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CP odd Observables for the tt System produced at pp and pp Colliders

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Abstract: We propose some CP odd observables to test CP invariance in the tt system produced at pp and pp colliders. Using these observables the effects of CP violation from the production and from the decay of the top quarks can be separated well. The asymmetry in the initial state at pp colliders unrelated to CP violation can only give suppressed contributions to our observables. To parametrize CP violating interactions we use an effective Lagrangian for the production and a general form factor approach for the decay.

1. Introduction

The top quark mass should not be smaller than 91 GeV according to recent experimental results[1]. Since a heavy top quark ($m_t > 120$ GeV) will on average decay too quickly to form hadronic bound states[2], t and $\bar{t}$ quarks produced simultaneously will decay separately into W^+b and $W^-\bar{b}$. In this paper we consider possible CP tests in the following processes:

$$\begin{aligned} p+\bar{p} &\to t+\bar{t}+X \\ p+p &\to t+\bar{t}+X\ , \end{aligned} \tag{1.1}$$

and the produced t and $\bar{t}$ quark will undergo the decay sequence:

$$\begin{aligned} t &\to W^+b,\ \ W^+ \to \ell^+\nu \\ \bar{t} &\to W^-\bar{b},\ \ W^- \to \ell^-\bar{\nu} \end{aligned} \tag{1.2}$$

According to the parton model the leading subprocesses for (1.1) are:

$$\begin{aligned} G+G &\to t+\bar{t} \\ q+\bar{q} &\to t+\bar{t} \end{aligned} \tag{1.3}$$

We will only consider these $2 \to 2$ subprocesses contributing to (1.1). CP violation can be detected in these subprocesses provided information about the spin polarization of the produced $t\bar{t}$ system can be obtained. Because the top quarks undergo weak decays dominantly, the polarization will reveal itself in the decays (1.2). Thus, the effect of CP violation in (1.3) can be detected. CP violation in the standard model leads to no effect in (1.3) up to two loop level at least and the effect is then small. CP violation can also appear in the decays. The standard model can give a nonzero effect of CP violation in (1.2) already at one loop level, but the effect is very tiny[3]. Therefore, any detected nonzero effect of CP violation in (1.3) and (1.2) will indicate some new sources of CP violation.

Based on the arguments mentioned above, CP violation in the $t\bar{t}$ system was studied [4,5,6] within the two Higgs doublet model[7]. In [4] CP violation in the decay (1.2) was studied. In [6] the effect of the absorptive parts in the production amplitude in (1.1) within the model was investigated. The production of $t\bar{t}$-pair at e^+e^--colliders was considered in [5,7]. We will study CP violation in (1.1) and (1.2) in a model-independent manner: For the production, we use an effective Lagrangian approach to parametrize CP violating effects in the dispersive part of the production amplitude. The purely gluonic vertex proposed in [9,10] and the chromo-dipole moment of top quarks can contribute to (1.3). For the decay a general form factor decomposition is used[3]. To test CP invariance, we will use the momenta of the charged leptons in (1.2) to construct CP odd observables, because these momenta can be easily measured in experiments. As CP violation can appear both in the production (1.3) and in the decays (1.2), it is important to separate the effects from each other by using different observables. These will be discussed in Sect. 2, and some comments on using the observables for pp colliders are also made. Numerical results for our observables are given in Sect. 3 together with their sensitivity to the new CP violating interactions. Sect. 4 gives a summary.

2. CP violating Interactions and CP odd Observables

We use the approach of an effective Lagrangian L_{CP} to parametrize possible CP violating interactions involved in the production (1.3). We impose $SU_C(3)$ gauge invariance on L_{CP} and neglect the masses of quarks except for top quarks. We collect in L_{CP} only operators with dimension $d \leq 6$. There are only two relevant operators under the above mentioned conditions:

$$L_{CP} = -\frac{i}{2} d_t \bar{t} \sigma^{\mu\nu} \gamma_5 \frac{\lambda^a}{2} t G^a_{\mu\nu} + i\lambda \mathrm{Tr}(G_{\alpha\beta} G_{\mu\nu} G_{\rho\sigma}) \varepsilon^{\alpha\beta\mu\rho} g^{\nu\sigma} \tag{2.1}$$

In (2.1) t denotes the top quark field and $G_{\mu\nu} = \frac{\lambda^a}{2} G^a_{\mu\nu}$ is the strength tensor of the gluon field. These two operators are well known. d_t is the chromo-dipole moment of the top quark. The other one is purely gluonic. The gluonic operator was found to give an important CP violating effect in 3 jet production at $p\bar{p}$ colliders[9] and was discussed extensively in [10,11,12] because it may give the main contribution to the electric dipole moment of the neutron. We will only consider the processes in (1.3) in the Born approximation of QCD. The contribution from L_{CP} to the density matrix for the production comes from the interference with the QCD amplitude and is given in the appendix. The corresponding Feynman diagrams are given in Fig. 1.

For CP violation in the decays we use a form factor approach. The CP violating form factors are defined in [3]. Neglecting the mass of b quarks only one form factor will contribute to our observables. This CP violating form factor gives a contribution to the $t \to W^+ b$ amplitude as follows

$$\frac{-e}{2\sqrt{2}\sin\theta_W} \epsilon^{*\mu}(p_{W^+}) \bar{u}(p_b) i\sigma_{\mu\nu} (p_t - p_b)^\nu \frac{1}{M_W} f_2^R (1+\gamma_5) u(p_t) \tag{2.2}$$

Its contribution to CP odd observables will be caused by interference with the decay amplitude from the standard model at tree level.

We propose two groups of observables to test CP invariance. One is:

$$\begin{aligned} O_o &= \hat{\mathbf{p}} \cdot (\mathbf{q}_+ \times \mathbf{q}_-) \\ O_L &= \hat{\mathbf{p}} \cdot (\mathbf{q}_+ \times \mathbf{q}_-) \hat{\mathbf{p}} \cdot (\mathbf{q}_+ - \mathbf{q}_-) \\ O_E &= \hat{\mathbf{p}} \cdot (\mathbf{q}_+ \times \mathbf{q}_-) \hat{\mathbf{p}} \cdot (E_- \mathbf{q}_+ - E_+ \mathbf{q}_-) \end{aligned} \tag{2.3}$$

Here, $\hat{\mathbf{p}}$ is the moving direction of the proton, $\mathbf{q}_+$ and $\mathbf{q}_-$ are the three-momenta of ℓ^+ and ℓ^- in (1.2), E_+ and E_- are the corresponding energies. All quantities refer to the c. m. system of the $p\bar{p}$ or the pp collider. The other group of observables is :

$$\begin{aligned} A_E &= E_+ - E_- \\ A_L &= (\hat{\mathbf{p}} \cdot \mathbf{q}_+)^2 - (\hat{\mathbf{p}} \cdot \mathbf{q}_-)^2 \end{aligned} \tag{2.4}$$

We calculated the expectation values of these observables analytically with the general form of the decay density matrix in [3] up to the integration over the parton distributions. Through these calculations we can show the following important properties for our observables:

1. Under CPT invariance, CP violation in the dispersive part of the decay amplitude can not contribute to the CP odd observables in (2.3). CP violation in the absorptive part can give contributions to these observables only in the presence of an absorptive part in the production amplitude. The absorptive parts are of higher order. Therefore, in comparison with CP violation in the production CP violation in the decay will only give suppressed contributions to the observables (2.3). The observables are only sensitive to CP violation in the production. Further, under the approach (2.1), $< O_o >= 0$.

2. Generally, a t or $\bar{t}$ quark produced at colliders can have a vector polarization in the production plane. If CP violation does not appear in such vector polarizations, the observables in (2.4) are only sensitive to $\mathrm{Re} f_2^R$. In QCD these vector polarizations are absent due to parity-conservation. In our approximation, the observables in (2.4) are sensitive only to CP violation in the decay.

3. Concerning the application of the observables to pp colliders: Becauce a pp collider is not an eigenstate of a CP transformation, there is in the initial state an asymmetry unrelated to CP violation. However, we can show through our analytical calculations that

this asymmetry does not lead to any contribution to the expectation value $< O_E >$ if one only considers reactions (1.3) and neglects transverse momenta of the colliding partons. The asymmetry affects the observable O_L only if an absorptive part in the $q + \bar{q} \to t + \bar{t}$ amplitude exists. The effect from this asymmetry is then suppressed by some coupling constants. Further, due to the dominance of the process $G + G \to t + \bar{t}$ at LHC and SSC, this effect is suppressed again. Property 2 concerning the observables in (2.4) holds when they are used at pp colliders and the asymmetry leads to no effect in the observables in leading order of QCD.

The integration over the parton distributions is done numerically. We use the distribution functions from the recent fit[13] to experimental results.

3. Numerical Results

To present our results, we use m_t to rescale our dimensionful observables and the dimensionful couplings so that all quantities are dimensionless:

$$
\begin{aligned}
\hat{O}_L &= \frac{1}{m_t^3} O_L, \quad \hat{O}_E = \frac{1}{m_t^4} O_E \\
\hat{A}_E &= \frac{1}{m_t} A_E, \quad \hat{A}_L = \frac{1}{m_t^2} A_L \\
\hat{d}_t &= \frac{m_t}{g_s} d_t, \quad \hat{\lambda} = \frac{m_t^2}{g_s} \lambda
\end{aligned}
\tag{3.1}
$$

g_s is the strong coupling constant. All quantities with a hat in (3.1) are dimensionless.

We have calculated $< \hat{O}_L >$, $< \hat{O}_E >$ and $< \hat{A}_E >$. The statistical errors of the observables can also be estimated. Let $< O^2 >$ be the variance of observable O, then the statistical error is $\delta < O >= \sqrt{< O^2 > /N}$. Here N is the number of the events used to measure O. We use $m_t = 130$ GeV.

For the $p\bar{p}$ collider Tevatron with $\sqrt{s} = 1.8$ TeV, we find:

$$
\begin{aligned}
< \hat{O}_L > &= -(0.012\hat{d}_t + 0.0066\hat{\lambda}), \quad & \delta < \hat{O}_L > &= \frac{0.086}{\sqrt{N}} \\
< \hat{O}_E > &= -(0.0081\hat{d}_t + 0.0034\hat{\lambda}), \quad & \delta < \hat{O}_E > &= \frac{0.058}{\sqrt{N}} \\
< \hat{A}_E > &= 0.113 \mathrm{Re} f_2^R, & \delta < \hat{A}_E > &= \frac{0.40}{\sqrt{N}}
\end{aligned}
\tag{3.2}
$$

For the proposed pp collider LHC with $\sqrt{s} = 16$ TeV:

$$\begin{aligned}
&< \hat{O}_L > = -(0.019\hat{d}_t + 0.18\hat{\lambda}), \quad \delta < \hat{O}_L >= \frac{0.42}{\sqrt{N}} \\
&< \hat{O}_E > = -(0.033\hat{d}_t + 0.15\hat{\lambda}), \quad \delta < \hat{O}_E >= \frac{0.42}{\sqrt{N}} \\
&< \hat{A}_E > = 0.228\mathrm{Re}f_2^R, \qquad \delta < \hat{A}_E >= \frac{1.48}{\sqrt{N}}
\end{aligned} \tag{3.3}$$

For the proposed pp collider SSC with $\sqrt{s} = 40$ TeV:

$$\begin{aligned}
&< \hat{O}_L > = -(0.05\hat{d}_t + 0.36\hat{\lambda}), \quad \delta < \hat{O}_L >= \frac{0.89}{\sqrt{N}} \\
&< \hat{O}_E > = -(0.098\hat{d}_t + 0.32\hat{\lambda}), \quad \delta < \hat{O}_E >= \frac{1.27}{\sqrt{N}} \\
&< \hat{A}_E > = 0.36\mathrm{Re}f_2^R, \qquad \delta < \hat{A}_E >= \frac{3.0}{\sqrt{N}}
\end{aligned} \tag{3.4}$$

With the numerical results in (3.2), (3.3) and (3.4), the sensitivity of our CP odd observables to $\hat{d}_t$, $\hat{\lambda}$ and $\mathrm{Re}f_2^R$ can be estimated if one knows how many events can be used. We take (3.4) as an example and assume there will be 10^6 events at SSC available for the observables. Measurable effects from the couplings and $\mathrm{Re}f_2^R$ arise only if:

$$\hat{\lambda} \geq 0.0025, \quad \hat{d}_t \geq 0.018, \quad \mathrm{Re}f_2^R \geq 0.0083 \tag{3.5}$$

or:

$$\lambda \geq 1.6 \times 10^{-7}(\mathrm{GeV})^{-2}, \quad d_t \geq 1.5 \times 10^{-4}(\mathrm{GeV})^{-1} \tag{3.6}$$

It is instructive to compare the accuracies in (3.6) with possible upper bounds from elsewhere. There is no constraint on d_t. The coupling constant λ can be constrained by the assumption that the gluonic operator in (2.1) gives the main contribution to the electric dipole moment d_n of the neutron. Using the present upper bound[14] for d_n one can get(see also [9]):

$$\lambda \leq 4.0 \times 10^{-11}(\mathrm{GeV})^{-2} \tag{3.7}$$

This is much smaller than the accuracy reachable in (3.4) and (3.6). However, one should bear in mind that in d_n many CP odd operators can contribute and cancellations are by no means excluded. On the other hand, this upper bound is obtained at low energy, while the CP test performed with (3.2) to (3.4) is at the energy of several hundred GeVs. Needless to say, experimental efforts to measure λ and d_t with the CP odd observables in (2.3) are very worthwhile. Assumming there is an energy scale Λ_{CP} characterizing possible new CP violating interactions, we can obtain at SSC with (3.4) a sensitivity to the new interactions with Λ_{CP} up to several TeVs. Here we assume, due to dimension:

$$d_t \sim \Lambda_{CP}^{-1}, \quad \lambda \sim \Lambda_{CP}^{-2} \tag{3.8}$$

4. Summary and Outlook

We have shown that the $t\bar{t}$ system produced at $p\bar{p}$ or pp colliders can be used for CP tests. At $p\bar{p}$ colliders our CP odd observables can receive nonzero contributions only from CP violating interactions. At pp colliders, apart from CP violating interactions, the asymmetry of the initial state unrelated to CP violation can fake CP violating effects. However, we showed that these effects are small and suppressed; especially, this asymmetry can not affect one of our CP odd observables via the dominant reactions for $t\bar{t}$ production (1.3). Using the observables in (2.3) and (2.4) the CP violating effects from the production and from the decays can be separated well. The standard model gives here only a very small effect. The possible new CP violating interactions can be studied at an energy scale Λ_{CP} up to several TeVs at SSC. The property that the polarization of the produced $t\bar{t}$ systems is measurable and can reliably be predicted plays an important role for the CP test. This property can be used not only for CP tests, but can also be used to test theories in other aspects as discussed in [15,16,17].

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Appendix

We give here the definition of the density matrix for the subprocesses in (1.3) in the partonic c.m.s.:

$$\begin{aligned} G(q_1) + G(q_2) &\rightarrow t(k_+) + \bar{t}(k_-) \\ q(q_1) + \bar{q}(q_2) &\rightarrow t(k_+) + \bar{t}(k_-) \\ \mathbf{q_1} + \mathbf{q_2} &= \mathbf{k}_+ + \mathbf{k}_- = 0 \end{aligned} \tag{A.1}$$

The q in (A.1) denotes u, d, s and c quarks, which we treat as massless.

The density matrix for $GG \rightarrow t\bar{t}$ in (A.1) can be defined as:

$$\begin{aligned} R^G_{\alpha_1\alpha_2,\,\beta_1\beta_2}(\mathbf{q_1},\mathbf{k}_+) = \sum_{\text{colour,gluon spin}} & < t(k_+,\alpha_1)\bar{t}(k_-,\beta_1)|T|G(q_1)G(q_2) >^* \\ & < t(k_+,\alpha_2)\bar{t}(k_-,\beta_2)|T|G(q_1)G(q_2) > \end{aligned} \tag{A.2}$$

Here α_1, α_2 (β_1, β_2) denote the z component of the top (antitop) quark spin in its rest frame which is related to the c.m.s. defined in (A.1) only through a Lorentz boost. Similary, one can also define R^q for $q\bar{q} \rightarrow t\bar{t}$. The density matrices obtain contributions from L_{CP} through the interference with the QCD amplitude at tree-level. The contributions are:

$$\begin{aligned} R^q(\mathbf{q_1},\mathbf{k}_+) &= \frac{1}{256}\frac{512}{9}g_s^3 d_t \vec{\sigma}_+ \times \vec{\sigma}_- \cdot \{\mathbf{q_1}\beta x + \mathbf{k}_+ \frac{-\beta^2x^2E_1 + \beta^2E_1 - m_t - E_1}{E_1 + m_t}\} \\ R^G(\mathbf{q_1},\mathbf{k}_+) &= \frac{1}{256}g_s^3\vec{\sigma}_+ \times \vec{\sigma}_- \cdot \{\frac{144\lambda}{\beta^2x^2 - 1}(-\mathbf{q_1}\beta x m_t + \mathbf{k}_+\frac{\beta^2x^2m_tE_1}{E_1 + m_t}) \\ &+ \frac{16d_t}{3(\beta^2x^2-1)^2}(\mathbf{q_1}\beta x(-9\beta^4x^4 + 9\beta^4x^2 - 7\beta^2x^2 + 23\beta^2 - 16) \\ &+ \mathbf{k}_+(9\beta^6x^6E_1 - 18\beta^6x^4E_1 + 18\beta^6x^2E_1 + 9\beta^4x^4m_t + 16\beta^4x^4E_1 \\ &- 18\beta^4x^2m_t - 43\beta^4x^2E_1 + 14\beta^4E_1 + 2\beta^2x^2m_t + 18\beta^2x^2E_1 \\ &- 14\beta^2m_t - 35\beta^2E_1 + 21m_t + 21E_1)\frac{1}{E_1 + m_t})\} \end{aligned} \tag{A.4}$$

with

$$E_1 = \frac{1}{2}\sqrt{(q_1+q_2)^2}, \quad \beta = \sqrt{1 - \frac{m_t^2}{E_1^2}}, \quad x = \frac{\mathbf{q_1}\cdot\mathbf{k}_+}{|\mathbf{q_1}||\mathbf{k}_+|}. \tag{A.5}$$

Here $\vec{\sigma}_+$ ($\vec{\sigma}_-$) are the Pauli matrices with indices referring to α_1, α_2 (β_1, β_2). From (A.4) we can see $R^G(\mathbf{q_1},-\mathbf{k}_+) = -R^G(\mathbf{q_1},\mathbf{k}_+)$ and the same for R^q leading to $< O_o >= 0$ as already mentioned in Sect. 2.

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Figure Caption

Fig.1: Feynman diagrams for reactions (1.3) induced by L_{CP} of (2.1).

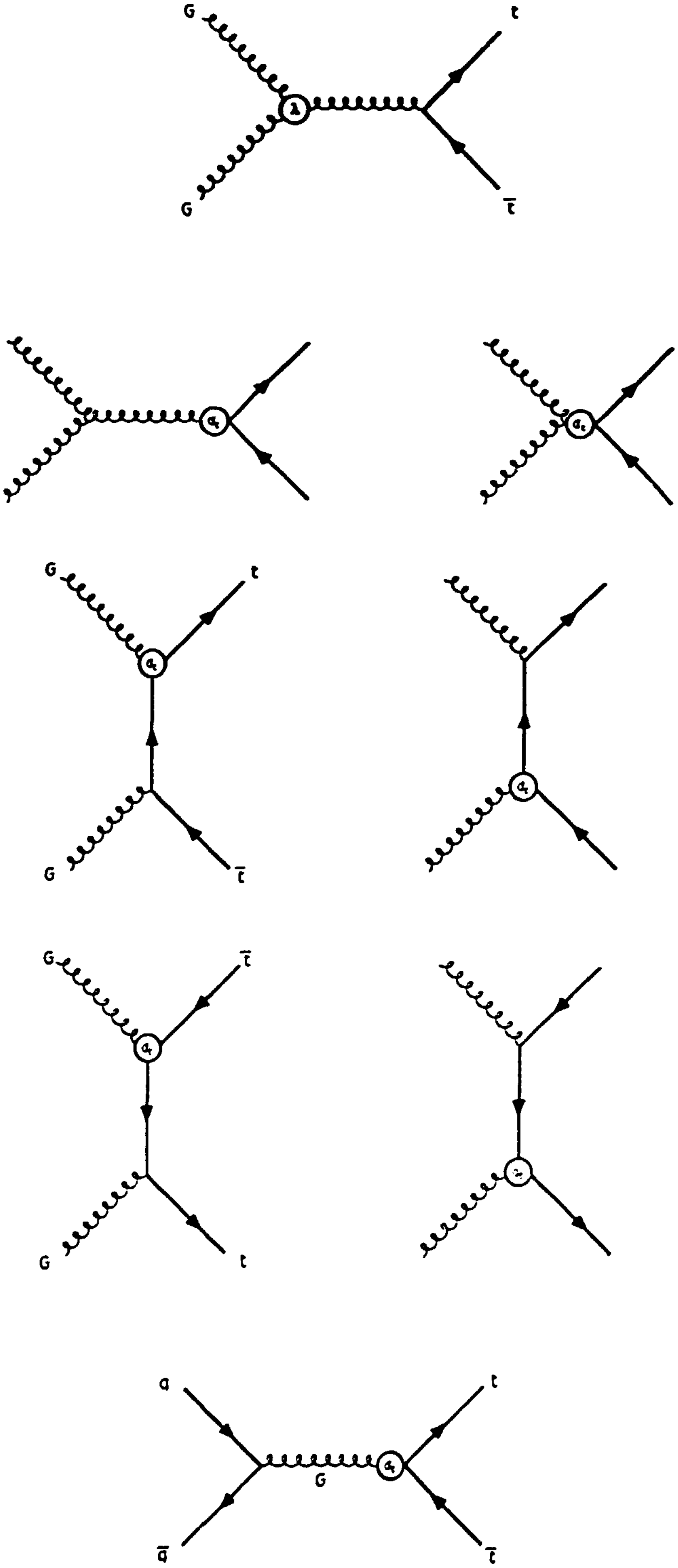

Fig. 1