IS NUCLEON DEFORMED?

AFSAR ABBAS
Institute of Physics
Bhubaneswar - 751005
India.

Let us start by giving the answer to the question in the title :yes, the nucleon in the ground state itself is deformed.

The non-relativistic quark model has been singularly successful [1]. If there are any shortcomings these are often ascribed to relativistic corrections. But these relativistic corrections lead to problems of their own [2]. It turns out that treating neutron and proton as deformed by including appropriate D-state admixture in the wavefunction can, for the same value of the admixture, lead to a very satisfying and global as well as individual fit to the experimental data. We will conclude that this must be the true description of the nature of the nucleon. This is my prediction and further signatures of this deformed nucleon be looked for.

WAVEFUNCTIONS:

Let us take the nucleon and the delta wavefunction defined as follows [3]

$$| N >= \sqrt{1 - P_D(N)}\, | N_S > + \sqrt{P_D}\, | N_D >$$

$$| \Delta >= \sqrt{1 - P_D(\Delta)}\, | \Delta_S > - \sqrt{\frac{P_D(\Delta)}{2}}\, | \Delta_D^{(1)} > + \sqrt{\frac{P_D(\Delta)}{2}}\, | \Delta_D^{(2)} >$$

The subscripts S and D represent the spherical S-wave and the deformed D-wave parts respectively. The superscripts (1) and (2) refer to the symmetry $(70, 2^+)$ and

$(56, 2^+)$ respectively for Δ. The D-state admixture can be induced by a tensor term in the interaction between quarks [2]. Let the tensor term be of the form

$$\Theta_D = \beta \Sigma_{i<j} S_{ij} \vec{\tau}_i . \vec{\tau}_j$$

where

$$S_{ij} = (\vec{\sigma}_i . \hat{r}_{ij})(\vec{\sigma}_j . \hat{r}_{ij}) - \frac{1}{3} \vec{\sigma}_i . \vec{\sigma}_j$$

If the same tensor term acts on the nucleon and the delta then the two D-state probabilities are related by $P_D(\Delta) = 2P_D(N)/(1 + P_D(N))$ leaving us with only one parameter $P_D(N)$.

SEMI-LEPTONIC DECAYS OF BARYONS:

Glashow [4] had used the above wavefunction to obtain the axial vector coupling constant and F/D ratio relevant in the decay $n \rightarrow pe^- \nu$:

$$G_A/G_V = 5/3 - 2P_D(N))$$

$$(D + F)/(D - F) = 5(1 - 6/5P_D(N))$$

Without deformation the non-relativistic quark model values are off the mark. A $P_D(N) \sim 0.2 - 0.25$ fits both these quantities very well.

I studied [2] all the kinematically allowed semileptonic decays of $1/2^+$ baryon octet with the assumption that each member is deformed with the same deformation parameter $P_D(N)$. Predictions were made for axial vector coupling constants and F/D ratio for semileptonic decays like $\Lambda \rightarrow pe^- \nu, \Sigma^- \rightarrow ne^- \nu, \Sigma^- \rightarrow \Lambda e^- \nu, \Xi^- \rightarrow \Lambda e^- \nu$ and many others. In 1988 [2] there were some differences between what was predicted with deformed baryon and what was observed experimentally. But the recently observed and better data [5] is in complete agreement with my deformed baryon predictions giving complete support to our model.

PION-NUCLEON-DELTA COUPLING CONSTANT:

The ratio of $\pi N\Delta$ to πNN coupling constant $r_{\pi N\Delta} = (f_{\pi N\Delta}/f_{\pi NN})^2$ is experimentally found to be 4 in good agreement with Chew-low theory. The quark model with the value 72/25 fails miserably. On including deformation one finds

$$r_{\pi N\Delta} = 72/25[(1 - \frac{1}{2}P_D(N)/(1 - \frac{6}{5}P_D(N)]^2/(1 + P_D(N))$$

Again the same $P_D(N) \sim 0.2 - 0.25$ gives a good fit to the experimental number.

DOUBLE DELTA COUPLING CONSTANT:

This coupling constant with the deformed nucleon turns out to be [6]

$$(\frac{f_{\pi\Delta\Delta}}{f_{\pi NN}})^2 = \frac{1}{25}[\frac{1 - P_D(N)}{(1 - \frac{6}{5}P_D(N))(1 + P_D(N))}]^2$$

The value without deformation is 1/25. Since $r_{\pi N\Delta}$ in the quark model (without deformation) fail so badly there is no reason why this value should be acceptable. Our prediction for $(f_{\pi\Delta\Delta}/f_{\pi NN})^2$ for $P_D(N) = 0.25$ is 0.029. Though this quantity has not been determined well experimentally it is of great relevance in hadronic physics. It turns out that it is needed for the reactions $^{12}C(\pi, 2\pi)$ and also for the softening of short range correlations in nuclei.

SPIN DEPENDENT STRUCTURE FUNCTION:

The spin dependent structure functionis defined as

$$g_1^p = \int_0^1 dx g_1^p(x) = \frac{1}{2}\int dx[\frac{4}{9}\Delta u(X) + \frac{1}{9}\Delta d(x) + \frac{1}{9}\Delta s(X)]$$

where for each feavour of quarks (u,d,s)

$$\Delta q(x) = q\uparrow(x) + \overline{q}\uparrow(x) - (q\downarrow(X) + \overline{q}\downarrow(X))$$

Here the arrows pointing up or down indicate whether the spin of the quark q or antiquark $\overline{q}$ is aligned parallel or antiparallel to the spin of the proton.

In our model there are no strange quarks and one obtains [7]

$$g_1^p = \int_0^1 dx g_1^p(x) = \frac{5}{18} - \frac{4}{9}P_D(N)$$

$$g_1^n = \int_0^1 dx g_1^n(x) = -\frac{1}{9}P_D(N)$$

For no deformation $g_1^p = 5/18 (\approx 0.278)$. This is much larger than the experimentally determined value of 0.126 by the EMC group [7]. This had led to the well known crisis for the quark model as to where the spin of the nucleon resides. For a deformed nucleon this is no problem. An appropriate amount of deformation can quite clearly explain the EMC result. It also turns out that with no other configuration mixture can one explain the EMC [8]. This gives additional support to the concept of a deformed nucleon.

CONSTITUENT QUARK MASS AND M1 TRANSITION MOMENT:

A constituent quark mass of 340 MeV is what gives all the acceptable results of the quark model [1]. What I have found recently [9] is that the constituent quark mass is not a fixed quantity; it changes with deformation. For $P_D(N) = 0.25$ the correct constituent quark mass to use is 255 MeV. This shows that the constituent quark mass goes down with deformation.

This fact has several nontrivial implications. The magnetic is given as

$$\mu(n) = -\frac{2}{3}(1 - P_D(N))\mu_q$$

$$\mu(p) = (1 - P_D(N))\mu_q$$

where $\mu_q = e\hbar/2m_qc$. The ratio does not change from the value without deformation - which is good. However the M1 transition moment for $\Delta^+ \to P\gamma$ with deformation becomes

$$\mu_{\Delta P} = \frac{2\sqrt{2}}{3}\left[\frac{(1 - P_D(N)(\frac{\sqrt{2}-1}{2\sqrt{2}})}{\sqrt{1 + P_D(N)}}\right]\frac{e\hbar}{2m_qC}$$

Without $P_D(N)$ the value $2\sqrt{2}/3$ was well known to fail [10]. Only by taking the corret mass are we able to fit the experimental value [9]. So a deformed nucleon again resolves a long standing problem.

E2/M1 RATIO:

Again in the transition $\gamma N \to \Delta$ $E2/M1$ ratio would be zero for $P_D(N)$=0. A nonzero value could be a signal of deformation [11]. Indeed the value turns out to be nonzero [12]. A small enough bag with large deformation can explain the experimental E2/M1 ratio value [11]. So this fact is compatible with deformation (though unfortunately there are other ways of explaining this ratio too [13]). Clearly the best way to measure deformation would be to measure the static quadrupole moment of Δ which ought to be accessible to mechanics like CEBAF.

DEFORMED NUCLEONS IN NUCLEI:

Let us assume that our nucleus consists of A such deformed nucleonic bags. We define the nuclear ground state by the state determinant of these nucleous [14,15]

$$|0> = \frac{1}{\sqrt{A!}}\mathcal{A}\Pi_{i=1}^{A} \mid (qqq)_i >$$

where $\mid (qqq)_i >, i = 1, 2... A$ are wave functions of deformed nucleon. The anti-symmetrizing operator $\mathcal{A}$ acts only within the nucleonic space. We will assume that the specific nuclear medium effects will modify the free nucleon parameter $P_D(N)$ to some other value $P_D(A)$ for a particular nucleus with mass number A. Let us define the Gamow-Teller β_- and β_+ operators and the M1 operator as follows

$$\hat{\beta}_{\mp} = \Sigma_{a=1}^{A}\{\Sigma_{a\in i=1}^{3}\Sigma_{\mu}\sigma_{\mu}(i)\tau_{\pm}(i)\}$$

$$\hat{M}_1 = \Sigma_{a=1}^{A}\{\Sigma_{a\in i=1}^{3}\Sigma_{\mu}(\sigma_{\mu}(i) + l_{\mu}(i))Q(i)\mu_q(i)\}$$

where $\tau_{\pm} = (\tau_1 \pm i\tau_3)/2$. μ labels the spherical components and i labels the quarks in a particular nucleon a. Q is the quark charge, σ and l represent the spin and the angular momentum parts respectively and $\mu_q = e\hbar/2m_q c$ where m_q is the constituent quark mass.

(a) MAGNETIC MOMENTS:

We can easily find the magnetic moment of closed shell plus or minus one nucleon. So for example for the trinucleon systems one expects that the magnetic moment of the ground state is obtained from that of the odd nucleon. Hence [15]

$$\mu(^3He) = -2/3[1 - P_D(^3He)]\mu_q$$

$$\mu(^3He) = [1 - P_D(^3He)]\mu_q$$

where $P_D(N)$ refers to the deformation of the odd nucleon for a particular nucleus. This is determined by fitting to the experiments the ratio $\mu(^3He)/\mu(n)$ and $\mu(^3H)/\mu(p)$. One finds $P_D(^3He)$=0.168 and $P_D(^3He)$=0.199. These are used to predict $\delta\mu(^3He)/\delta\mu(^3H) = [\mu(^3He) - \mu(n)]/[\mu(^3H) - \mu(p)]$ to be -1.08 which compares very well with the experiments.

Note that $P_D(A) < P_D(N)$. One of the ways of explaining the EMC effect is to encrease the effective confinement size of the nucleons in the nuclear medium.

If we accept this then we can easily explain $P_D(A) < P_D(N)$. In a deformed nucleon there is a larger surface area in the flat region than at the edges of (say) the pumpkin. If the pull on the surface is uniform, greater expansion will take place perpendicular to the flat part and so the deformation will decrease.

(b) MAGNETIC DIPOLE STRENGTHS:

The operators defined above are strangeness conserving and so can excite only N and Δ degrees of freedom in nuclei. The total strength including the N and the Δ sectors is

$$S^{N+\Delta} = \Sigma_n |< n \mid \Sigma_{a=1}^{A} \times (a) \mid 0 >|^2$$

$$= T_r(\overline{X^+X}) + (T_r X^+).(T_r X) - T_r(X^+X) - (T_r X)^2$$

where the traces are meant with respect to the A-dimensional nucleonic subspace. $\overline{X^+X}$ means it is for the same nucleon while in X^+X the nucleons may be different. Since excitations to the Δ sector are not subject to the Pauli blocking they are obtained in a straight forward manner. This taken away from $S^{N+\Delta}$ gives the strength sitting in the low-energy N sector. In units of $\mu_q^2.(\frac{3}{4}\pi)$ we obtain [15]

$$S_{M1}^{N+\Delta} = [\frac{8}{3} + \frac{5}{9}P_D(N) - \frac{4}{3}P_D(N)^2](N+Z) + P_D(N)[\frac{17}{9} - \frac{5}{3}P_D(N)]Z$$

$$+\frac{8}{9}[1 - P_D(N)]^2\delta_{(N-1)/2,I} + 2[1 - P_D(N)]^2\delta_{(Z-1)/2,I}$$

$$S_{M1}^{\Delta} = \frac{2}{3}[2\sqrt{[1 - P_D(N)][1 - P_D(\Delta)]} + \left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)\sqrt{P_D(N)P_D(\Delta)/2}]^2(N+Z)$$

$$S_{M1}^{N} = S_{M1}^{N+\Delta} - S_{M1}^{\Delta} ...and... \{I = (0, 1, 2, 3, ...)\}$$

For ^{28}Si and ^{48}Ca a $P_D(^{28}Si) \sim 0.04$ and $P_D(^{48}Ca) \sim 0.02$ are needed to match the experimentally determined strengths at low energies. Infact we find that $P_D(A) \to 0$ as $A \to \infty$ (or very large). Hence the individual nucleons become more and more spherical as the nucleus becomes heavier.

(c) GAMOW-TELLER STRENGTHS:

The strengths obtained with the Gamon-Teller operators are [14] For β_-

$$S_{\beta_-}^{N+\Delta} = (6+3g_A)N + [3+3g_A(1-g_A)]Z$$

$$S_{\beta_-}^{\Delta} = g_A^2 r_{\pi N\Delta}(\frac{1}{3}N + Z)$$

$$S_{\beta_-}^{N} = (6+3g_A - \frac{1}{3}g_A^2 r_{\pi N\Delta})N + [3+3g_A(1-g_A) - g_A^2 r_{\pi N\Delta}]Z$$

For β_+

$$S_{\beta_+}^{N+\Delta} = (3+3g_A)N + [6+3g_A(1-g_A)]Z$$

$$S_{\beta_+}^{\Delta} = g_A^2 r_{\pi N\Delta}(N + \frac{1}{3}Z)$$

$$S_{\beta_+}^{N} = (3+3g_A - g_A^2 r_{\pi N\Delta})N + [6+3g_A(1-g_A) - g_A^2 r_{\pi N\Delta}/3]Z$$

where $r_{\pi N\Delta}$ and $g_A = G_A/G_V$ in $n \to pe^-\nu$ were obtained earlier.

The strengths S_β^N and S_β^Δ are obtained in (p,n) and (n,p) reactions and also in heavy ion charge exchange reactions [14]. These can be compared to what has been obtained experimentally say at Arogonne and TRIUMF and one finds in agreement with what was discussed earlier that $P_D(A) \to 0$ as $A \to \infty$ (or very large). Note also that the advantage of this technique has been to obtain each strength individually for β_- and β_+ as well as N and Δ sectors separately. This is in contrast to the very simplistic 3(N-Z) sum rule [14] and as such has great predictive potential.

In summing up we can state that quite clearly our method of study of a deformed nucleon whether free or in a nuclear medium has nontrivial and interesting implications for hadronic physics.

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