by a factor of ten.

3. COMPUTATIONS OF REACTOR POWER

Results of reactor response calculations following positive step reactivity insertions both with and without an extraneous source showed that the agreement between the present model and the exact kinetics method was within a few per cent. For negative step changes in reactivity, the agreement was even better. Fig.1 gives plots of computed reactor power vs. time/reactivity in the ^{233}U fuelled thermal system at equilibrium critical for positive linear reactivity insertion rates of 20 cents/s and 10 dollars/sec. Dots and crosses were obtained by the present model. It is seen that upto Δk of about 80 cents, agreement between the present model and the exact kinetics calculations is to within a few per cent. Beyond that, predictions of the present model are unreliable.

4. INSTANTANEOUS REACTOR PERIOD

Fig.2 shows plots of computed instantaneous inverse reactor period as a function of reactivity in a ^{239}Pu fuelled experimental fast reactor for three different ramp reactivity insertion rates. The points (dots, crosses and squares) were obtained using the present model. It is seen that in all these cases, the agreement between the present method and the exact kinetics method is very good being to within a few per cent. Similar agreement was observed for step reactivity changes.

In eq.(6), the first term represents the prompt neutron part and the second term the delayed-neutron part. If variation of delayed-neutron source during the time reactivity change is made is small compared to the total neutron level, eq.(6), to a first approximation, becomes:

$$\bar{\alpha}(t) = a/\|\Delta k_p(t)\|$$

The above equation shows that the instantaneous period of a reactor is independent of the kinetic parameters l_p and β_{eff}, and depends only on the rate of reactivity variation and the instantaneous state of sub-prompt criticality of the reactor. Since it is customary for the reactor period trip to be set at 20 s, we have the inequality

$$a(t) < 0.05\ \|\Delta k_p(t)\|$$

The above relation shows that as a reactor moves towards delayed critical, i.e., as $\Delta k_p \longrightarrow \beta_{eff}$, reactivity addition rate should continuously decrease to avoid period scrams.

5. ESTIMATION OF REACTOR REACTIVITY

Fig.3 shows plots of reactivity obtained by the present method of analysis (dotted curve) and the exact inverse kinetic method (continuous curve) by analyzing the same experimental linear channel current data in a kinetic experiment in which the coarse control rod was withdrawn in the Purnima zero energy fast reactor at 468 mW delayed critical and then inserted back symmetrically so as to restore delayed criticality, thereby, subjecting the reactor to a V-type of reactivity perturbation. An extraneous neutron source of effective strength 4.5×10^6 n/s, which was appropriate to the assembly, was used in the analysis. It is seen from the figure that the agreement between the present result and the result obtained from the inverse kinetic analysis [5] is excellent, the maximum discrepancy being about 3% which occurs around the time the control rod is in its minimum reactivity position. In the present analysis, reactor power data were read at equal time intervals of 5 s and data at intermediate time points were computed using a quadratic interpolation formula.

6. ENERGY RELEASE IN A CONTROLLED NUCLEAR POWER EXCURSION

In dynamics and safety analysis of nuclear reactors, one of the most important quantities that determines the consequences of a criticality accident is the magnitude of the total energy released in the accident. For the safe design and operation of nuclear reactors, it is necessary that in the event of an inadvertent reactivity input into the system followed by shut-down mechanism action, the total energy released into the system should be less than a certain threshold value so that damage to the core does not occur. To gain insight into the physics of the system dynamics, we divide the total energy released into the reactor during the controlled power excursion as sum of three components:

$$Q_{total} = Q_1 + Q_2 + Q_3$$

where, Q_1 = the energy released from the time reactivity accident is initiated upto the time reactivity removal by safety/control system commences, Q_2 = the energy released during reactivity removal by the fast shut-down system and Q_3 = the post shut-down energy liberated after the safety mechanism action is complete. Although, these quantities are usually computed, it is possible to derive closed form expressions for Q_2 and Q_3. In this paper, we derive an expression for Q_3.

Once the safety mechanism action is complete and the reactor is brought under control, the system is left at a steady sub-delayed critical level Δk (t=0). Thereafter, reactor power falls due to the decay of delayed-neutron precursors (c_{i0}) which were produced during the accident. Neglecting changes in degree of subcriticality due to temperature variations in the shut-down state and neglecting the extraneous source power (S_{ex}) which is negligible compared to the total reactor power, we write using eq.(2):

$$Q_3 = \sum_{i=1}^{6} \int_0^{\infty} (\text{power due to the ith group of delayed-neutron precursors}) \exp(-\lambda_i t) dt$$

$$= \sum_{i=1}^{6} \int_0^{\infty} (\lambda_i c_{i0}/\Delta k_p \bar{\nu}_p) \exp(-\lambda_i t) dt$$

If the reactor shut-down mechanism is sufficiently fast to bring the power excursion under control quickly, decay of delayed-neutron precursors during the excursion can be ignored and c_{i0} can be assumed to be proportional to the total integrated fissions (Q_1+Q_2) upto that time. Therefore,

$$Q_3 = \sum_{i=1}^{6} \int_0^{\infty} (\lambda_i/\Delta k_p \bar{\nu}_p)(Q_1+Q_2)\bar{\nu}_p \beta_{ieff} \exp(-\lambda_i t) dt$$

Integrating and simplifying,

$$= [(Q_1+Q_2)/\Delta k_p] \sum_{i=1}^{6} \beta_{ieff}$$

$$= (Q_1+Q_2)/\Delta k_p(\$) \qquad (10)$$

where, $\beta_{eff} = \sum_{i=1}^{6} \beta_{ieff} = 1\$$

Eq.(10) shows that if the energy released in the sub-delayed-critical phase(Q_3) is to be less than the energy released in the super-delayed-critical phase(Q_1+Q_2), the shut-down reactivity margin $[\Delta k_p(\$)]$ should be greater than 1\$. This explains the importance of the first few dollars of reactivity removal by the safety mechanism. Using eq.(10), we write

$$Q_{total} = Q_1+Q_2+(Q_1+Q_2)/\Delta k_p(\$)$$

$$= (Q_1+Q_2)[1+1/\Delta k_p(\$)]$$

It follows from the above relation that by making (Q_1+Q_2) sufficiently small in magnitude, it is possible to ensure safety of a nuclear reactor by making it only a fraction of a dollar sub-delayed critical.

7. DETERMINATION OF THE FICTITIOUS DELAYED-NEUTRON FRACTION, β' AND THE SENSITIVITY OF A PULSED FAST REACTOR

A pulsed reactor of periodic operation is a facility for producing controlled and recurrent bursts of nuclear fission to do neutron beam research. It is different from a steady-state reactor in that it is controlled by prompt neutrons. The reactor is pulsed periodically by the periodic movement of the position of a neutron-reflecting block which makes the reactor prompt supercritical by a small amount for a short duration. The neutron source before the pulse, coming from the decay of delayed-neutron precursors born in all the previous pulses, is multiplied during this time to give a fast neutron power pulse. During the pulse, new delayed neutron precursors are produced. Between the pulses, the delayed-neutrons coming from the decay of precursors are prompt multiplied by the reactor giving rise to the background power. Also, the increment in the precursor concentration gained during the pulse decays. When the precursor production during a pulsation period is equal to the precursor decay during the same period, an equilibrium operating cycle is established and the reactor is said to be pulsed critical.

For small reactivity changes, the mean power kinetics of such a repetitively pulsed reactor is similar to that of a conventional constant power reactor provided we use a fictitious delayed-neutron yield, β' [6]. The quantity β_{eff}/β' is called the sensitivity of a periodically pulsed reactor and is an index of how much more sensitive is a reactor to small reactivity changes near criticality in pulsed mode than in steady-state mode of operation. This increased sensitivity is taken into consideration while designing a repetitively pulsed reactor.

Consider a pulsed reactor operating at pulsed criticality with a mean fission power, $\bar{P}$. Using eq.(2), we write

$$\bar{P}=(S_{ex}+S_d)/\beta'$$

Let $\bar{P}'$ be the mean power immediately after the prompt-jump following the introduction of a small positive step reactivity of $\Delta\epsilon_m \ll \beta'$. During this time interval, which is of the order of milliseconds or less, the concentration of the delayed-neutron precursors is practically invariant and we again write using eq.(2)

$$\bar{P}'=(S_{ex}+S_d)/(\beta'-\Delta\epsilon_m)$$

Combining the above two equations and simplifying, we get

$$\beta'=\Delta\epsilon_m[\bar{P}'/(\bar{P}'-\bar{P})] \qquad (11)$$

Table I gives β' and the sensitivity of the earlier proposed ^{239}Pu fuelled Kalpakkam Pulsed Fast Reactor (KPFR) design for two different sets of reactor parameters l_p and α (parabolic coefficient of reactivity). They were calculated using eq.(11) knowing $\Delta\epsilon_m$ and $\bar{P}'$. $\bar{P}'$ was computed numerically by inserting step reactivity in the range of 0.2-30 pcm (β_{eff}=1\$=200 pcm in the present calculation). It is seen from the table that the fictitious delayed-neutron fraction, β' of the KPFR lies between 35 and 46 pcm and the sensitivity between 4 and 6. This makes KPFR the least sensitive of the three reactors shown. Our calculations showed that β' varied by a small amount with $\Delta\epsilon_m$. Therefore, the value of β' used in calculating sensitivity was actually an average of these β' values.

Table I. Computed fictitious delayed neutron yield, β' and sensitivity of the earlier proposed KPFR design

Reactor	$\bar{P}$ (kW)	ϵ_0 (\$)	β_{eff} (pcm)	β' (pcm)	β_{eff}/β'
KPFR 1	30	-10	200	35	5.71
KPFR 2	30	-10	200	46	4.35
IBR			350	11	31.80
SORA			200	24	8.30

REFERENCES

(1) G.I. BELL and SAMUEL GLASSTONE: Nuclear Reactor Theory (Van Nostrand Reinhold Company, New York and London, 1970) Chaps. 9&10.

(2) R.J. TUTTLE: Nucl. Sci. Eng. 56 (1975) 37.

(3) P.A. SECKER (Jr.): Report LA-4252(1969).

(4) S. DAS: Report BARC/1992/E/014, Bhabha Atomic Research Centre, Bombay, India.

(5) S. DAS: Comp. Phys. Commun. 29 (1983) 321.

(6) I.I. BONDARENKO and YU. YA. STAVISSKII: Atomnaya Energiya 7 (1959) 417.

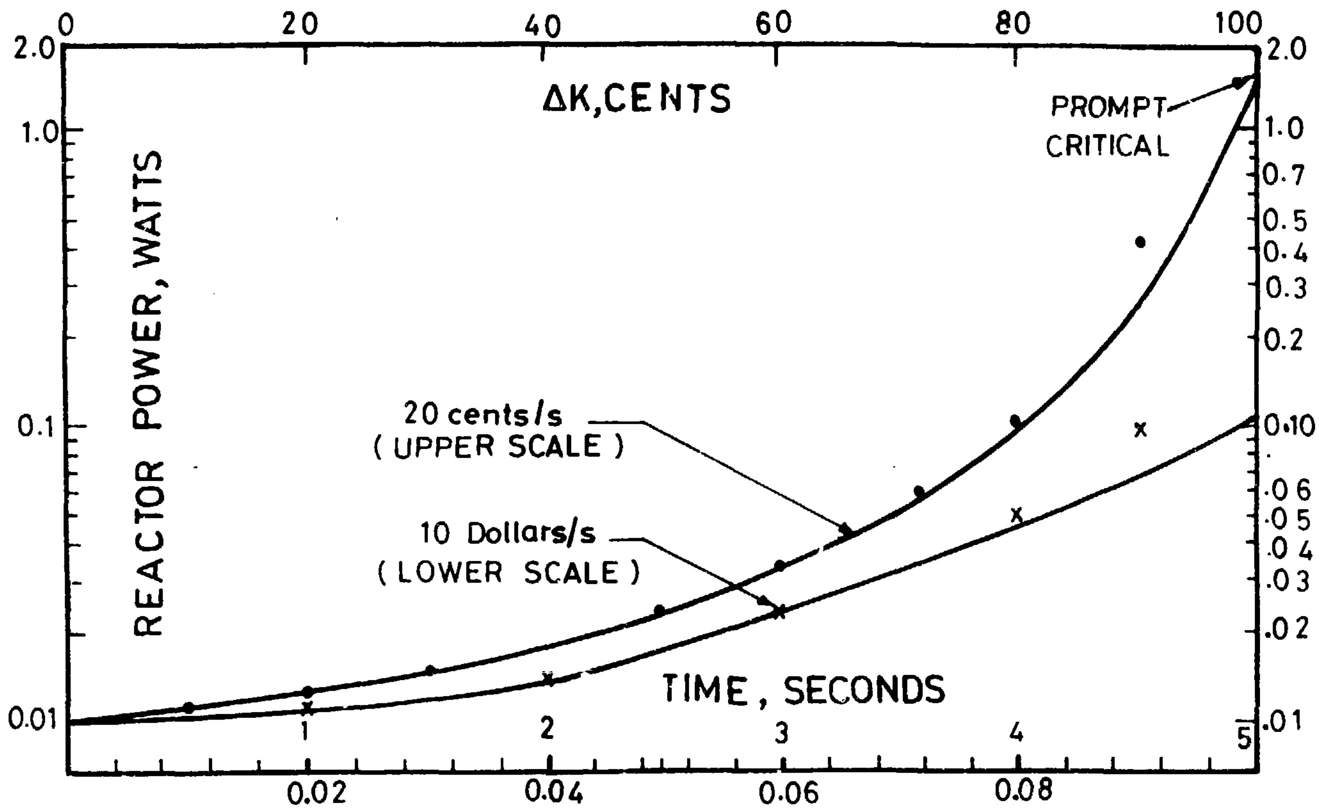

Fig. 1. Computed kinetic response to linear time variation of reactivity in a ^{233}U fuelled thermal system (l_p = 40 μsec, β_{eff} = 0.00325)

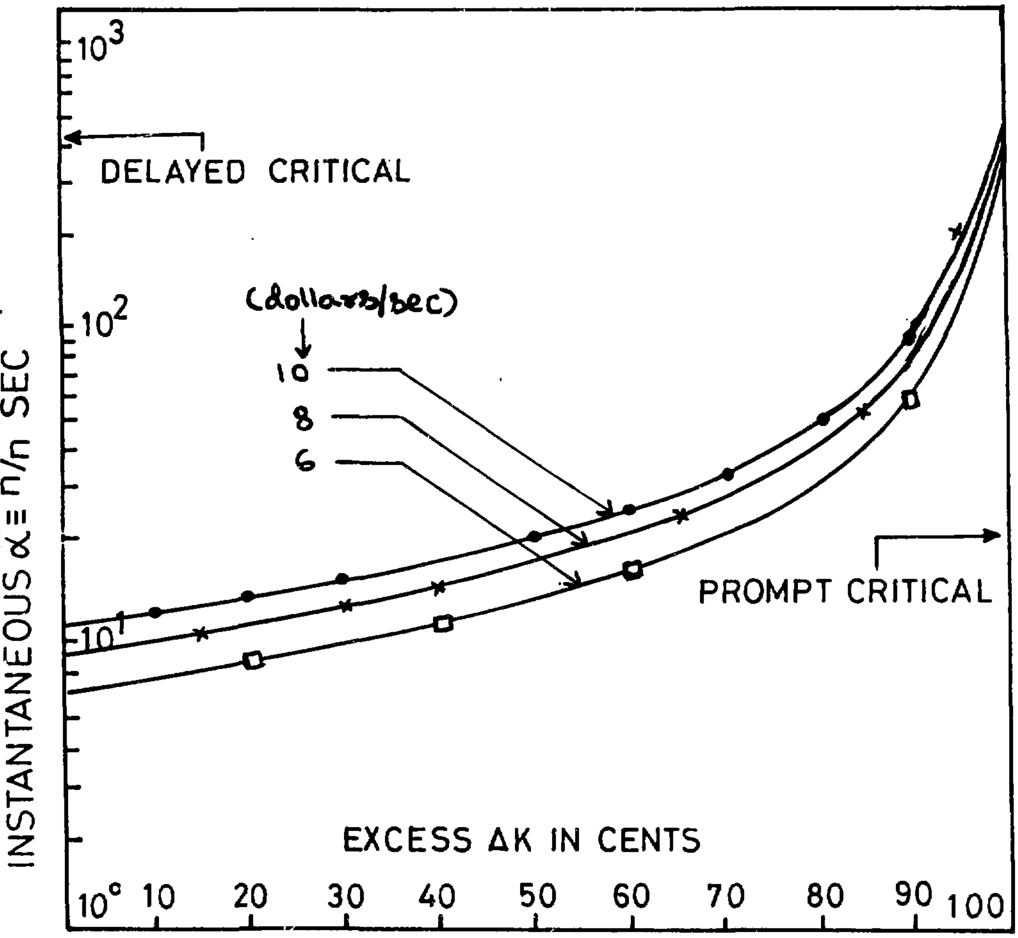

Fig. 2. Instantaneous inverse reactor period vs. reactivity for various ramp rates in a ^{239}Pu fuelled fast system (l_p = 50 n sec and β_{eff} = 0.002).

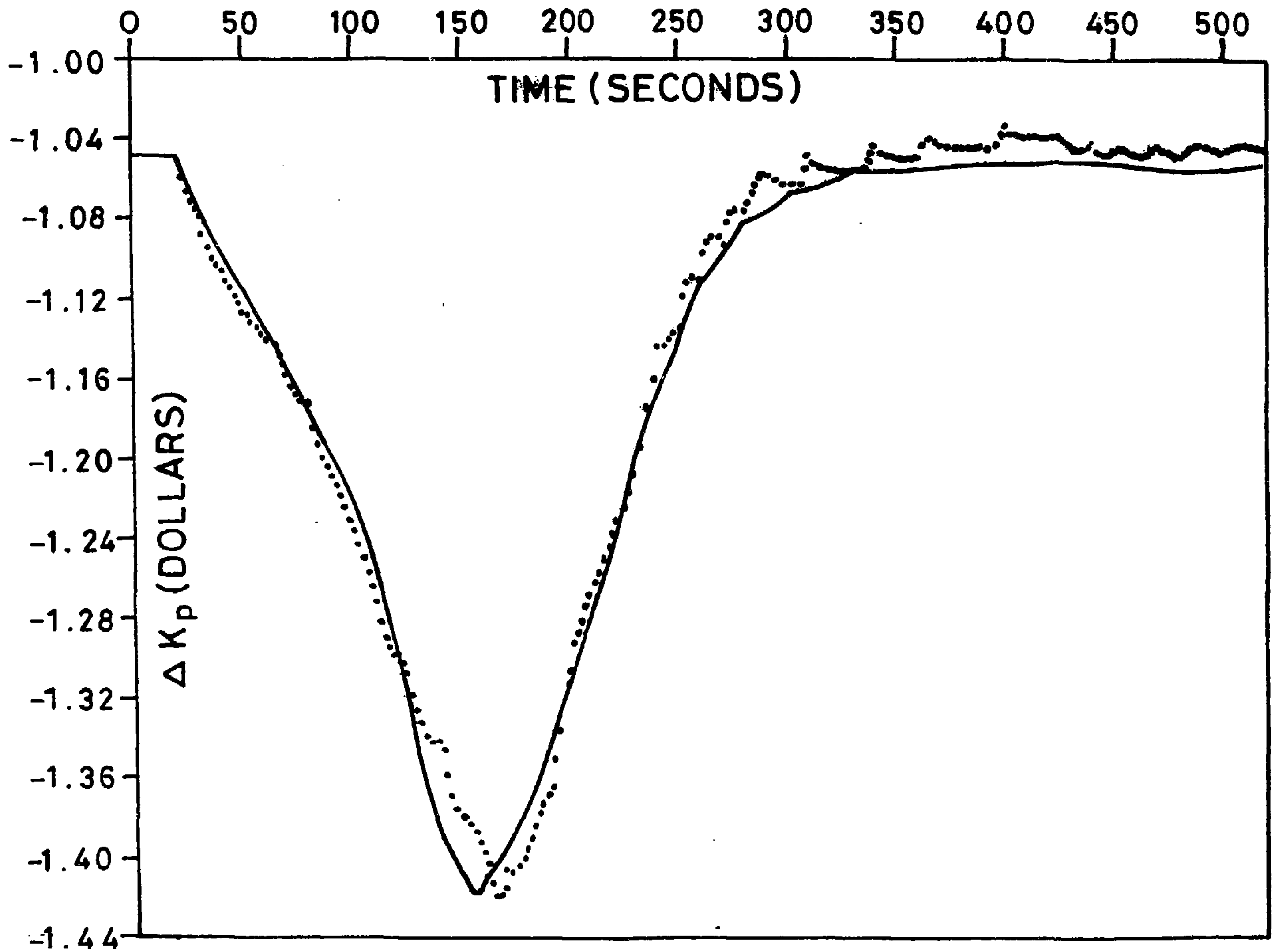

Fig.3. Computed time variation of a V-type reactivity perturbation from experimental channel current in PURNIMA fast reactor.

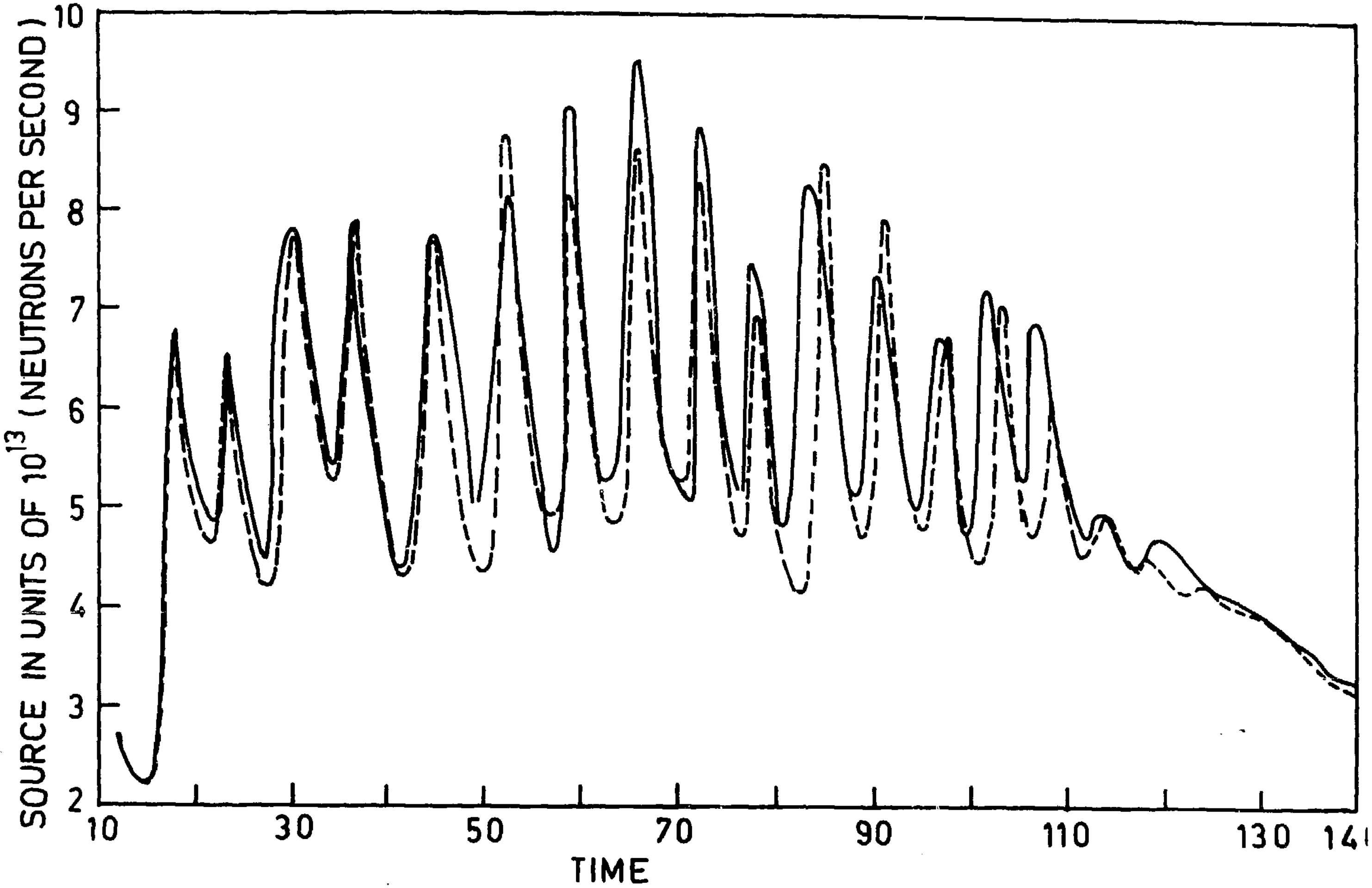

FIGURE A: COMPUTED TIME VARIATION OF EFFECTIVE SOURCE STRENGTH IN CRAC-23 EXPERIMENT

Published by : M. R. Balakrishnan Head, Library & Information Services Division
Bhabha Atomic Research Centre Bombay 400 085