VIETNAM NATIONAL ATOMIC ENERGY COMMISSION

VAEC - E - 010

THE SEMI-EMPIRICAL LOW-LEVEL BACKGROUND STATISTICS

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1992

ABSTRACT

A semi-empirical low-level background statistics was proposed. The one can be applied to evaluate the sensitivity of low background systems, and to analyse the statistical error, the "Rejection" and "Accordance" criteria for processing of low-level experimental data.

Vietnam National Atomic Energy Commission, Hanoi, 1992.

The most difficult point in applying statistical method to evaluate equipment, to lay out measuring methods, to process results in order to improve low-level background measurements is choosing distribution functions. It remains in so far a discussing problem [1-5].

In this work, we propose a semi-empirical low-level background statistics which is comfortable in applications for a large measuring group. The conditions to choose Counting time, Equipments, and procedures of sample and background measurement in order to replace distribution function $f(x)$ of counting rates by semi-empirical Gaussian $f_{em}(z)$ of a statistical fluctuation $z = (x - \bar{x})$ are the criteria for applying the statistics. The mean-square d_{em} of $f_{em}(z)$ will be defined by counting rates from the same considered single measurement. The theory was applied to calculate the statistical error, to evaluate the sensitivity of low background sysytems, and to analyse the "Rejection" and "Accordance" criteria for processing of low-level experimental data.

A. THEORY

I. Criteria for application.

We let x_f, x_t are respectively the background, the total counting rates measured during the time intervals t_f, t_t; $x_o = (x_t - x_f)$ is their difference; $n_i = n_{if}$, n_{it} are the decay rates of the radiators having respective contribution into x_f, x_t; ε_i - their respective recording efficiencies. In fact, for a large group of low-level background measurements the following conditions are acceptable:

1. All partial counting efficiencies ε_i are practically equal and the number of separate radiations q is less than 5.

$$\varepsilon_i = \varepsilon \; ; \; x = x_f, x_t = \varepsilon \sum_{i=1}^{q} n_i, \; q \leq 5$$

2. The distribution functions $f(\varepsilon)$ is so sharp a Gaussian that the relative error δ_ε is less than 15%.

$$f(\varepsilon) = [\exp\text{-}(\varepsilon\text{-}\bar{\varepsilon})^2/2d\varepsilon / \sqrt{2\pi d\varepsilon} \; ;$$

$$(\varepsilon - \bar{\varepsilon}) = -\infty \div +\infty \;, \; \delta\varepsilon = \sqrt{\bar{d\varepsilon}}/\bar{\varepsilon} \leq 15\%$$

3. The counting rates x_o, x_f satisfy the relation: $x_o \geq 6\sqrt{x_f/t_t}$.

4. The counting number $X_f = x_f.t_f$ is greater than 100 and $X_t = x_t.t_t$ is greater than 160.

II. The distribution function of counting rates f(x).

We show that the distribution function of counting rates $f(x)$, $x = x_o$, x_f, x_t can be approximated by Gaussians:

$$f(x) = [\exp-(x - \bar{x})^2/2d_x] / \sqrt{2\pi d_x}; \; (x - \bar{x}) = -\infty \div +\infty \qquad (1)$$

$$d_x = d_f, d_t, d_o; \; d_f = \mu\bar{x}_f/t_f; \; d_t = \mu\bar{x}_t/t_t; \; d_o = d_f + d_t; \; \mu = 1 + \bar{\varepsilon}$$

According to the criteria 3, 4 we get the relations:

$$X_o = x_o t_t \geq 60; \; X^*_f = x_f t_t \geq 100; \; X_t = x_t t_t \geq 160; \; X_f = x_f t_f \geq 100 \quad (2)$$

Then according to the criterion 1, all of the separate decay numbers N_i will be greater than 20. So that, all of distribution functions $f(N_i)$ can be approximated by Gaussians:

$$f(N_i) = [\exp-(N_i - \bar{N}_i)^2/2\bar{N}_i] / \sqrt{2\pi\bar{N}_i}, \; N_i - \bar{N}_i = -\infty \div +\infty$$

Due to the fact $f(\varepsilon)$ is Gaussian and the magnitudes (n_{if}, ε) or (n_{it}, ε) are statistically independent, the following relations will be valid:

$$\mu = (\bar{X}_f.\delta^2\varepsilon + \bar{\varepsilon}), \; (\bar{X}_t.\delta^2\varepsilon + \bar{\varepsilon})$$

Taking into account the ralations $\delta\varepsilon \leq 15\%$, X_f, $X_t \geq 100$ we get a basis to approximate all of distribution functions $f(x_f)$, $f(x_t)$ by Gaussians (1).

It should be kept in mind that the counting rates x_f, x_t in general are not statistically independent. It is the criterion 3 that is the condition for these magnitudes to be statistically independent. Physically it means that the sample component X_o^* of the total count X_t is twice the maximum statistical fluctuation $|X_f^* - \bar{X}_f^*|_{max}$ of the background component X^*_f. In that case, we can approximate x^*_f by x_f and x^*_o by x_o and get the results (1).

III. The semi-empirical distribution function.

In fact, the counting rates x_f, x_t of repeated measurements vary considerably depending on working conditions. Besides, each measurement often lasts from 1 to 10 hours. For these reasons, the idea to evaluate statistical errors by semi-empirical method seems to be rational [4]. Though, the statistics had been based on a distribution having unclear physicomathematical meanings and the obtained results have got a lot of difficulties in application. To correct the mistakes, we introduce a semi-empirical distribution function of the statistical fluctuation $z = (x - \bar{x})$:

$$f_{em}(z) = [\exp{-z^2/2d_{em}}] / \sqrt{2\pi\, d_{em}};\ z = x - \bar{x} = -\infty \ldots +\infty$$

$$d_{em} = d_{emf},\ d_{emt},\ d_{emo};\ d_{emf} = \mu x_f/t_f; \qquad (3)$$

$$d_{emt} = \mu x_t/t_t;\ d_{emo} = d_{emf} + d_{emt}$$

The following relation for d_x and d_{em} is obvious:

$$d_x - 2\mu\sqrt{d_x/t} \le d_{em} \le d_x + 2\mu\sqrt{d_x/t};\ \sqrt{d_{em}} - \sqrt{d_x} \le \mu\,/t \qquad (4)$$

Figure 1 shows 4 curves of the Gaussians $f(z)$ having the mean-squares: 1) $d_z = d_x - 2\mu\sqrt{d_x/t}$; 2) $d_z = d_x + 2\mu\sqrt{d_x/t}$; 3) $d_z = d_x$; 4) $d_z = d_{em}$. The 4-th curve is identified by that the mean-square d_{em} is a statistical magnitude.

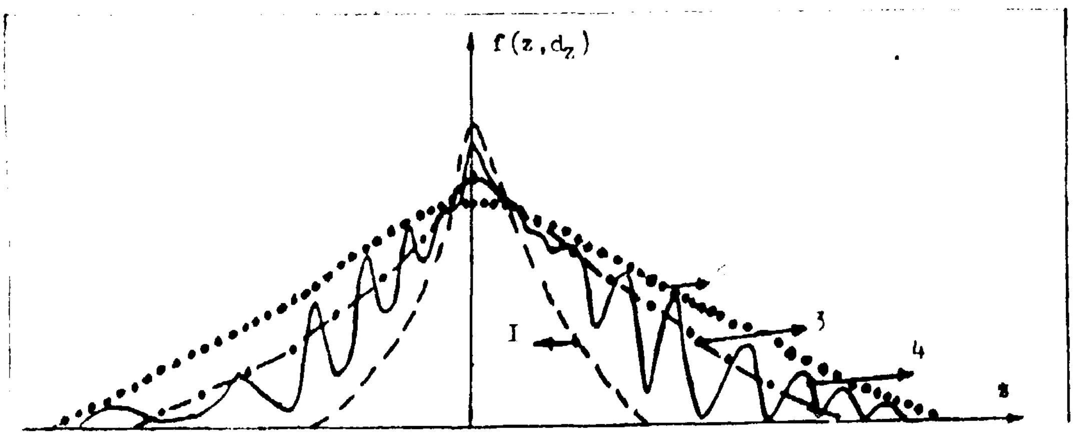

Figure 1.

B. APPLICATION OF THE THEORY

1. The semi-empirical error.

Basing on the Gaussian f(x) of the counting rates (1), we get the "Normalized" statistical error: $\sigma_x = \sqrt{d_x}$, $= \sigma_x/x$.

Basing on the semi-empirical Gaussian $f_{em}(z)$, we introduce a semi-empirical error

$$\sigma_{em} = \sqrt{d_{em}};\ \delta_{em} = \sigma_{em}/x.$$

Using the lower limit condition (2) we get the upper limits for δ_{em} as follows:

$$\delta_{emf} \leq 14\%,\ \delta_{emt} \leq 11\%,\ \delta_{emo} \leq 38\%.$$

The difference of the two standards can be valued by the parameter S:

$$S = |\ \delta_{em} - \delta_x\ | \leq \mu/xt.$$

From the relations (2), (4) we get: S_f, S_t, $S_o \leq$ 2%, 1.26%, 3.4%.

2. Sensitivity of low background equipment.

Sensitivity of low background systems can be defined by minimum beta-gamma activity A_{min} measured by the system during the preset time interval $T = t_f + t_t$ and with the relative error δ_{emo} of the sample counting rate x_o.

* A_{min} of the stable system.

It should be mentioned that the definition of a stable low-level system is still an opening problem. In this work, the system will be considered as stable when the modal values $x_{f,t}$ can be determined by measuring $x_{f,t}$ repeatedly.

A_{min} of the stable system has been calculated as follows:

$$A_{min} = \mu[1 + \sqrt{1+4.\overline{x}_f.\ \delta^2{}_{emo}.t_t/\mu}\ ] \times 1.7\times10^{-2}/\ 2\ \varepsilon\delta^2{}_{emo}.t_t\ (Bq)$$

where x_o is in numbers of pulses per minute; t - in minutes.

* A_{min} of the unstable system.

A_{min} of the unstable system has been determined by optimizing the ratio t_f/t_t in order to get the minimum absolute error δ_{emo}:

$$A_{min} = \mu\ [1 + 2\,\delta_{emo}.\sqrt{Tx_f/\mu}\ ] \times 1.7\times10^{-2}/2.\ \varepsilon\delta^2_{emo}.T\ (Bq).$$

3. "Accordance" criterion.

We suppose that a statistical magnitude Y is a function of an exact magnitude X. A set of numberical values (Y_i, X_i) was measured experimentally and another one $(Y_{thi} = Y(X_i))$ was calculated by theory. If the statistical fluctuation $Z_i = (Y_i - Y_i)$ obeys the semi-empirical Gaussians, we get the "Accordance" criterion for comparing theoretical and experimental results:

$$\chi^2 = \sum_{i=1}^{q} (Y_i - Y_{thi})^2/\mu Y_i,\ P_q(\chi^2) = \chi^{q-2}.\exp(-\chi^2)/2^{q/2}.\Gamma(q/2),$$

$$|\chi^2 - q| \leq a.\ \sqrt{2q};\ a = \int_{\chi_a}^{\infty} P_q(\chi^2)d\chi^2$$

The probability F1 of the first type mistake (when the criterion has been satisfied but the theory has been wrong) may be evaluated by the relation:

$$F1 \leq \mu / Y_i \leq 2\%$$

The probability F2 of the second type mistake (when the criterion has not been satisfied but the theory has been right) is equal to the previously given constant a. The value χ_a can be found in tables.

4. "Rejection" criterion.

We have considered the low background systems used to detect weak active sources [3] and supposed $x_1 \geq x_2$ are the two counting rates measured during the same time interval t but at two different moments. The non-statistical rejection criterion or the criterion for detecting radioactive traces is:

$$|x_1 - x_2| \geq q.\ \sqrt{x_2/t}.$$

It is the non-equality of the counting rates x_1, x_2 in the right side that is a weak point of this criterion. In the semi-empirical statistics this can be corrected. Assuming that x_1, x_2 obey the semi-empirical Gaussians and generally are not statistically independent, we get the semi-empirical

Rejection criterion:

$$|x_1 - x_2| \geq q \sqrt{\mu(x_1 + x_2).t}$$

The probability F1 is determined by the ralation $F1 \leq \mu / x_2.t \leq 2\%$. Whereas F2 is determined by the integral F(q):

$$F2 = F(q) = 2 \int_{qa}^{5a} f_{em}(z_o) dz_o ; f_{em}(z_o) = \int_{-b}^{b} f_{em1}(z_o + z_2).f_{em}(z_2) dz_2$$

$$a = \sqrt{\mu(x_1 + x_2)/t};\ b = 2\sqrt{\mu\, x_2/t}$$

Where q is a *constant and numerical* values of F(q) can be found in tables [5].

References

[1] Harrison, B.P.; Nucl. Instrum. and Methods; V.A.238, No.2-3, p.500; 1985

[2] Perkov A.I. and the others; Evaluating the Probability for Detecting Radioactive Traces; PTE. No.5; 1989 (in Russian)

[3] Verovkin A.D. and the others; Studying Gamma Traces of the Automatical Systems; Moscow, Nauka, 1983 (in Russian)

[4] Dementiev B.A.; Measuring Low-level Activities of Radioactive Preparations; Moscow, Atom, 1967 (in Russian)

[5] Prais B.; Recording Nuclear Radiations; In-lit., p.38; 1960 (in Russian)

Received on June 6, 1992

Published on June 21, 1992

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