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BARYON NUMBER VIOLATION AND PARTICLE COLLIDER EXPERIMENTS *

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Abstract

Baryon number non-conservation, due to non-perturbative effects (sphalerons) in the standard model, may have been important in the early Universe. In this talk we discuss the possibilty that similar effects could show up at future particle collider experiments.

* to be published in J. Tran Thanh Van (ed.) PARTICLE ASTROPHYSICS, IV-th Rencontre de Blois, 1992

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ABSTRACT

Baryon number non-conservation, due to non-perturbative effects (sphalerons) in the standard model, may have been important in the early Universe. In this talk we discuss the possibilty that similar effects could show up at future particle collider experiments.

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INTRODUCTION

The last few years have seen a renewed interest in non-perturbative physics at the electroweak scale and potential implications for cosmology. The crucial observation is that the configuration space of the theory has non-trivial structure at an energy scale set by the Sphaleron $E_S = O(M_W/\alpha_w)$, or, in simpler terms, that there is more to the standard model than the usual elementary particles [1]. This "non-standard physics in the standard model" suggests that electroweak baryon and lepton number non-conservation may have been important in the early Universe [2]. Remarkably, this occurs at relatively low temperatures (T $\gtrsim$ 100 GeV) and without the need for drastic modifications of the standard model. Precisely how the cosmic baryon (B) and lepton (L) number change is not known at this moment. Different scenarios have been discussed at this conference by Shaposhnikov and Cohen and we refer to them for further details.

More down to earth, we could ask wether or not similar effects could show up at future particle colliders as well [3,4]. Most likely, these would be multiparticle events with average transverse momenta of the order of $p_t \sim M_W$. Of course, we do not rule out the possibility of genuine new physics showing up at large p_t. The effects we are concerned with here are benchmark results, in that they could occur already in the standard model itself. An additional motivation to look for these effects in laboratory experiments is to be sure of the physics that goes into the early Universe (and not just to speculate about it).

To be specific, we consider the B+L violating inclusive cross-section for two quarks (q) into antiquarks ($\bar{q}$) and antileptons ($\bar{l}$)

$$q + q \rightarrow 3\,\bar{l} + 7\,\bar{q} + \mathrm{X}\,, \qquad (1)$$

where X stands for, typically, $O(\alpha_w^{-1})$ W and Z vector-bosons and Higgs scalars, provided the center-of-mass energy is large enough. This is a particularly clean probe for non-perturbative effects, since Feynman diagrams do not contribute to any order (all vertices conserve B and L). From a standard instanton calculation [5,6,7] it follows that the total cross-section, at low center-of-mass energies $\epsilon \equiv \sqrt{s}/E_S << 1$, behaves as [4,8,9]

$$\sigma_{\Delta(B+L)} \propto \exp\left(\frac{4\pi}{\alpha_w} F(\epsilon)\right), \tag{2}$$

where the function $F(\epsilon)$ increases with ϵ from $F(0) = -1$. This shows that, at low center-of-mass energy, the cross-section is exponentially suppressed ($4\pi/\alpha_w \sim 400$), but also that it increases exponentially with energy. Concretely, we are faced with two questions :

1. what is the value ϵ_{max} where $F(\epsilon)$ reaches its maximum, or, in other words, what is the threshold energy ?

2. what is the value $F(\epsilon_{max})$, or, in other words, what is the cross-section at threshold ?

The instanton calculation gives $F(\epsilon)$ only in terms of a power series

$$F_I = -1 + c_2 (\epsilon^{2/3})^2 - c_3 (\epsilon^{2/3})^3 + O(\epsilon^{8/3}), \tag{3}$$

with $c_{2,3}$ certain numerical coefficients [9,10], and cannot be used to answer these questions.

So, the big problem is to calculate $F(\epsilon)$ reliably at large center-of-mass energies. At this moment nobody has succeeded in doing this. The general feeling is that there should be some kind of master field, just as the Sphaleron in the case of high temperature. We refer the reader to an extensive review [11] of ongoing work, but here we have the prerogative of discussing some work of our own [12], which consists of two parts. First, we point out the possible existence of a special 4-dimensional Euclidean configuration (I^*) and present some new numerical results for its parameters. Second, we use this configuration to saturate the path integral of the forward elastic scattering amplitude, which, by the optical theorem, gives the total cross-section. In this way we obtain an estimate of the threshold energy, which turns out, not unexpectedly, to be close to the Sphaleron energy $E_S \sim 10\,\text{TeV}$. We thus have a preliminary answer to the first question listed above, but the second, even more important question remains entirely open.

We conjecture [12] the existence of a new non-self-dual constrained instanton I^* in fundamental $SU(2)$ Yang-Mills-Higgs theory. By 'instanton' we mean any localized, finite action solution of the Euclidean field equations, not just the original instanton I of Belavin, Polyakov, Schwartz and Tyupkin [5]. This new instanton has topological charge and action

$$q_{I^*} = 0 \ , \ \ A_{I^*} \sim 4\pi/\alpha_w \,. \tag{4}$$

Presumably, I^* is the lowest action (non-trivial) solution in the vacuum sector, hence its importance. The existence of I^* follows from the construction of a suitable non-contractible loop of configurations.

The existence of non-contractible loops (spheres etc.) in configuration space implies, as Taubes [13] has shown in a somewhat different context, the existence of new solutions to the classical field equations. Also in our case, it is not too difficult to construct a non-contractible loop of 4-dimensional configurations, which implies the existence of a new solution, different from the vacuum and with an action less or equal to the maximum action over the loop. However, we have to exclude the possibility that this so-called new solution is merely a superposition of two previously known solutions at infinite separation, i.e. a constrained instanton and anti-instanton with total action $2A_I$. Obviously, this loophole is closed if we manage to construct a non-contractible loop for which

$$\max A_{\rm NCL}(\omega) < 2A_I \,, \tag{5}$$

where ω parametrizes the position along the loop. We have obtained numerical and analytical results for $A_{\rm NCL}(\omega)$, which indicate that (5) can be satisfied. If correct, this would imply the existence of a new constrained instanton I^*.

Before we proceed with the non-contractible loop we should clarify what we mean by a 'constrained' instanton. A simple scaling argument shows that any 4-dimensional solution in Euclidean Yang-Mills-Higgs theory will collapse to a point. Hence, we have to fix a scale $\bar{\rho}$ and later integrate over it. This can be done by introducing a constraint in the path integral [7]. For the numerical results we present here we have used the following dimension-8 operator

$$\left[-\frac{1}{2}\,\mathrm{Tr}\left(\frac{1}{2}\epsilon_{\kappa\lambda\mu\nu}W_{\mu\nu}W_{\kappa\lambda}\right)\right]^2 , \tag{6}$$

with $W_{\mu\nu}$ the $SU(2)$ field strength. With a Lagrange multiplier this term is added to the standard Yang-Mills-Higgs action and will prevent collapse of the scale. It is in *this* theory that the non-contractible loop argument leads to a new solution I^*, provided the inequality (5) holds. The present, dynamical treatment of the constraint is an improvement compared to our earlier paper, where, for simplicity, we had fixed the scale by hand [12].

We can now sketch the particular non-contractible loop used to look for I^*. The general structure of the loop is as follows : we start by extracting from the vacuum an instanton–anti-instanton pair with such relative isospin as to give Yang-Mills attraction, separate this pair to a maximum distance d_{max}, rotate their relative isospin so that the Yang-Mills forces become repulsive and then attractive again, and, finally, collapse the pair back to the vacuum. The specific fields are given in eqs. (6–9, 13) of our paper [12], where further details can be found. Furthermore, we have chosen to work with quartic Higgs coupling constant $\lambda = 0$. In order to find the action profile over this non-contractible loop, we have solved numerically the variational equations (PDEs) for the attractive ($\omega = \pm\pi/2$) and repulsive ($\omega = 0$) Yang-Mills channel, which we sketch in Fig. 1. The general behaviour is easy to understand : at large distances ($d >> M_W^{-1}$) only the attractive Higgs force is operative, whereas for smaller distances the Yang-Mills interaction joins in, which can be either attractive or repulsive. The action profile along the non-contractible loop is obtained as follows : start in Fig. 1 at $d = 0$ on the $\omega = \pi/2$ curve and move out to $d = d_{max}$, then go straight up to the $\omega = 0$ curve and back, and, finally, return to $d = 0$. Clearly, the best choice for d_{max} is at the point where the two curves effectively meet, which we designate by d^*, with a corresponding value of the action A^*. In that case, the inequility (5) is satisfied,

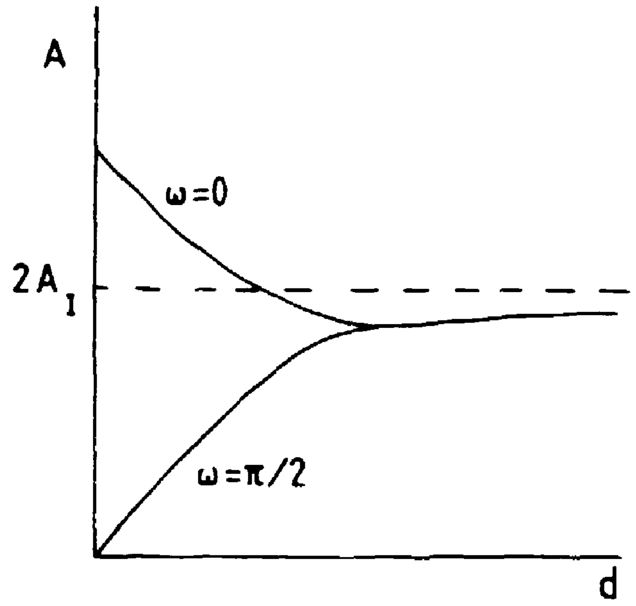

Figure 1: Sketch of the action A vs. the core separation d for the configurations in the repulsive ($\omega = 0$) and attractive ($\omega = \pi/2$) Yang-Mills channels, at vanishing Higgs coupling constant $\lambda = 0$. The action is understood to include a higher dimension term (6), with given Lagrange multiplier, to fix the scale.

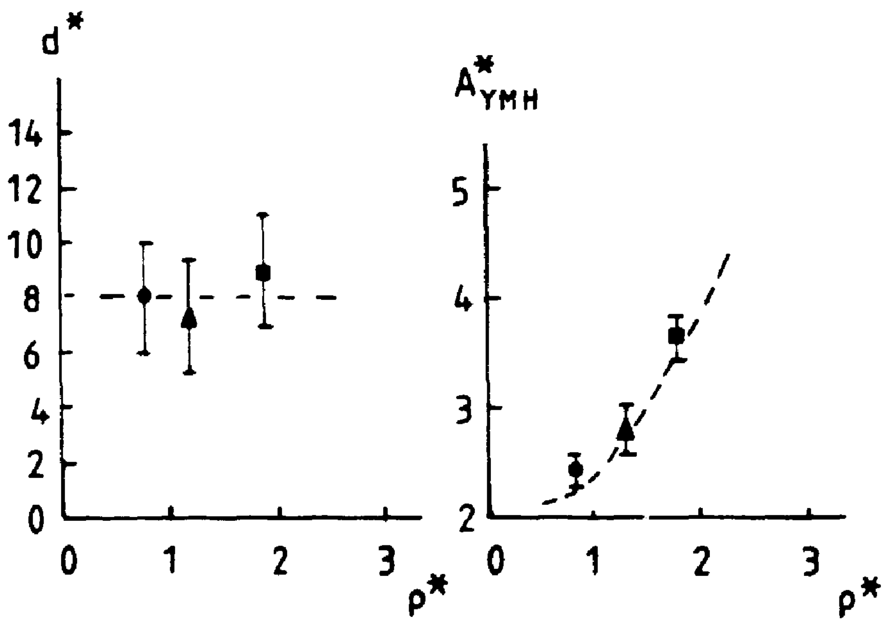

Figure 2: Estimates of the distance parameter d^* and corresponding action $A^*_{\rm YMH}$ of the new instanton I^*, both as a function of its scale ρ^*. The distances d^* and ρ^* are in units M_W^{-1} and the action $A^*_{\rm YMH}$ in units $2\pi/\alpha_w$. The dashed curves are the parametrization (7).

essentially because of the attraction due to the Higgs. Also, for this value of the distance parameter d^*, we can solve for the Lagrange multiplier and obtain the scale ρ^* and the true Yang-Mills-Higgs action A^*_{YMH}.

In the way outlined above we have determined numerically the optimal maximum configuration ($\omega = 0$, $d = d^*$) of the non-contractible loop, which should give a reasonable approximation of I*. Concretely, the configuration is cigar-shaped with a length and width determined by d^* and ρ^*, respectively, and has an action A^*_{YMH}. Figure 2 shows these numerical estimates, which should be considered preliminary. The numerical results for d^* and A^*_{YMH} can be parametrized as follows (dashed curves in Fig. 2)

$$\begin{aligned} d^* &= d_0^* \, M_W^{-1} \\ A^*_{\mathrm{YMH}} &= \left(a_0^* + a_1^* \, (\rho^* M_W)^2 \right) 2\pi/\alpha_w \, , \end{aligned} \tag{7}$$

with approximate values $d_0^* \sim 8.0$, $a_0^* \sim 2.0$ and $a_1^* \sim 0.5$. We will now turn to a possible application of this conjectured new instanton I*.

THRESHOLD ENERGY

Consider the fermion number violating total cross-section (1), which, by the optical theorem, follows from the imaginary part of the forward elastic scattering amplitude $\mathcal{T}$. It is rather straightforward to calculate the contribution of the approximate I* configuration to the Euclidean Green's function for the forward elastic scattering amplitude. Making the analytic continuation from Euclidean to Minkowski space-time we find, for $\sqrt{s}/M_W >> 1$,

$$\mathcal{T} \;\propto\; \exp\left[\frac{4\pi}{\alpha_w} \left(F_{\Gamma}(\epsilon) + \mathrm{O}(\alpha_w) \right) \right] \, , \tag{8}$$

$$F_{\Gamma}(\epsilon) = \epsilon \, \frac{d_0^*}{4} - \frac{a_0^*}{2} \, , \tag{9}$$

where we have defined

$$\epsilon \equiv \sqrt{s}/E_{\mathrm{S}}$$

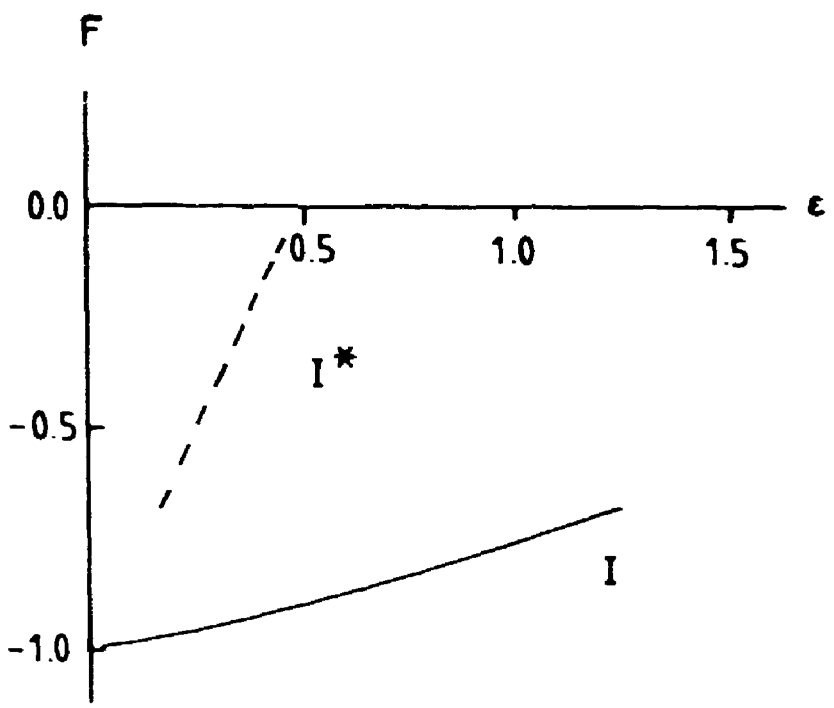

Figure 3: Exponent factor F (dashed curve), calculated with an approximate I* configuration (9). The BPST instanton result (3) is shown also (full curve).

and

$$E_{\mathrm{S}} \equiv \pi \, \frac{M_W}{\alpha_w}$$

which is close to the true value $3.04 \, M_W/\alpha_w$ for the sphaleron energy at $\lambda = 0$. Thus, the exponent factor $F_{\mathrm{I}^*}(x)$ grows *linearly* with the center-of-mass energy (dashed curve in Fig. 3). This behaviour contrasts with that of the BPST instanton calculation (3), shown as the full curve in Fig. 3.

To obtain an estimate of the threshold center-of-mass energy we look for a vanishing exponent $F_{\mathrm{I}^*}(\epsilon)$ in (9) and find

$$(\sqrt{s})_{\mathrm{threshold}} \sim (2\, a_0^*/d_0^*)\; E_{\mathrm{S}} \sim 0.5\, E_{\mathrm{S}} \,, \tag{10}$$

where we have used the parameter values quoted below (7). Even without numerical results, we know that, in our approach, the general order of magnitude of the threshold energy is

$$(\sqrt{s})_{\mathrm{threshold}} \sim \; A^*_{\mathrm{YMH}}/d^* \sim [A^*_{\mathrm{YMH}}/(16\pi^2/g^2)]\, [4\,/(d^* M_W)]\, E_{\mathrm{S}} \,.$$

Regardless of its precise value, the fact remains that at the threshold energy our classically evaluated cross-section (8) violates unitarity, which should be restored by multi-instanton induced processes [14], for example by iteration of the effective I^* vertex in the forward elastic scattering amplitude. These effects make it difficult to calculate the *value* of the fermion number violating cross-section and it may very well turn out to be negligably small [15]. Still, we expect that, if 'non-perturbative' effects manifest themselves in one way or another, this should be at a parton center-of-mass energy of order $E_S \sim 10\,\mathrm{TeV}$, which may be within reach [16] of SSC and perhaps LHC.

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