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FLAVOR CONNECTIONS AND NEUTRINO MASS HIERARCHY IN VARIANT INVISIBLE AXION MODELS WITHOUT DOMAIN WALL PROBLEM

C.Q. Geng and John N. Ng
TRIUMF Theory Group, 4004 Wesbrook Mall, Vancouver, B.C., Canada V6T 2A3

Abstract

A new type of invisible axion models based on the recent variant axion models are presented. They belong to $N = 1$ type model and hence are free of domain wall problems. The Peccei-Quinn symmetry transformations are not totally generation- and flavor-blind which may help in understanding the small values of electron and u-quark and large t-quark masses. The light neutrino mass pattern in the two Higgs singlets models can have a very different hierarchy which differs from the other type of invisible axion models.

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1. Introduction

A serious problem in the standard model is the strong CP-problem[1] of why the parameter θ which involves a P and T, or CP-violating term in QCD lagrangian is so small, $\theta < 10^{-9}$. A natural and elegant solution of this problem is the Peccei-Quinn (PQ) mechanism[2] which yields $\theta = 0$ dynamically. The spontaneous breakdown of the chiral global $U(1)_{PQ}$ symmetry gives rise to a pseudo-Goldstone boson called the axion.[3] However, the original PQ axion with symmetry breaking scale, f_a, equal to the electroweak scale (EWS) $v = (\sqrt{2}G_F)^{-1/2} \approx 250$ GeV has not been found experimentally. Recently, the observation of a narrow positron line in the positron energy spectrum from heavy ion collisions at GSI[4] has stimulated speculation of the existence of a short-lived EWS axion. This leads to the construction of variant axion models.[5,6] Unfortunately, they are also ruled out by recent experiments.[7–9] Thus, further modification of these models must be considered. One well-known possibility is to make PQ $f_a \gg v$ by adding an $SU(3)_c \times SU(2)_L \times U(1)_Y$ complex Higgs singlet field.[10,11] Then the axion becomes very light and weakly coupled and therefore it eludes experimental detection. In this sense it is dubbed "invisible" axion. Astrophysical and cosmological considerations[12] require f_a to lie between 10^8 and 10^{12} GeV.

Broadly speaking there are two minimal types of invisible axion models due to Dine-Fischler-Srednicki-Zhitnitskii[10] (DFSZ) and Kim-Shifman-Vainshtein-Zakharov (KSVZ).[11] The DFSZ model has two doublets $\phi_i (i = 1, 2)$ and one singlet χ Higgs field, while the KSVZ model has one ϕ and one χ Higgs field plus a superheavy exotic quark which is a $SU(2) \times U(1)$ singlet. The DFSZ axion model, suffers from a difficulty called the "domain wall" problem.[13] The model with N generations of the quarks and leptons has N degenerate disconnected vacua,[14] leading to the formation of cosmologically unacceptable domain walls. One interesting solution of the problem is to construct a model with a unique vacuum, i.e., $N = 1$, or have N vacua that are not topologically distinct. To date, models that evade the domain wall problem make use of either an exotic quark such as the KSVZ model or an enlarged gauge (or global) symmetry.[15] In this letter, we will build a new type of the minimal invisible axion models with $N = 1$ by introducing extra singlet Higgs fields to the variant EWS axion models. We shall call these variant invisible axion (VIA) models. The $N = 1$ property of variant EWS axion models was first pointed out by Krauss and Wilczek,[6], but they have not constructed explicit viable models.

The most unsatisfactory feature of the standard model is the lack of understanding of the Higgs sector and the origin of the intriguing family and mass patterns of the quarks and leptons which certainly seem to be connected. The recent measurement of $B_d^0 - \overline{B}_d^0$ mixing[16] requires the quark mass $m_t > 50$ GeV in the standard model. Although when charged Higgs are present one can accommodate the large $B_d^0 - \overline{B}_d^0$ mixing with $m_t < 50$ GeV/c^2, the Yukawa coupling of the t-quark is still uncomfortably large.[17] This deepens the mystery of fermion masses. Why are the electron and the u-quark masses m_e and m_u small and m_t so large? Why are all the neutrinos nearly massless? Any suggestion which sheds light on the question is welcome. Clearly such models must have some symmetries which distinguish both the flavors and families. Naturally one hopes that the PQ-symmetry can be employed for this purpose since the strong CP problem is related to the quark mass matrix. In the DFSZ model, the PQ transformations are generation-blind and they distinguish only the up-type and down-type quark flavors while the KSVZ model does not incorporate any flavor information at all. In constructing VIA models, we make use of the freedom that different quarks can have different PQ charges and we assign different PQ charges to different families of quarks as well as different PQ charges for u and d-type quarks within a familiy. Hence, $U(1)_{PQ}$ is no longer family- and flavor-blind.[18] This is an attempt to relate the fermion mass hierarchy and the strong CP problem. Recently, several authors hypothesized that f_a can be identified as the scale $\Lambda \simeq \langle\chi\rangle$ of the "see-saw" mechanism[19] for generating small neutrino masses. This entails adding a right-handed singlet lepton N_R to the DFSZ or KSVZ model[20]. The resulting neutrino mass hierarchy behaves as

$$M_{\nu_e} : M_{\nu_\mu} : M_{\nu_\tau} \sim m^2_{u,e} : m^2_{c,\mu} : m^2_{t,\tau} , \tag{1}$$

and the masses of the three types of neutrinos have the following ranges:

$$\begin{aligned} 10^{-10}\ \mathrm{eV} \le M_{\nu_e} \le 10^{-4}\ \mathrm{eV} \\ 10^{-5}\ \mathrm{eV} \le M_{\nu_\mu} \le 10^{-1}\ \mathrm{eV} \\ 10^{-3}\ \mathrm{eV} \le M_{\nu_\tau} \le 10\ \mathrm{eV}\ . \end{aligned} \tag{2}$$

The neutrino masses in (2) automatically satisfy the current experimental limits:[21]

$$M_{\nu_e} < 18\ \mathrm{eV}\ , \quad m_{\nu_\mu} < 250\ \mathrm{keV}\ \ and \quad M_{\nu_\tau} < 35\ \mathrm{MeV}\ , \tag{3}$$

and cosmological constraints for stable neutrinos[22]:

$$\sum M_{\nu_i} \le 65\ \mathrm{eV}\ . \tag{4}$$

Even if an extra singlet scalar χ' with $\langle\chi'\rangle \sim \Lambda'$ is introduced in the DFSZ or KSVZ model,[23,24] the mass hierarchy in (1) still holds, but now f_a is given by $f_a \sim (\Lambda^2+\Lambda'^2)^{1/2}$. A priori there is no lower limit of Λ anymore. However, the constraints of (3) and (4) rule out the values of $\Lambda < 10^8$ GeV. Hence, one still has $10^8\ \mathrm{GeV} \le \Lambda \le f_a^{max} \sim 10^{12}$ GeV and neutrinos masses still lie in the range of given by (2). We show by explicit construction that VIA models with two doublets and two singlets can have neutrino mass pattern different from (1) and (2).

The models are based on the $N = 1$ variant EWS axion models but adds $SU(3)_c \times SU(2)_L \times U(1)_Y$ singlet Higgs fields to them. First we examine the minimal two doublets $\phi_i (i = 1, 2)$ and one χ_1 Higgs field VIA models, hereafter referred to as Model I. Then we present the two doublets $\phi_i (i = 1, 2)$ and two singlets $\chi_i (i = 1, 2)$ VIA models denoted as Model II.

Model I: The Higgs fields and their VEVs are written as

$$\begin{aligned} \phi_1 &= \begin{bmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}} e^{i\theta_1} (v_1 + R_1 + iI_1) \end{bmatrix}, \quad \phi_2 = \begin{bmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}} e^{i\theta_2} (v_2 + R_2 + iI_2) \end{bmatrix}, \\ \chi_1 &= \frac{1}{\sqrt{2}} e^{i\alpha} (\Lambda_3 + R_3 + iI_3), \end{aligned} \tag{5}$$

and

$$\langle \phi_1 \rangle = \frac{1}{\sqrt{2}} e^{i\theta_1} v_1, \quad \langle \phi_2 \rangle = \frac{1}{\sqrt{2}} e^{i\theta_2} v_2, \quad \langle \chi_1 \rangle = \frac{1}{\sqrt{2}} e^{i\alpha_1} \Lambda_1, \tag{6}$$

respectively, and $v = \sqrt{v_1^2 + v_2^2}$. The scalars and fermions are given the following $U(1)_{PQ}$ transformations:

$$\begin{aligned} \phi_1 &\rightarrow e^{-i\alpha \cdot x} \phi_1, \quad \phi_2 \rightarrow e^{i\alpha \cdot 1/x}, \quad \chi_1 \rightarrow e^{i\alpha \cdot (x + 1/x)} \chi_1 \\ u_R^i &\rightarrow e^{i\alpha Z_u^i} u_R^i, \quad d_R^i \rightarrow e^{i\alpha Z_d^i} d_R^i, \quad l_L^i \rightarrow e^{1/2 i\alpha (x - 1/x)} l_L^i, \\ N_R^i &\rightarrow e^{i\alpha Z_N^i} N_R^i, \quad e_R^i \rightarrow e^{i\alpha Z_e^i} e_R^i, \end{aligned} \tag{7}$$

where $x = v_2/v_1$ and $i = 1, 2, 3$ are generation indices. There are many possible $N = 1$ VIA models for different choices of Z_u^i, Z_d^i and Z_e^i. Here, we show two of them

Model Ia:

$$(Z_u^1, Z_u^2, Z_u^3) = \frac{1}{x} (-x^2, 1, 1)$$

$$
\begin{aligned}
(Z_d^1, Z_d^2, Z_d^3) &= -\frac{1}{x}\,(1,1,1) \\
(Z_e^1, Z_e^2, Z_e^3) &= -\frac{1}{2}\,\left(x+\frac{1}{x}\right)\,(-1,1,1) \\
(Z_N^1, Z_N^2, Z_N^3) &= -\frac{1}{2}\,\left(x+\frac{1}{x}\right)\,(1,1,1)\;,
\end{aligned} \tag{8}
$$

and Model Ib:

$$
\begin{aligned}
(Z_u^1, Z_u^2, Z_u^3) &= \frac{1}{x}\,(1,1,-x^2) \\
(Z_d^1, Z_d^2, Z_d^3) &= -\frac{1}{x}\,(1,1,1) \\
(Z_e^1, Z_e^2, Z_e^3) &= (Z_N^1, Z_N^2, Z_N^3) = -\frac{1}{2}\,\left(x+\frac{1}{x}\right)\,(1,1,1)\;.
\end{aligned} \tag{9}
$$

Thus, we have the following Yukawa couplings:

$$
\begin{aligned}
\mathcal{L}_Y(\text{Model Ia}) &= \sum_i \left(h_{i1}^u\,\overline{q}_L^i\,\tilde{\phi}_1\,u_R^1 + h_{i1}^e\,\overline{l}_L^i\,\phi_1\,e_R^1\right) + \sum_i\sum_{j\neq 1}\left(h_{ij}^u\,\overline{q}_L^i\,\tilde{\phi}_2\,u_R^j + h_{ij}^e\,\overline{l}_L^i\,\phi_2\,e_R^j\right) \\
&\quad + \sum_{i,j}\left(h_{ij}^d\,\overline{q}_L^i\,\phi_2\,d_R^j + h_{ij}^\nu\,\overline{l}_L^i\,\tilde{\phi}_1\,N_R^j + h_{ij}^N\,N_R^{iT}\,C N_R^j\,\chi_1\right) + \text{h.c.}\;,
\end{aligned} \tag{10}
$$

and

$$
\begin{aligned}
\mathcal{L}_Y(\text{Model Ib}) &= \sum_i\sum_{j\neq 3} h_{ij}^u\,\overline{q}_L^i\,\tilde{\phi}_2\,u_R^j + \sum_i h_{i3}^u\,\overline{q}_L^i\,\tilde{\phi}_1\,u_R^3 + \\
&\quad + \sum_{i,j}\left(h_{ij}^d\,\overline{q}_L^i\,\tilde{\phi}_2\,d_R^j + h_{ij}^e\,\overline{l}_L^i\,\phi_2\,e_R^j + h_{ij}^\nu\,\overline{l}_L^i\tilde{\phi}_1\,N_R^j + \right. \\
&\quad \left. h_{ij}^N\,N_R^{iT}\,C N_R^j\,\chi_1\right) + \text{h.c.}\,.
\end{aligned} \tag{11}
$$

The Higgs potential for both models is given by

$$
V_I = \sum_i m_i^2 \phi_i^\dagger \phi_i + \sum_{i\leq j} a_{ij}(\phi_i^\dagger\phi_i)(\phi_j^\dagger\phi_j) + b_{12}(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + [c\,\phi_1^\dagger\phi_2\,\chi_1^\dagger + \text{h.c.}]\,. \tag{12}
$$

where $\phi_3 \equiv \chi_1$. Spontaneous symmetry breaking takes place at two different scales Λ_1 and v. It is easily seen that there are two neutral Goldstone bosons. One is the would-be-Goldstone boson

$$
G^0 = \frac{v_1 I_1 + v_2 I_2}{v}\;, \tag{13}
$$

which becomes the longitudinal component of the Z-boson and the other is the axion

$$a = \frac{1}{f_a} \left(v_2 I_1 - v_1 I_2 - \frac{v^2 \Lambda_1}{v_1 v_2} I_3 \right) , \tag{14}$$

where

$$f_a = \frac{v}{v_1 v_2} (v_1^2 v_2^2 + v^2 \Lambda_1^2)^{1/2} , \tag{15}$$

is the axion decay constant. The PQ-symmetry current is

$$\begin{aligned} J_\mu^{\rm PQ} &= f_a \partial_\mu a - \sum_{i=1}^{N_g} \left(Z_u^i \overline{u}_R^i \gamma_\mu u_R^i + Z_d^i \overline{d}_R^i \gamma_\mu d_R^i \right) + \\ &\quad + \text{(lepton currents)} \end{aligned} \tag{16}$$

which has a color anomaly

$$\partial^\mu J_\mu^{\rm PQ} = \left(x + \frac{1}{x} \right) \cdot \frac{g_3^2}{32\pi^2} G_{\mu\nu}^A \tilde{G}^{A\mu\nu} , \tag{17}$$

where $G_{\mu\nu}^A$ is the color field strength tensor. Following Ref. 9, we define a anomaly-free current $\tilde{J}_\mu$ and consider only the u and d quarks as being light. One has

$$\tilde{J}_\mu = J_\mu^{\rm PQ} - \frac{1}{2} \cdot \left(x + \frac{1}{x} \right) \frac{1}{1+Z} \left[\overline{u} \gamma_\mu \gamma_5 u + z \overline{d} \gamma_\mu \gamma_5 d \right] , \tag{18}$$

where $Z = m_u/m_d$. The mass of VIA is calculated to be

$$\begin{aligned} m_a &= m_\pi \frac{f_\pi}{f_a} \left(x + \frac{1}{x} \right) \frac{\sqrt{Z}}{1+Z} \\ &\approx 25 \text{ keV} \cdot \left(x + \frac{1}{x} \right) \left(\frac{250 \text{ GeV}}{f_a} \right) , \end{aligned} \tag{19}$$

for $Z \approx 0.56$.

From (10) and (11), the following fermion mass matrices emerge:

Model Ia,

$$\begin{aligned} (M_u)_{ij} &= \left(h_{ij}^u v_k^* \right) , \quad (M_d)_{ij} = \left(h_{ij}^d v_2 \right) \\ (M_e)_{ij} &= \left(h_{ij}^e v_k \right) , \quad (M_\nu^D)_{ij} = \left(h_{ij}^\nu v_1^* \right) \end{aligned} , \tag{20}$$

with $v_k = v_l$ for $j = 1$ and $v_k = v_2$ for $j = 2, 3$ and Model Ib,

$$\begin{aligned} (M_u)_{ij} = \left(h^u_{ij} v^*_k\right) \;, \quad (M_d)_{ij} = \left(h^d_{ij} v_2\right) \\ (M_e)_{ij} = \left(h^e_{ij} v_2\right) \;, \quad \left(M^D_\nu\right)_{ij} = \left(h^\nu_{ij} v^*_1\right) \end{aligned} \;, \tag{21}$$

with $v_u = v_2$ for $j = 1, 2$ and $v_k = v_1$ for $j = 3$ where M_u, M_d, M_e and M^D_ν stand for up-type quark, down-type quark, charged lepton, and Dirac neutrino mass matrices respectively. In Model Ib, a small value of $v_1 < v_2$ gives a small mass for m_u and m_e. On the other hand, if $v_1 \gg v_2$ in Model Ib, we may understand why the top quark is heavier than other flavors.

Now let us study the neutrino masses. The Yukawa couplings of Eqs. (10) and (11) give the following general form for the neutrino mass matrix:

$$M_\nu = \begin{pmatrix} 0 & M^D_\nu \\ M^{D^T}_\nu & M_R \end{pmatrix} \;, \tag{22}$$

where

$$M_R = h^N \frac{\Lambda_1}{\sqrt{2}} \sim \Lambda_1 \;, \tag{23}$$

is the Majorana mass of the right-handed neutrino. Considering the structures of mass matrices in (20) and (21), it is reasonable to assume that $M^D_{\nu_i} \sim m^i_d$ for Model Ia, and $M^D_{\nu_i} \sim m^i_e$ for Model Ib. By using the "see-saw" formula,[19] one finds

$$M_{\nu_i}(\mathrm{Ia}) \sim \frac{(M_{\nu_i})^2}{M_R} \sim \frac{(m^i_d)^2}{\Lambda_1} \;, \tag{24}$$

and

$$M_{\nu_i}(\mathrm{Ib}) \sim \frac{(m^i_e)^2}{\Lambda_1} \;, \tag{25}$$

This places the light neutrino masses in the ranges:

$$\begin{gathered} M_{\nu_e} \sim 6.4 \times (10^5 - 10^{-8})\ \mathrm{eV} \;, \; M_{\nu_\mu} \sim 2.3 \times (10^{-2} - 10^{-5})\ \mathrm{eV} \;, \\ M_{\nu_\tau} \sim 2.0 \times (10 - 10^{-2})\ \mathrm{eV} \;, \end{gathered} \tag{26}$$

when we take $\Lambda_1 \sim 10^9 - 10^{12}$ GeV, and a slightly extended range

$$M_{\nu_e} \sim 2.5 \times (10^6 - 10^{-10}) \text{ eV} \ , \ M_{\nu_\mu} \sim 1.1 \times (10^{-1} - 10^{-5}) \text{ eV} \ ,$$
$$M_{\nu_\tau} \sim 3.2 \times (10 - 10^{-3}) \text{ eV} \ , \tag{27}$$

with $\Lambda_1 \sim 10^8 - 10^{12}$ GeV for Models Ia and Ib respectively.

Model II: we extend the Model I by adding another singlet complex Higgs field χ_2 which can be rewritten as

$$\chi_2 = \frac{1}{\sqrt{2}} e^{i\alpha_2} (\Lambda_2 + R_4 + iI_4) \ , \quad \langle \chi_2 \rangle = \frac{1}{\sqrt{2}} e^{i\alpha_2} \Lambda_2 . \tag{28}$$

The PQ symmetry transformation for χ_2 is

$$\chi_2 \to e^{i\alpha(x+1/x)} \chi_2 \ . \tag{29}$$

Except for χ_2 and N_R^i, the other scalar and fermion fields have the same PQ charges as the Model (Ia-b). Here we call them Model IIa and IIb with

$$(Z_N^1, Z_N^2, Z_N^3) = -\frac{1}{2} \left(x + \frac{1}{x} \right) (1, -1, -1) \ , \tag{30}$$

and

$$(Z_N^1, Z_N^2, Z_N^3) = -\frac{1}{2} \left(x + \frac{1}{x} \right) (1, 1, -1) \ , \tag{31}$$

respectively. The Higgs potential is given by

$$\begin{aligned} V \ = \ & \sum_i m_i^2 \phi_i^+ \phi_i + \sum_{i \le j} a_{ij} (\phi_i^+ \phi_i)(\phi_j^+ \phi_j) + b_{12} (\phi_1^+ \phi_2)(\phi_2^+ \phi_1) + \\ & + \Big[c_1 \chi_1 \chi_2 + c_2 \chi_1 (\chi_1 \chi_2)^2 + D_1 \phi_1^+ \phi_2 \chi_1^+ + D_2 \phi_1^+ \phi_2 \chi_2 + \\ & + e_1 \phi_1^+ \phi_1 \chi_1 \chi_2 + e_2 \phi_2^+ \phi_2 \chi_1 \chi_2 + \text{h.c.} \Big] \ , \end{aligned} \tag{32}$$

where $\phi_{i+2} = \chi_i$ $(i = 1, 2)$ and their Yukawa terms are given as follows

$$\begin{aligned}
\mathcal{L}_Y(\text{Model IIa}) &= \sum_i \left(h^u_{i1}\, \overline{q}^i_L\, \tilde{\phi}_1\, u^1_R + h^\nu_{i1}\, \overline{l}^i_L\, \tilde{\phi}_1\, N^1_R + h^e_{ij} \overline{l}^i_L \phi_1 e^1_R \right) + \\
&+ \sum_i \sum_{j\neq 1} \left(h^u_{ij}\, \overline{q}^i_L\, \tilde{\phi}_2\, u^j_R + h^\nu_{ij}\, \overline{l}^i_L\, \tilde{\phi}_2\, N^j_R + h^e_{ij} \overline{l}^i_L \phi_2 e^j_R \right) + \\
&+ \sum_{i,j} h^d_{ij}\, \overline{q}^i_L\, \phi_2\, d^j_R + h^N_{11}\, N^{1T}_R\, C N^1_R\, \chi_1 + \\
&+ \sum_{i,j\neq 1} h^N_{ij} N^{iT}_R\, C N^j_R\, \chi_2 + \text{h.c.}\,,
\end{aligned} \tag{33}$$

and

$$\begin{aligned}
\mathcal{L}_Y(\text{Model IIb}) &= \sum_i \sum_{j\neq 3} \left(h^u_{ij}\, \overline{q}^i_L\, \tilde{\phi}_2\, u^j_R + h^\nu_{ij} \overline{l}^i_L \tilde{\phi}_2 N^j_R \right) + \\
&+ \sum_i h^u_{i3}\, \overline{q}^i_L\, \tilde{\phi}_1\, u^3_R + h^\nu_{ij}\, \overline{l}^i_L\, \tilde{\phi}_1\, N^3_R) + \\
&+ \sum_{i,j} \left(h^d_{i,j}\, \overline{q}^i_L\, \phi_2\, d^j_R + h^e_{ij}\, \overline{l}^i_L\, \phi_2\, e^j_R \right) + \\
&+ \sum_{i,j\neq 3} h^N_{ij}\, N^{iT}_R\, C N^j_R\, \chi_1 + h^N_{33}\, N^{3T}_R C N^3_R\, \chi_2 + \text{h.c.}\,.
\end{aligned} \tag{34}$$

The axion field is

$$a = \frac{1}{f_a} \left[v_2 I_1 - v_1 I_2 - \frac{v^2}{v_1 v_2} \left(\Lambda_1 I_3 - \Lambda_2 I_4 \right) \right] \,, \tag{35}$$

with the decay constant

$$f_a = \frac{v}{v_1 v_2} \left[v_1^2 v_2^2 + v^2 (\Lambda_1^2 + \Lambda_2^2) \right]^{1/2} \,. \tag{36}$$

The invisible axion properties of Model I carry over to these models. However, the neutrino mass pattern can now be very different. Here one has

$$M_{\nu_e} : M_{\nu_\mu} : M_{\nu_\tau} \sim \frac{m_e^2}{\Lambda_1} : \frac{m_\mu^2}{\Lambda_2} : \frac{m_\tau^2}{\Lambda_1} \,, \tag{37}$$

and

$$M_{\nu_e} : M_{\nu_\mu} : M_{\nu_\tau} \sim \frac{m_u^2}{\Lambda_2} : \frac{m_c^2}{\Lambda_2} : \frac{m_t^2}{\Lambda_1} \,, \tag{38}$$

for Model IIa and Model IIb, respectively, which differ from (24) and (25). For example, one estimates that

$$M_{\nu_e} \sim 0.05 \text{ eV} , \quad M_{\nu_\mu} \sim 0.11 \text{ eV} , \text{ and } \quad M_{\nu_\tau} \sim 32 \text{ eV} , \tag{39}$$

by taking $\Lambda_1 \sim 5$ TeV and $\Lambda_2 \sim 10^8$ GeV in Model IIa and

$$M_{\nu_e} \sim 2.5 \times 10^{-4} \text{ eV} , \quad M_{\nu_\mu} \sim 20 \text{ eV} , \text{ and } \quad M_{\nu_\tau} \sim 25 \text{ eV} , \tag{40}$$

for $\Lambda_1 \sim 10^{11}$ GeV and $\Lambda_2 \sim 10^8$ GeV in Model IIb. The neutrino masses in (39) and (40) satisfy both (3) and (4). Clearly, the usual neutrino mass hierarchy[25] is lost here.

We have presented several new types of invisible axion models without the domain wall problem. The models look like the models between the DSFZ and KSVZ models. The domain wall problem is solved by recognizing the given special status to some flavors in the standard model rather than introducing the exotic quark. One may speculate that this may also lead to a new way of looking at the charged fermion mass hierarchy. The two doublets and two singlets VIA models successfully avoided the usual hierarchy of neutrino masses without having to extend the gauge group. This distinguishes them from other invisible axion models.

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