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FLAVOR CHANGING STRINGS AND DOMAIN WALLS

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ABSTRACT

We consider the cosmological consequences of a spontaneous breaking of non-abelian discrete symmetries, which may appear as a natural remnant of a continuous symmetry, such as a family symmetry. The result may be a stable domain wall across which an electron would turn into a muon (or ν_e into ν_μ) or a flavor analogue of an Alice string–domain wall structure with the same property.

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A. INTRODUCTION

It is quite natural that the spontaneous breaking of a continuous symmetry leads at some stage to a discrete one. If the original symmetry is nonabelian, there is no reason why the discrete subgroup should not be nonabelian as well. It is our aim in this paper to investigate the issue of topological defects in the case of a spontaneously broken nonabelian discrete symmetries, both as basic symmetries and also as a stage of symmetry breaking of some continuous symmetry.

In the case of pure nonabelian discrete symmetry, we find an interesting possibility of domain walls, light enough to be stable without spoiling the big-bang scenario. They would have a spectacular property of changing an electron into a muon or ν_e into ν_μ as they travel across the wall. If a discrete symmetry is embedded into a continuous one, one obtains a nice example of nonintercommuting strings [1], which eventually become boundaries of domain walls.

The essential feature of our result is the fact that continuous symmetry could be almost any horizontal symmetry, in particular the popular family symmetry. The nonintercommuting nature of the string–domain wall structures are to be expected in a large class of such theories. In some cases the strings become flavor analogues of Alice strings [2], i.e. they may have a spectacular property of an electron turning into a muon as it travels around the string.

We now turn our attention to phenomenologically relevant examples of a non-abelian discrete symmetry.

B. QUATERNION GROUP AND LIGHT DOMAIN WALLS

A few years ago, in an attempt to achieve a small neutrino mass m_ν and a large magnetic moment μ_ν, Voloshin [3] suggested a $SU(2)$ global (or local) symmetry between ν and ν^c, which becomes even more appealing as a flavor symmetry [4] between ν_e and ν_μ. Under such a symmetry, μ_ν being antisymmetric is a singlet, whereas the mass is a triplet and thus forbidden. The trouble is that such $SU(2)_H$ symmetry must be broken above M_W and thus cannot play its role at energies much below the weak scale relevant for neutrino mass considerations.

It was shown [5], however, that an equally important role can be played by a discrete subgroup of $SU(2)_H$, such as quaternion symmetry Q

$$Q = \{\pm 1, \pm i\,\sigma_1, \pm i\,\sigma_2, \pm i\,\sigma_3\} \tag{1}$$

where σ_i are Pauli matrices. Clearly, the 8-element group Q belongs to $SU(2)$ and equally clearly under Q, m_ν is forbidden, while μ_ν is allowed. We focus here on the lepton flavor version of this idea, where the basic doublet representation of Q is $\binom{e}{\mu}$ and $\binom{\nu_e}{\nu_\mu}$. To ensure that $m_e \neq m_\mu$, we need the scale of Q breaking Λ_Q to satisfy $\Lambda_Q \gtrsim (10-100)$ MeV. On the other hand, the upper limit on Λ_Q is also $10-100$ MeV, otherwise the resulting domain walls would carry too much energy density and distract the observed isotropy of the universe [6].

In other words, we predict

$$\Lambda_Q \simeq (10-100)\ \text{MeV}\ . \tag{2}$$

In any case, for the mechanism of the suppression of m_ν to work, Λ_Q should be as low as possible and the condition (2) is highly desirable. This is our claimed result of possibly stable domain walls in a realistic model of weak interactions (it is possible, though that gravity destabilizes the walls [7]).

Under the element $i\,\sigma_2 \in Q, e \to \mu$ $(\nu_e \to \nu_\mu)$ and $\mu \to -e$ $(\nu_\mu \to -\nu_e)$ a lepton transversing this domain wall would actually change its flavor! The astrophysical and cosmological consequences of such objects deserve further study and we will return to them in a future publication. Suffice it to mention that the usual argument [8] of percolation theory leading to the existence of a large (stretching over entire horizon) domain wall may not be automatically applicable to the case of a nonabelian discrete symmetry.

C. EMBEDDING INTO A CONTINUOUS SYMMETRY

As we mentioned before, quaternion group Q is a subgroup of $SU(2)$ and its phenomenological role was in fact inspired by this fact. Therefore, it is certainly natural to consider the scenario with $SU(2)_H$ symmetry being broken down to Q at some higher energy. More generally, let us imagine that your preferred continuous horizontal symmetry G_H (global or local) breaks down to a nonabelian discrete symmetry Q at some scale Λ_H. The most natural candidate for G is a family symmetry.

Properties of the resulting topological structures depend crucially on how the fermions transform under Q. This in turn determines the embedding of Q and its covering continuous group $SU(2)_H$ in a family symmetry G_H.

Here we study two simplest and most natural possibilities.

(a) ***Regular embedding***

Imagine fermions belonging to a two dimensional representation of Q, i.e. spinor under $SU(2)_H$. Therefore, $SU(2)_H$ is operating among two families, not a strange fact in view of the third family being much heavier than the first two. This would simply amount to the original family symmetry $SU(3)_H$ (or $SU(N)_H, N > 3$, if the number of families $N > 3$) being hierarchically broken down to Q

$$SU(3)_H \to SU(2)_H \to Q \to \mathbb{1}\ . \tag{3}$$

This picture reflects all the features of Voloshin and Chang *et al.* scenarios. The first stage of the breaking of $SU(3)_H$ can produce some interesting topological structures [9] which are not of interest here. For us the crucial point is to notice the possibility of a nonabelian discrete channel which, as we emphasized before, may play an important role in neutrino physics.

At the scale Λ_H when $SU(2)_H$ breaks down to Q, according to the standard scenario, strings are produced. These strings carry an important topological property of being nonintercommuting, i.e. the two strings corresponding to the noncommuting elements of the homotopy group cannot pass through each other [1]. As far as we know, this is the first realistic example of such structures. Since normally the major mechanism that determines the evolution of the string network is attributed to their intercommutavity [7], we expect the cosmology of the above system to be different from the conventional one.

However, as discussed before the discrete symmetry Q must be broken at the scale $\Lambda_Q \gtrsim 10$ MeV, so that strings become boundaries of the domain walls [10]. Now the flavor Alice effect will take place as the particle crosses the wall. Of course, the string–wall system is no longer topologically stable, since it can decay through the hole formation. This decay rate is exponentially suppressed by the unknown ratio Λ_H/Λ_Q.

One knows that independently of whether G_H is global or local the scale Λ_H must lie much above M_W, due to the flavor changing processes. In the local case this is triggered by the exchange of horizontal gauge bosons with $\Lambda_H \gtrsim 10^6$ GeV, whereas in the global case the bound is due to the possible emission of the resulting Goldstone bosons. The astrophysical limit due to the production of Goldstone bosons in stellar object is $\Lambda_H \gtrsim 10^9$ GeV [11]. In other words, we can conservatively assume $\Lambda_H \gtrsim 10^6$ GeV. In general we have no information on Λ_Q, but if we take seriously the large neutrino magnetic moment hypothesis Λ_Q should not exceed 1 GeV or so. In this case, of course, the string–wall structures would be stable for all practical purposes.

The desired symmetry breaking $SU(2)_H \underset{\Lambda_H}{\rightarrow} Q$ can be naturally achieved by the four index symmetric representation of $SU(2)_H$, i.e. with $SU(2)_H$ family spin $s_H = 2$, while the next stage of symmetry breaking of Q should be realized by the spin 1 scalars needed to give a mass to the fermions. We should admit though that the presence of the $s_H = 2$ fields is not automatic and so we postpone its discussion for the case of the special embedding of $SO(3)_H$ into $SU(3)$ when we are forced to introduce it.

(b) *Special embedding*

Let us now consider another natural embedding of the quaternion group Q in

the family symmetry, with fermions being vector rather than spinor representation of the covering group $SU(2)_H = SO(3)_H$. In this case family symmetry may be represented by $SO(3)_H$ itself or by a larger group $SU(3)_H$ containing $SO(3)_H$ as a special subgroup. By special subgroup we mean such an embedding of $SO(3)_H$ into $SU(3)_H$ when the fundamental triplet of $SU(3)_H$ coincides with a triplet of $SO(3)_H$ subgroup.

Note that the $SU(3)_H$ symmetry admits both vector and chiral realizations, depending on the transformation properties of left–handed and right–handed fermions. The concrete realizations were discussed already, here we shall try to perform a maximally model independent analysis.

Let us focus first on the case of $SO(3)_H$ family symmetry. Since quarks and leptons are triplets under $SO(3)_H$, the fermion mass terms transform as $3 \times 3 = 5 + 3 + 1$, and so the Higgs fields which break $SO(3)_H$ and give masses to fermions must transform in the same manner. We do not address here the detailed mechanism of the generation of fermion mass matrices, since we wish to keep our analysis as model independent as possible.

The bottom line in this approach is the fact that the horizontal family structure is projected in the fermion mass sector through higher dimensional effective operators, since the family scale must lie much above the weak scale. Of course, in general one can introduce extra Higgs fields, decoupled from fermions, but such complications appear neither necessary nor reasonable.

Notice that the diagonal mass terms can be generated only by 5 and 1 representations (3 is antisymmetric). Neglecting the contribution of 3–plets first, let us consider a most general vev of a 5–plet

$$< 5 > = \begin{pmatrix} a & & \\ & b & \\ & & -(a+b) \end{pmatrix} \tag{4}$$

which leaves unbroken group Q. In the case of a single 5–plet, we can show that the most general solution is $a = b$, in which the surviving symmetry is actually a semidirect product of $U(1)$ and Z_2. This would imply the existence of monopoles and the well known Alice strings. In contrast with the grandunification case [12], these flavor monopoles are not necessarily cosmologically harmful. This is related to the fact that the scale Λ_H may be as low as 10^6 GeV in the local case, whereas in the global case rapid annihilation [13] of monopoles renders them harmless even for large Λ_H. Of course, we must break the degeneracy $a = b$ by introducing at least one extra 5–plet. Now, monopoles will become connected by the nonintercommuting strings and are no more topologically stable.

As in the previous case, the stability of these structures depend on the ratio of the vev's of the two 5–plets. If the respective Yuhawa couplings are of the same order, then the electron–muon mass ratio indicates that these vev's must be of the same order. In this case one expects that these structures can decay by nucleation into monopole–antimonopole pairs which get rapidly annihilated [14] and one is left with the topologically stable string network.

In short the original symmetry is broken down to Q. Actually, since there are no spinors, Q acts as Z_4 on the field context of the theory. The elements of Z_4 are

$$\begin{aligned} g_1 &= diag(1,-1,-1); \qquad g_2 = diag(-1,1,-1); \\ g_3 &= diag(-1,-1,1); \qquad g_4 = \mathbb{1} \ . \end{aligned} \tag{5}$$

As before, the above symmetry breaking leads to the formation of nonintercommuting strings. As one travels around such a string, the vev of 5–plet winds continuously by the $SO(3)_H$ group transformation

$$< 5(\theta) > = \Omega^T(\theta) \ < 5(0) > \ \Omega(\theta) \quad (0 \le \theta \le 2\pi), \tag{6}$$

where $\Omega(\theta)$ interpolates between different elements $g_i \in Z_4$ as θ runs from 0 to 2π. Therefore, different strings can be labelled by a pair of group elements (g_i, g_j) connected by the path $\Omega(\theta)$ in question. For example, for the string (g_4, g_3), $\Omega_3(\theta)$ has the form

$$\Omega_3(\theta) = \begin{pmatrix} \cos\theta/2 & \sin\theta/2 & 0 \\ -\sin\theta/2 & \cos\theta/2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{7}$$

which is just a rotation around a third axis in the flavor space (in the same manner one obtains elements $\Omega_1(\theta)$ and $\Omega_2(\theta)$).

As in our previous discussion, these strings have a flavor changing property, but in a different manner. After a half a circle an electron turns into say a muon, but at the end of the journey it is again an electron, with just an opposite sign of a wave function. This change cannot be observed by a local experiment, since all the vev's are simultaneously changing by the same group transformation. However, the flavor changing interaction of the string will manifest itself through the scattering of the fermions on the string. This is due to the $SO(3)_H$ gauge field whose magnetic flux is trapped in the core (or the Nambu–Goldstone mode in the case of a global symmetry). This scattering amplitude can be strongly enhanced due to the Aharonov–Bohm effect [15].

The rest of the story depends on whether the quarks and leptons share the same horizontal symmetry or not. In the more conventional scenario when the family group is common, in order to generate realistic mass matrices for the quarks, their off–diagonal elements must be switched on. This in turn necessarily leads to the breaking of Z_4 symmetry. The natural candidates for this effect are 3–plet Higgs or nondiagonal elements of 5–plets. Once again, we end up inevitably with a string–wall network. The measure of its stability is, of course, the ratio of off–diagonal to diagonal elements of the fermion mass matrix, or in other words the amount of mixing among different families. This means that for all practical purposes these structures could be considered as stable. Here and everywhere by stability we mean stability under nucleation. Of course, these structures will tend to disappear through collapse and a more careful and model dependent analysis is needed to quantify this.

It is the very existence of the wall that now allows observable flavor transitions. To see this, consider, for example, a wall bounded by (g_4, g_3) string (described by $\Omega_3(\theta)$). This wall can be formed through the nonvanishing triplet vev $< 3 > \propto (1,0,0)$, since $g_3 < 3 > \neq < 3 >$. The winding condition $< 3(\theta) > = \Omega_3(\theta) \ < 3(0) >$ cannot thus be satisfied in the wall. Therefore, the fermion mass matrix is modified nontrivially inside the wall and this change is no longer a gauge artifact. In other words, when a fermion is passing through the wall (nonadiabatically) a flavor changing transition will necessarily take place.

Strictly speaking, the quark and lepton family groups may be distinct. If there is no mixing in the lepton sector, we would be left with stable strings we described here.

Similarly, we can consider a case of $SU(3)_H$ family symmetry. The main difference from the previous case is the new unshrinkable paths which connect elements of Z_4 with identity and belong to $SU(3)/SO(3)$ manifold. For example, consider again a minimal string corresponding to $g_3 = diag(-1,-1,1)$. This element can be obtained by an $SO(3)$ rotation as well as the $SU(3)/SO(3)$ rotation $\Omega(\theta) = \exp(i\ \theta/_2\lambda_8)$, where $\lambda_8 = diag(1,1,-2)$ is a generator of $SU(3)_H$. This provides a new path with $\Omega(\theta) = 1, \Omega(2\pi) = g_3$. Which path would correspond to the minimal energy depends on the pattern of symmetry breaking. For example, in the case of the hierarchical breaking $SU(3)_H \to SO(3)_H \to Q$, clearly the minimal energy path is the one belonging to the $SO(3)$ manifold.

D. DISCUSSION AND OUTLOOK

In this paper we studied the cosmological consequences of the spontaneous breaking of nonabelian discrete symmetries, both as basic symmetries and as a stage in symmetry breaking of continuous symmetries. The nonabelian discrete symmetry case is inspired by the quaternion group Q used to keep neutrino mass naturally small, while allowing a large magnetic moment. The resulting domain walls produced once the discrete symmetry gets broken can be stable, present today since the scale of the breaking of Q, Λ_Q can be as low as $1-100$ MeV. The most interesting characteristic of these domain walls is that the fermion changes its flavor when crossing the wall, i.e. electron would turn into a muon, electron neutrino into a muon neutrino etc. The astrophysical and cosmological consequences of such objects deserve further study and we postpone it for a future publication.

Now, independent of how much you believe in nonabelian discrete symmetry, it can certainly result naturally in the process of a breaking of a continuous symmetry. We suggest that a large class of horizontal family symmetries do break at some stage to a nonabelian discrete symmetry, such as the quaternion group studied in this paper. The result is a string–wall system with a fermion changing its flavour in the wall. Clearly, the effects associated with these systems are quite fascinating and, stable or not, their cosmological and astrophysical role is likely to be very important.

Finally, we wish to mention that there are other nonabelian discrete subgroups of $SU(3)$ which may be phenomonologically relevant [16], such as e.g. a dicyclic group which can be embedded into $SU(2) \times U(1)$.

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