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### Introduction

An undulator has two properties which make it an extremely attractive source of electromagnetic radiation.[1] The first is that the radiation is concentrated in a number of narrow energy bands known as harmonics of the device. The second characteristic is that under favorable operating conditions, the energy of these harmonics can be shifted or "tuned" over an energy interval which can be as large as two or three times the value of the lowest energy harmonic.

Both the photon energy of an undulator as well as its tunability are determined by the period,  $\lambda$ , of the device, the magnetic gap,  $G$  (which is larger than the minimum aperture required for injection and operation of the storage ring), and the storage ring energy,  $E_R$ . Given the photon energy,  $E_p$ , the above parameters ultimately define the limits of operation or tunability of the undulator.

In general, the larger the tunability range, the more useful the device. Therefore, for a given required maximum photon energy, it is desirable to find the operating conditions and device parameters which result in the largest tunability interval possible.

With this in mind, we have investigated the question of undulator tunability with emphasis on the role of the ring energy in order to find the smallest  $E_R$  consistent with the desired tunability interval and photon energy. As a guideline, we have included a preliminary criteria, concerning the tunability requirements for the Advanced Photon Source (APS) to be built at Argonne. The analysis is aimed at X-ray undulator sources on the APS [2,3] but is applicable to any storage ring.

### Analysis

The energy in keV of the  $i^{\text{th}}$  harmonic of an undulator for an observation point along the midplane axis of the device is given by:

$$E_{Pi} = \frac{0.949 E_R^2 i}{\lambda(1 + K^2/2)} \quad (1)$$

Where  $E_{Pi}$  is photon energy in keV,  $E_R$  is the ring energy in GeV,  $\lambda$  is the undulator period in cm and  $i$  is the harmonic number. We will consider only the first harmonic  $i = 1$  and  $E_{P1} = E_p$ . The deflection parameter  $K$  is given in terms of the undulator period (in cm) and the peak magnetic field  $B_0$  (in Tesla) by

$$K = 0.934 \lambda B_0 \quad (2)$$

For hybrid magnets based on permanent magnet blocks and vanadium permendur pole tips,  $B_0$  is given by [4]

$$B_0 = 0.95 a \exp(-G/\lambda(b-cG/\lambda)) \quad (3)$$

where  $G$  is the magnet gap of the undulator in cm.

In Eq. 4, the factor 0.95 represents the "filling factor" which takes into account the packing factor of high-permeability blocks in the undulator assembly. The constants  $a$ ,  $b$  and  $c$  depend on the magnetic material and are given in Table 1 for two permanent magnet candidate materials,  $\text{SmCo}_5(\text{REC})$  and the Nd-Fe-B alloy

TABLE 1  
Constants used in Eq. 3 for Hybrid Undulators

	REC	Nd-Fe-B
a (T)	3.33	3.44
b	5.47	5.08
c	1.8	1.54

Equation 3 for the peak field is valid in the interval  $0.07 < G/\lambda < 0.7$ . Although the upper limit for  $G/\lambda$  does not define the maximum operational gap of the device, we have taken this as the maximum gap in our calculations. The main reason is that the  $K$  values encountered for gaps much larger than  $0.7 \lambda$  become small resulting in reduced intensity of the photon beam. We have taken the ratio  $R = G/\lambda = 0.7$  as a conservative upper limit in the example that follows.

Equations 1, 2, and 3 form a set of coupled equation which determine the photon energy of a given harmonic as a function of gap, device period and ring energy. At a given ring energy  $E_R$ , the undulator period and gap, determine the on-axis 1st harmonic energy. Decreasing the gap increases  $B_0$  and hence the value of  $K$  resulting in a lower photon energy.

In summary, the largest photon energy  $E_U$  occurs at the largest gap for a given undulator period and has both the smallest  $K$  and intensity. The energy may be shifted down from this maximum by decreasing the gap. The tunability from the maximum to minimum photon energy is limited by the maximum gap determined by  $R=0.7$  and the minimum gap determined by the ring aperture. In addition, the tunability interval in the photon energies depends on the storage ring energy.

### Results

At any given  $E_R$ , the maximum desired photon energy,  $E_U$ , and minimum deflection parameter,  $K_U$ , at the open gap ( $G_U$ ) position determine the required device period at each ring energy. This value of  $\lambda$  is the maximum one at  $E_R$  capable of producing  $E_U$  and is also the one with the largest tunability range consistent with  $E_U$  and  $K_U$  because it has the smallest closed-gap to  $\lambda$ -ratio. Once this period is determined for  $E_U$ , then the gap at each  $E_R$  required to produce any  $E_p$  smaller than  $E_U$  can be determined.

For reasons mentioned above, the value of  $K_{\text{U}}$  was taken as that given by the open gap relation of  $K=0.7$ . Following this procedure, the gap values as a function of ring energy were obtained for a 14 KeV undulator planned for the APS. The gaps necessary to achieve the photon energies of 14, 7 and 4.7 KeV are shown in Fig. 1 for REC-hybrid magnet undulators. The period of the "optimum" device determined at 14 keV and  $G/\lambda = 0.7$  is shown for each  $E_{\text{R}}$  on the top of the figure. The minimum gap is determined by the ring stay-clear aperture, coupled with the vacuum chamber wall thickness. This minimum gap in turn set the lower limit on the tunability at a given  $E_{\text{R}}$ .

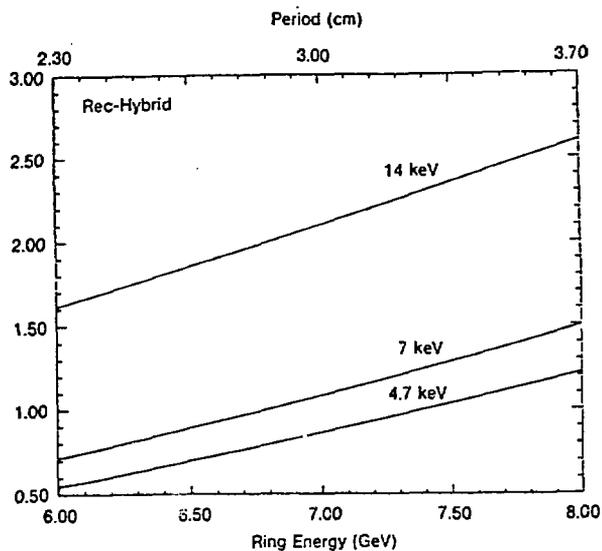


Fig. 1. Constant photon energy (KeV) plots of the gap values (cm) as a function of ring energy (GeV) needed by a 14 KeV undulator to obtain the photon energies shown. The period (cm),  $\lambda$ , of the device is the maximum one permitted by the maximum gap condition  $G/\lambda = 0.7$ .

The intersection of any constant gap line and one of the constant photon energy curves (7 or 4.7 keV in this case) corresponds to the minimum ring-energy necessary at this gap to achieve the tunability between the maximum  $E_{\text{p}}$  and the selected lower photon energy. For example, at a minimum gap of 1.4 cm, expected for the APS in its initial operating phase  $E_{\text{R-min}}$  is approximately 7.8 GeV for a REC-Hybrid device. At a final phase minimum gap of 1.0 cm, tunability between 14 and 4.7 keV will occur at 7.4 GeV minimum ring energy. These are upper limits and considerable reductions in ring energy can be achieved using Nd-Fe-B at larger open gaps (see below).

The results of the analysis is shown for a 20 keV undulator in Fig. 2. These curves show that at 1.5 cm gap, the minimum ring energy necessary to produce 20 keV photons in the 1st harmonic is approximately 7 GeV for the REC magnet material. At the final phase with a minimum gap of 1.0 cm, the lowest photon energy achievable at 7 GeV is approximately 16.5 KeV. At higher ring energies, the tunability interval increases.

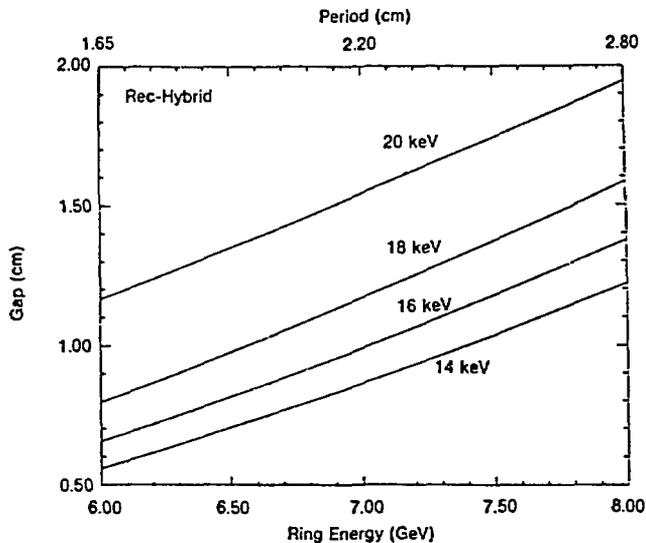


Fig. 2. Constant photon energy (keV) plots of the gap values (cm) as a function of ring energy (GeV) needed by 20 KeV undulators. The period (cm),  $\lambda$ , of the devices is that permitted by the maximum gap condition,  $G/\lambda = 0.7$ .

The results depend on the maximum gap/period ratio considered and the minimum K-value acceptable. For example, in the case of the 14 KeV undulator and a 1.4 cm minimum gap, increasing the R from 0.7 to 0.8 results in a decrease in  $E_{\text{R}}$  from 7.8 to 7.5 GeV for a REC-Hybrid device. However, the minimum K-value also drops from 0.55 to 0.41, thus lowering the photon flux. A further increase of R to 0.9 results in an  $E_{\text{R}}$  of 7.31 GeV and a  $K_{\text{U}}$  of 0.32. Lower  $E_{\text{R}}$  values can be achieved by using Nd-Fe-B magnet material. For the case of 1 cm gap expected for the mature phase of the APS, a ring energy of 7 GeV rather than 7.4 GeV will be sufficient to provide the required 4.7 to 14 KeV tunability for a Nd-Fe-B based undulator. In this case, the  $K_{\text{U}}$  value at 14 KeV is approximately 0.3 and the period is close to 3.3 cm. Again, the final minimum ring energy necessary depends on the acceptable operating point of the device.

In conclusion, the preceding analysis can be useful for determining the minimum ring energy needed for a desired tunability photon energy range. The analysis shows that unique solutions are possible if the the minimum gap is specified and a maximum gap to period ratio (R) or minimum K-value is provided.

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#### References

- [1] For a recent review of undulator properties see S. Krinsky, M. L. Perlman and R. E. Watson in Handbook on Synchrotron Radiation, Vol. 1a, edited by E. E. Koch, North-Holland, 1983, p. 65.

- [2] G. K. Shenoy and P. J. Viccaro, "An Overview of the Characteristics of the 6-GeV Synchrotron Radiation: A Preliminary Guide to Users", Argonne National Laboratory Report, ANL 85-69 (October 1985).
- [3] "Characteristics of Insertion Devices for the 6-GeV Synchrotron Source", Argonne National Laboratory Light Source Document, LS-52, March 1986.
- [4] K. Halbach, J. Appl. Phys. 57, 3605 (1985) and references therein.