Electric and Magnetic Polarizabilities of Hadrons via Elastic Compton Scattering at KAON

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The static properties of hadrons, such as charge and spin and mass, have been extensively and generally successfully studied in a variety of quark model calculations. Dynamical properties present somewhat more of a challenge. Among the most basic of these are the electric ($\alpha$) and magnetic ($\beta$) polarizabilities describing the electromagnetic structure of hadrons. They characterize the induced transient dipole moments of hadrons in an external electromagnetic field. With $E$ and $H$ the electric and magnetic external field on the hadron, for Gaussian units the electric dipole moment $\vec{d}$ is given by $\vec{d} = \alpha \vec{E}$ and the magnetic dipole moment $\mu$ is given by $\mu = \beta \vec{H}$, while $d = 4\pi\varepsilon_0\alpha E$ in MKS units. During gamma-hadron Compton scattering, the lowest order scattering is determined by the charge and magnetic moment. The next order scattering is determined by the induced dipole moments. The dipole polarizabilities probe the rigidity of the internal structure of baryons and mesons, the dipole moments being induced by the rearrangement of the hadron constituents driven by the presence of the electric and magnetic fields of the photon during scattering. They are important physical attributes of the hadrons that should and can be studied experimentally and theoretically in order to understand hadron structure. A sophisticated understanding of hadrons within the framework of QCD will be tested, in part, by the prediction of these quantities. For the light charged pion, chiral symmetry leads to a precise prediction for the polarizabilities. For the heavier charged kaon, chiral perturbation theory can be applied to predict the polarizabilities. For these cases, the experimental polarizabilities subject the underlying chiral symmetry and chiral perturbation techniques of QCD to new and serious tests. Here the physics of electromagnetic polarizabilities is first described, followed by a review of previous experimental and theoretical polarizability results for the proton, neutron, pion, and kaon. A brief description is then given how polarizabilities for these hadrons can be studied at the proposed TRIUMF KAON facility.

The essential physics underlying electric polarizability can be seen by considering the one-dimensional problem of an electron bound by a harmonic force, subjected
to a harmonic electric field \( E = E_0 e^{-i\omega t} \) in the positive x-direction. This problem including a damping term leads to:

\[
\ddot{x} + \Gamma \dot{x} + \omega_0^2 x = \frac{eE_0}{m} e^{-i\omega t}; \quad x = \frac{eE_0}{m \omega_0^2 - \omega^2 - i\Gamma \omega}.
\]  

(1)

With \( d = eR(x) = \alpha R(E) \), where \( R \) designates the real part, we have:

\[
\alpha = \frac{e^2}{m} \frac{1}{\omega_0^2 - \omega^2 - i\Gamma \omega}.
\]  

(2)

For sufficiently small \( \omega \), we obtain \( \alpha \sim e^2/(m\omega_0^2) \), with the polarizability independent of the exciting frequency. This solution for \( \alpha \) describes the classical scattering of light from bound atomic electrons; for example, in He\(^4\) gas at STP. With \( N \) the number of (electron) oscillators per unit volume, the dielectric constant and index of refraction are given in terms of the polarizability \( \alpha \) by:

\[
\epsilon = 1 + 4\pi Na; \quad n = \sqrt{\epsilon}; \quad n \sim 1 + 2\pi Na.
\]  

(3)

For scattering at optical wavelengths, the incident photon energies \( h\omega \) are of the order of 4 eV. The low frequency condition discussed above is then satisfied, this energy being much smaller than typical \( h\omega_0 \) atomic binding energies. Since a changing dipole moment radiates energy at the rate \( P \sim \langle \vec{d} \rangle^2 \), we have \( P \sim \alpha^2 \omega^4 \sim \alpha^2 \lambda^{-4} \). This leads to a cross section for the scattering of light that depends on \( \lambda^{-4} \); so that the intensity of scattered and transmitted sunlight is dominated by blue and red respectively. Such scattering is denoted by Rayleigh scattering, following Rayleigh's explanation of blue skies and red sunrises and sunsets.

In the literature, hadron polarizabilities are usually quoted in Gaussian units as \( \text{cm}^3 \). The Gaussian units of \( 10^{-43} \text{ cm}^3 \) are used in most of the following discussions; such that a pion polarizability of \( 2.8 \times 10^{-43} \text{ cm}^3 \) is written as \( 2.8 \). It is useful to make a semi-classical estimate (in MKS units here) of the maximum magnitude of the electric field associated with a 100 MeV gamma ray incident on a target pion, in order to better understand the size of the induced moments. The average Poynting energy density in units of energy per unit volume is given in terms of the maximum value of the oscillating electric field by \( S/c = (\sqrt{2})e_0 E_m^2 \). By the uncertainty principle, the smallest volume in which one can hope to localize a gamma ray whose direction is completely unknown is of the order of \( (1/8)\lambda^3 \), where \( \lambda \) is the wave length. From the equation:

\[
(1/16)e_0 E_m^2 \lambda^3 \sim E_7,
\]  

(4)

one finds for such a gamma ray (and therefore others of the same energy) the rather large value \( E_m = 6.2 \times 10^{21} \text{ volts/meter} \), corresponding to approximately 82 MV potential drop across the diameter of a pion. For an intrinsic pion polarizability of \( 3. (3. \times 10^{-49} \text{ meter}^3 \text{ in MKS}), the induced electric dipole moment corresponds to an oscillation of the pion's quarks over a distance scale of 0.1% of the pion's diameter. This is a true dynamical property, similar to the displacement of protons from neutrons in the giant dipole resonance excitation of nuclei by gamma rays.

The hadron polarizabilities \( \alpha \) and \( \beta \) can be obtained from precise measurements of the Compton cross section. The Compton scattering amplitude \( F_C \) for a zero-spin hadron is written in terms of the incident (unprimed) and scattered (primed)
gamma ray energy $\omega$, polarization direction $\hat{e}$, momentum direction $\hat{k}$, and transverse direction vector $\hat{s} = \hat{k} \times \hat{e}$, as \(^1\):\(^3\):

$$F_C = F_T + F_R; \quad F_T = -\frac{e^2}{m} \hat{e} \cdot \hat{e}, \quad F_R = \omega'((\tilde{\alpha} \hat{e} \cdot \hat{e}' + \tilde{\beta} \hat{s} \cdot \hat{s}')).$$ \(5\)

Here $F_T$ and $F_R$ represent respectively the Thomson scattering from a point charge $e$, and the Rayleigh scattering amplitude arising from the polarization interaction of the target hadron with the gamma ray electromagnetic field. The $\tilde{\alpha}$ and $\tilde{\beta}$ polarizabilities implicitly include \(^1\)\(^-\)\(^3\) hadron form factor terms involving the mean square radius of the hadron’s charge distribution ($r^2$). The Thomson amplitude for proton Compton scattering ($\sim 2 \times 10^{-16} \text{cm}$) sets the scale by which one can gauge whether or not the Rayleigh polarizability amplitude is large enough to yield observable effects. The unpolarized Compton scattering cross section depends on the square of the Compton amplitude, including a Thomson-Rayleigh interference term. For proton Compton scattering with 100 MeV gamma rays, the $\omega^2$ term in the Rayleigh amplitude is about 1% of the Thomson term, so that it is the interference term that carries most of the polarizability information. For neutron Compton scattering, there is no Thomson amplitude, and therefore no corresponding interference term; but there is a magnetic moment term. The polarizabilities then enter in higher order $\omega^4$ cross section terms. Finally, the cross section data determines the Compton polarizabilities $\tilde{\alpha}$ and $\tilde{\beta}$, which are defined \(^1\)\(^-\)\(^3\) as the sum of the intrinsic polarizabilities $\alpha, \beta$ and form factor terms $\Delta \alpha, \Delta \beta$. It is the intrinsic polarizabilities that satisfy $d = \alpha E$ and $\mu = \beta H$. We have therefore:

$$\tilde{\alpha} = \alpha + \Delta \alpha, \quad \tilde{\beta} = \beta + \Delta \beta.$$ \(6\)

Perturbation theory \(^1\) gives the relationship:

$$\tilde{\alpha} = 2 \sum \frac{|\langle 0|d|n\rangle|^2}{E_n - E_0} + \frac{e^2}{3m}(r^2),$$ \(7\)

where the first and second terms give the intrinsic and form factor contributions, respectively. Here $d$ is the electric dipole operator $d = \sum \epsilon r_k$. Similarly \(^1\)\(^,\)\(^3\),

$$\tilde{\beta} = 2 \sum \frac{|\langle 0|\mu|n\rangle|^2}{E_n - E_0} - \frac{e^2}{m}(r^2)(1 - \eta).$$ \(8\)

The first and second terms respectively give the intrinsic and form factor contributions, and $\mu$ is the magnetic dipole operator $\mu = (2mc)^{-1} \sum \epsilon r_k \times \vec{p}_k$. The parameter $\eta$ is a measure of the size of a quark. It is the ratio of the mean square quark radius to the hadron mean square charge radius. Here we estimate $\eta \sim 0.32$ in order to achieve agreement with the dispersion sum rule \(^1\): $\tilde{\alpha} + \tilde{\beta} = 14.2$.

Theoretical calculations for $\alpha$ and $\beta$ have been made for hadrons in a variety of quark models. Consider a simple non-relativistic quark model calculation for proton polarizabilities \(^1\)\(^,\)\(^3\). We describe this calculation here in order to illustrate the ingredients of such calculations, even though this is clearly not the best quark model calculation available. The intrinsic contribution of eqn. 7 can be evaluated using closure by saturating the sum over odd parity dipole excitations near 660 MeV excitation for the proton, giving:

$$\alpha_p \sim \frac{2}{\Delta E} \langle p| (\sum q_i x_i)^2 |p \rangle \sim \frac{2}{\Delta E} \sum q_i^2 \langle p | x_i^2 | p \rangle \sim \frac{2}{3} \frac{(r^2)}{\Delta E} e^2 (\frac{4}{9} + \frac{4}{9} + \frac{1}{9})$$ \(9\)
With $\Delta E \sim 660$ MeV and $\langle r^2 \rangle \sim 0.74$ fm, we obtain:

$$\alpha_p = \frac{2 \cdot 0.74}{3 \cdot 660} \frac{e^2}{\hbar c} = 10.8, \quad \Delta \alpha_p = \frac{1}{3} \frac{\hbar c e^2}{\hbar c 938.3} = 3.8, \quad \bar{\alpha}_p = \alpha_p + \Delta \alpha_p = 14.6. \quad (10)$$

The magnetic polarizability can similarly be evaluated by saturating the magnetic dipole excitations with the N to $\Delta$ transition, giving $^{1,3}$:

$$\beta_p \sim \frac{2}{M_\Delta - M_p} |\langle p | \hat{\mu}_z | \Delta \rangle|^2 \sim \frac{2}{M_\Delta - M_p} \left( \frac{2\sqrt{2}}{3} \mu_p \right)^2 \sim 7.4, \quad (11)$$

where $\mu_p$ is the proton magnetic moment. Here, the proton to $\Delta$ magnetic dipole transition matrix element is written $^1$ in terms of the proton magnetic moment, following the nonrelativistic quark model. We estimate:

$$\Delta \beta_p \sim -\frac{\hbar c \mu_p}{\hbar c 938.3} (1 - 0.32) = -7.8, \quad \bar{\beta}_p \sim \beta_p + \Delta \beta_p \sim -0.4 \quad (12)$$

From above, the intrinsic electric polarizability depends on the bag radius $^{3,4}$ as:

$$\alpha \sim \frac{\langle r^2 \rangle}{\Delta E} \sim R^3, \quad (13)$$

since $E \sim p$ for relativistic energies; and $p \sim R^{-1}$ by the uncertainty principle. Therefore, the size of the bag in quark models has a strong influence on the calculated polarizabilities. This has led Schafer et al. $^5$ to suggest that the polarizabilities may offer a possibility to measure the effective size of nucleon bags inside of nuclei.

For the proton, calculations are available in a variety of quark $^4$-$^{10}$ models. The proton electric polarizability $\bar{\alpha}$ can be separated as in eqn.7 into a classical part related to the electromagnetic size, and an intrinsic part. In the calculation of Weiner and Weise $^4$, the intrinsic part is understood as being mainly due to the role of the charged pion cloud surrounding the proton core. The relative contributions to the polarizabilities from the quark core and pion cloud depend sensitively on the quark core radius in their model. A small quark core radius is required in their model for a variety of reasons, including the need to reproduce the position of the excited odd parity states of the nucleon. Recent calculations have also shown that the difference in the electric polarizabilities for the proton and the neutron are sensitive to the polarization of the quark sea $^7$.

For the nucleon case, with photon energies below the nucleon resonances ($k \leq 100$ MeV), the Compton cross sections are small and difficult to measure: $\sigma_p \sim 10 - 20$ nb/sr and $\sigma_n \sim \sigma_p / 25$. Above the pion threshold, the cross sections are larger but their dependence on the polarizabilities becomes model dependent, due to the increasing need to take into account amplitudes with intermediate $\pi$-nucleon or $\Delta$ vertices. Lvov nonetheless claims $^2$ that the neutron polarizabilities should be studied optimally with 60 - 120 MeV gamma-ray energies, and also surprisingly in the higher energy range of 200 - 260 MeV. The case of the neutron is further complicated by the requirement of a deuterium target and the detection of the recoil neutron in order to completely describe the reaction. The gamma-proton $^{1,11}$ and gamma-neutron $^{12,13}$ Compton scattering were measured with incident gamma rays on proton and deuterium targets. A fit $^1$ to these data was carried out including a very reliable sum rule constraint for $\bar{\alpha} + \bar{\beta}$, where the constraint is based on a
dispersion relation calculation using the total photoproduction cross section. The fit yields $\alpha_p = (11.3 \pm 2.5)$ and $\beta_p = (2.9 \pm 2.5)$. One can also relate $\alpha$ and $\beta$ to the results of deep inelastic electron scattering $^{11}$, although this relationship is more model dependent, considering the need to extrapolate from high momentum transfer to real photons. The resulting $\alpha_p = (9.3 \pm 2.0)$ is barely within the lower limits of the errors quoted in Ref. 1. The neutron results $^{13}$ are $\alpha_n = 11.7(\pm 4.3) - 11.7$). These deduced polarizabilities for the proton do not yet provide a stringent test of model calculations, and the information on the neutron is so poor that the interesting prediction for $\alpha_p - \alpha_n$ due to quark sea effects cannot yet be examined. Improved gamma-nucleon experiments are being planned $^{14}$ with polarized tagged gamma rays at the BNL LEGS facility $^{15}$. With linearly polarized photons, it is possible to measure separately the cross sections for photons polarized parallel and perpendicular to the reaction plane. Such data will allow the determination of $\alpha$ and $\beta$ with significantly reduced error bars.

For the pion polarizability, the $\gamma - \pi$ scattering was measured $^{16-19}$ with 40 GeV pions via radiative pion scattering in the nuclear Coulomb field ($\pi^- + Z \rightarrow \pi^- + Z + \gamma$); where the incident pion Compton scatters from a virtual photon in the Coulomb field of a nucleus of atomic number $Z$; and the final state gamma ray (typically 30 GeV) and pion (typically 10 GeV) were detected in coincidence. This reaction (equivalently $\gamma + \pi^- \rightarrow \gamma + \pi^-$ scattering for a laboratory gamma ray of several hundred MeV incident on a target $\pi^-$ at rest) is an example of the well tested Primakoff formalism $^{18,19}$ that relates processes involving real photon interactions to production cross sections involving the exchange of virtual photons. The spectral distribution of equivalent virtual photons is given by the Weizsacker-Williams distribution; which can therefore provide a consistency check on the data analysis. The pion electric polarizability $\alpha_\pi$ was initially deduced $^{16}$ to be:

$$\alpha_\pi = 6.8 \pm 1.4,$$

where it was assumed in the analysis that $\alpha_\pi + \beta_\pi = 0$. In a subsequent analysis $^{17}$, the values given are:

$$\beta_\pi = -7.1 \pm 2.8_{stat} \pm 1.8_{syst}, \quad \alpha_\pi + \beta_\pi = 1.4 \pm 3.1_{stat} \pm 2.5_{syst}.$$ (15)

Although a value for $\alpha$ was not given in this later analysis, the results given imply:

$$\alpha_\pi \sim 8.5 \pm 4_{stat} \pm 4_{syst},$$ (16)

somewhat higher than originally given. We note however that a similar experiment $^{18,19}$ at the Fermi Laboratory at 200 GeV with limited statistics found that the data are consistent with the simplest predictions of a point like pion, with no indication of pion structure effects such as pion polarizability. The careful experimental checks of this latter group are well documented $^{18-20}$. But since this latter experiment did not determine statistical upper limits for the polarizabilities; it is difficult to compare the results of this experiment to theory. The different results for the pion from the groups demonstrate the need for much higher quality data.

In the radiative pion scattering experiments, it was shown (experimentally $^{16-19}$ and theoretically $^{21}$) that the Coulomb amplitude clearly dominates and yields sharp peaks in t-distributions at very small four momentum transfers $t \leq 6 \times 10^{-4}(GeV/c)^2$. The gamma-pion Compton scattering for these experiments corresponds to gamma-ray energies in the range 60-600 MeV in the pion rest frame. The
cross sections corresponding to the sharp peaks in the \( t \)-distributions for targets with different atomic number \( Z \) scaled as \( Z^2 \), further demonstrating the correspondence with the electromagnetic Compton effect. Background from other processes could easily be estimated and subtracted by extrapolating in \( t \) from events in the region of flat \( t \)-distribution of \( 3.8 \times 10^{-3} \text{(GeV/c)}^2 \). The sources of these backgrounds are described below. More theoretical work is needed to understand the different backgrounds, and to determine the optimum conditions for these experiments.

For gamma-pion Compton scattering, the Compton amplitude in principle may include contributions of pion (or rho) rescattering. The rescattering diagram includes two vertices, one for the pion strong interaction (with two real and two virtual pions), and one for the Compton effect with the virtual pions. These contributions for the conditions of the 40 GeV pion experiment have been evaluated theoretically \(^{22}\) and were claimed to affect the determination of the pion polarizabilities at the level of 10%. If these calculations are improved and made accurate to 10%, then any rescattering effects will be negligible.

Another strong background is the coherent process of pion elastic scattering accompanied by gamma emission. Because the squared four-momentum transfer \( t \) to the nucleus is so small, incoherent pion inelastic scattering with or without gamma emission will be negligible. The radiative pion elastic scattering will not have the sharp \( t \)-distribution peak of the Compton scattering. This strong process is described theoretically in terms of the exchange with the nucleus of a neutral meson such as a \( \pi^0, f_2, \rho \). Following this exchange, an intermediate state is formed which then decays to a gamma ray and a pion. If the value \( \sqrt{s} \) (the pion-gamma invariant mass) is kept sufficiently small via experimental cuts, the intermediate state will be only a pion; otherwise one needs to sum over all possible accessible intermediate states. The amplitudes for these strong processes need to be added coherently to the Compton amplitude in order to give the total amplitude for the radiative scattering process. For an incident pion at 40 GeV and \( \sqrt{s} \leq 3.2m_\pi \), Gal'perin et al. \(^{21}\) estimated that these strong cross sections account for only 2.5% of the cross section below \( t = 2 \times 10^{-4} \text{(GeV/c)}^2 \) for a carbon target, the remainder being the interesting Compton channel. This estimate is consistent with the carbon data of Antipov et al. \(^{16,17}\). These backgrounds are not expected to vary significantly with incident energy, although they should decrease with decreasing \( s \), and change with target atomic number. The data for heavier targets \(^{16,17}\) in fact show smaller percent backgrounds from the strong processes. The coherent radiative pion elastic channel also contributes an interference term with the Compton amplitude, but this term should be very small as the elastic and radiative elastic amplitudes are dominantly imaginary, so that the Coulomb-nuclear scattering phase shift is near 90 degrees. The sign of different parts of the interference term would change with \( \pi^+ \) and \( \pi^- \) scattering, so that it could be a useful test to take data with both positive and negative pions. Also, the isovector part of the interference term would depend on the target isospin, being zero only for an isospin zero target. Considering the expected \( Z \)-dependence of the background contributions discussed in this paragraph, it should be valuable in these experiments to use a variety of targets, ranging from light isospin zero targets such as C\(^{12}\) to heavy targets such as Pb\(^{208}\) and \( \text{U}^{238} \).

Other backgrounds come from the very interesting higher cross section inelastic channels \( \gamma + \pi^- \rightarrow \rho^- \rightarrow \pi^- + \pi^0 \); and \( \gamma + \pi^- \rightarrow A_1(1260) \rightarrow 3\pi \). These were studied in their own right, the former \(^{19}\) to measure the radiative decay of the \( \rho^- \); and the latter \(^{20}\) to study the radiative decay of the \( A_1(1260) \). Background events result when only one of the gamma rays from a \( \pi^0 \) decay reaches the detector,
or when the detector cannot separate the two $\pi^0$ decay gamma rays. For a kaon polarizability experiment, backgrounds will come similarly from resonances such as the $K^*(892)$, whose radiative decays are also generally interesting.

One must also evaluate electromagnetic corrections to the process of radiative pion scattering of high energy pions in the Coulomb field of a nucleus, where the requirement is to measure only single photon bremsstrahlung emission. Here the detailed properties of the gamma detector are important, such as the photon detector threshold, $t$-resolution, and the two-photon angular resolution. Calculations are also needed as a function of the target atomic number to determine the optimum $Z$ to maximize the signal compared to the double bremsstrahlung background. For the setup of Antipov et al. $^{16,17}$ at 40 GeV with $t \leq 2 \times 10^{-4}$ (GeV/c)$^2$ for a carbon target, electromagnetic double bremsstrahlung corrections were estimated $^{23}$ to affect polarizability determinations at the level of 30%. These corrections can be made smaller with improved detectors, and they can be most likely made more reliably with improved calculations. After correcting for the double bremsstrahlung events, the remaining two-photon events would be due to $\pi^0$ decays. For the $\pi^0$ symmetric decays, it will be easy to resolve the two gamma rays, as their separation distance at the gamma detector will be maximal. From the yield of such large separation events, it will be possible to use the phase space distribution to estimate the small separation $\pi^0$ decay contributions.

To specifically illustrate some of the kinematics germane to a KAON experiment, the reaction:

$$\pi + Z \rightarrow \pi' + Z' + \gamma$$  \hspace{1cm} (17)

is considered for a 20 GeV incident pion, where $Z$ is the nuclear charge. The 4-momentum of each particle is $P_\pi$, $P_Z$, $P_{\pi'}$, $P_{Z'}$, $k'$, respectively. In the one photon exchange domain, eqn. 17 is equivalent to:

$$\gamma + \pi \rightarrow \gamma' + \pi'$$  \hspace{1cm} (18)

and the 4-momentum of the incident virtual photon is $k = P_Z - P_{Z'}$. The cross section for the reaction of eqn. 17 can be written as:

$$\frac{d\sigma}{dtdsd\Omega} = \frac{Z^2 \alpha_f |F(t)|^2}{\pi} \frac{t - t_0}{s - m_{\pi^0}^2} \frac{d\sigma_{\pi\gamma}}{dt}$$  \hspace{1cm} (19)

where $d\sigma_{\pi\gamma}/d\Omega$ is the unpolarized differential cross section for eqn.18 (for real photons), $t$ is the square of the four-momentum transfer to the nucleus, $F(t)$ is the nuclear form factor (essentially unity at small $t$-values), $\sqrt{s}$ is the mass of the $\pi\gamma$ final state, and $t_0$ is the minimum value of $t$ to produce a mass $\sqrt{s}$. The quantities $s$ and $t$ are Lorentz invariants. We have

$$t = k^2 \equiv -M(V)^2,$$  \hspace{1cm} (20)

where $k$ is the 4-momentum transferred to the nucleus, and $M(V)$ is the virtual photon mass. Since $t = 2M_Z(E_{Z',\text{lab}} - M_Z) > 0$, the virtual photon mass is imaginary. To approximate real pion Compton scattering, the virtual photon must be almost real; and so $M(V) < 0.0167$ GeV corresponding to $t < 2.8 \times 10^{-4}$ (GeV/c)$^2$ can be required in the experiment. In addition,

$$-s = (P_{\pi'} + k')^2 \equiv -M(\pi\gamma)^2,$$  \hspace{1cm} (21)
where $M(\pi \gamma)$ is the pion-photon invariant mass. The minimum value for $t$ is given by $t_0 \sim (s - m_\pi^2)^2/4|\vec{P}_\pi|^2$, corresponding to $t_0 \sim 10^{-6}(GeV/c)^2$ for $\sqrt{s} = 1.75m_\pi$ at 20 GeV incident energy. If the gamma detector threshold is set very low, the minimum measurable transfer will approach $t_0$. With the lead glass detectors described below, a $t$-resolution of the order of $\sim 6.0 \times 10^{-4}(GeV/c)^2$ was achieved at 40 GeV, where the experimental $\gamma$-ray energy resolution was the main limiting factor. This $t$-resolution set the experimental maximum in $t$ for accepted events. With an improved $\gamma$-ray detector, it should be possible to achieve significantly better $t$-resolution, and to obtain a major fraction of the Compton cross section using an experimental maximum limit for accepted events of $t = 2.8 \times 10^{-4}$ (GeV/c)$^2$, or even lower. The maximum of the differential cross section for reaction (17) occurs at $t = 2t_0$ and grows as $|t_0|^{-1} \sim (\vec{P}_\pi)^2$, where $\vec{P}_\pi$ is the laboratory incident pion 3-momentum. The integrated Compton cross section up to $t = 2.8 \times 10^{-4}$ (GeV/c)$^2$ grows less slowly, as $\ln(\vec{P}_\pi)$. With this experimental limit and backgrounds that are not expected to vary rapidly with incident energy, the percent background would increase from the estimated 21 2.5% at 40 GeV discussed previously to roughly 3.3% at 20 GeV and 4.3% at 10 GeV. One could then consider experiments at KAO at both 20 and 10 GeV with manageable backgrounds at both energies. The cross section of eqn. 19 assumes that the atomic electrons do not screen the nuclear Coulomb field. This is valid as long as the incident energy is not too high ($E < 100 GeV$), else $t_0$ will be so small so as to correspond to an impact parameter $b$ within the electron cloud of the atom ($t_0 \sim b^2$).

The energy of the incident photon in the pion rest frame is:

$$E(V) = (s - m_\pi^2 + t)/2m_\pi \sim (s - m_\pi^2)/2m_\pi$$

(22)

at small $t$; so that the energy of the virtual photon is determined by $s$. The elemental cross section at low $E(V)$ is a function of $E(V)$, $\cos(\theta)$, $\alpha$, $\beta$; where $\theta$ is the Compton scattering angle in the pion rest frame. In this frame, the nucleus $Z$ represents a beam or cloud of virtual photons sweeping past the pion. At large $E(V)$ ($\geq 140$ MeV), strong interaction effects increase, and these will enter in a model dependent manner. Thus, a desirable experimental requirement would be to keep $M(\pi \gamma)/m_\pi < 1.75$, thereby achieving $E(V) \leq 140$ MeV, as seen in Fig. 1. This constitutes a hadronic low frequency condition, in analogy to that discussed for eqn. 2. This condition was not achieved in the work of Antipov et al. 16,17, which rather accepted $M(\pi \gamma)/m_\pi < 3.2$. Consider the case of 20 GeV incident laboratory pions. Figure 1 shows as a function of $M(\pi \gamma)/m_\pi$, the maximum and minimum values of $M(V)$ and $E(V)$. Also shown with stars in Fig. 1 is the largest value of $M(V) = 0.0167 GeV$ that should be accepted in the experiment. The range of values is associated with the recoil polar angles of the nucleus. Figures 2-4 show as a function of final photon laboratory angle for $M(\pi \gamma)/m_\pi = 1.75$ and $M(V) = 0.0167$ GeV the ranges of Compton scattering angle in the pion rest frame (Fig.2); the lab angle of the final pion and the pion-photon lab opening angle (Fig. 3); and the laboratory kinetic energy of the final pion and photon (Fig.4). The ranges are due to the azimuthal angle ranges of the recoiling nucleus and the final photon. The figures show results in the laboratory frame for outgoing $\gamma$-rays emitted up to 1 degree, and the corresponding outgoing pions emitted up to 0.5 degrees. The gamma ray energies considered then range from 4 - 14 GeV, and the corresponding outgoing pion energies range from 6-16 GeV. The corresponding Compton scattering angular range is $\sim 40 - 180$ degrees in the $\pi$ rest frame. The Compton scattering at angles
less than 40 degrees could be measured in separate runs with the \( \gamma \)-ray detector positioned off the beam axis to accept \( \gamma \)-rays emitted at angles greater than one degree in the laboratory frame. Such extra data runs were not carried out in previous experiments \(^{16-18}\), which is why only partial Compton scattering angular distributions were obtained.

It is useful to describe briefly the experimental apparatus used in previous experiments \(^{16-19}\), even if future experiments may have improved equipment. In the Fermi Lab experiment \(^{19}\), the beam was 1 cm in diameter and particle trajectories were defined by two sets of proportional chambers, giving the trajectory angle to a precision of \( \pm 0.03 \) mrad. The integrated beam flux in the entire experiment was \( 2.2 \times 10^9 \) \( \pi^- \), roughly the flux available in a few seconds at KAON. A finely segmented liquid-argon calorimeter (LAC) was used for high energy gamma rays, with a spatial resolution \( \sigma \) of \( \sim 0.7 \) mm per projected coordinate (X and Y), and the ability to detect the position of multiple gamma ray hits with good two-photon resolution. The LAC was positioned 24 meters downstream of the target, centered on the beam axis, with an angular acceptance of approximately \( \pm 1.5 \) degrees. The LAC energy resolution in GeV was \( \sigma^2 = 0.18 + (0.20)^2 E \), which would give \( \sigma \sim 0.9 \) GeV at 15 GeV. Charged particle trajectories and momenta were measured using drift chambers and proportional chambers together with an analyzing magnet. The drift chamber resolution was \( \sigma = 0.20 \) mm per plane, giving an angular resolution of \( \sim 0.06 \) mrad; and the momentum resolution was \( \delta p / p \sim 8 \times 10^{-5} p \) with \( p \) in GeV/c; which is only 20 MeV/c at 15 GeV. In the Serpukov \(^{16}\) experiment, beam particles were detected with an angular resolution of \( \sim 0.06 \) mrad. Pions were detected using an analyzing magnet and photons with a lead glass array; their angles defined with \( \sigma = 0.12 \) mrad. The lead glass energy resolution was the main component in giving an overall energy resolution of \( \sigma \sim 1.5 \) GeV for the summed 40 GeV energy of the detected pion and \( \gamma \)-ray. The data selection criteria required one photon and one charged particle in the final state, their total energy consistent with the beam energy, small \( t \), and other position, angle, and energy conditions. The limiting factor in these experiments \(^{16,19}\) was primarily beam quality and quantity, not detector technology. But improved rate capability of the detection system will be valuable in order to make optimal use of the \( \sim 10^9 \) sec\(^{-1}\) flux available at KAON. And improved \( \gamma \)-ray energy resolution would be valuable in order to improve the \( t \)-resolution. This would then allow setting a lower experimental limit on \( t \), thereby decreasing the background contribution from strong processes.

Recently, calculations of the pion polarizability have been carried out by Bernard, Hiller, and Weise \(^{24}\); and by Holstein and Donoghue \(^{25,26}\). Bernard, Hiller, and Weise \(^{24}\) compare the experimental pion polarizability to several models in which vector meson dominance diagrams play an important role. They find values for \( \alpha_\pi \) ranging from \( \sim 7 - 14 \). Another theoretical approach is via dispersion relations \(^{27}\). The s-channel \(^1 (\gamma + \pi \rightarrow \gamma + \pi) \) and t-channel \(^{27} (\gamma + \gamma \rightarrow \pi + \pi) \) dispersion sum rules yield, respectively:

\[
\tilde{\alpha}_\pi + \tilde{\beta}_\pi = 0.39 \pm 0.04, \quad \tilde{\alpha}_\pi - \tilde{\beta}_\pi \sim 10.8, \quad (23)
\]

with the uncertainty in the latter values not given \(^{27}\), but most likely at least 25\%. These sum rules then imply \( \alpha_\pi \sim 5.6 \) and \( \beta_\pi \sim -5.2 \), values that should be considered model dependent theoretical estimates. Other calculations are also available \(^{28-31}\), but will not be discussed in detail here. In most of these calculations, the pion electric polarizability can be separated into two terms, as in eqns. 6. The classical term is related to the pion electromagnetic size, and is roughly twice as large.
as the empirical $\alpha_\pi$ quoted above. The second term is an intrinsic part associated with possible $1^+$ excited states of the pion reached by electric dipole transitions, and with vacuum polarization effects. The large positive size of the classical term implies that the intrinsic polarizability must be negative. This feature was pointed out $^{24,25}$ to be in sharp contrast to the proton case, where the intrinsic contributions were larger than the classical, with their summed positive contributions close to the empirical values quoted above. These results indicate striking differences in the electric dipole response of the proton and pion.

Holstein and Donoghue $^{25,26}$ take a more fundamental approach than the two calculations described above. They describe the $\gamma - \pi$ interaction with a chiral symmetric effective Lagrangian, one which does not explicitly incorporate vector meson dominance. Unitarity is achieved by adding pion loop corrections to lowest order, and the resulting infinite divergences are absorbed into physical (renormalized) coupling constants. The physical coupling constants are determined using empirical data for the pion charge radius and for the axial structure constant in radiative pion decay. The electric and magnetic polarizabilities are fixed by the physical coupling constants. The results:

$$\alpha_\pi + \bar{\beta}_\pi = 0, \quad \alpha_\pi = -\bar{\beta}_\pi = 2.8 \quad (24)$$

are then precise predictions of chiral symmetry. Holstein $^{25}$ emphasizes that there are no free parameters or model assumptions in this calculation, other than the assumption of chiral symmetry and low momentum transfer. But it is known that there are other successful forms for effective Lagrangians, as described recently by Meissner $^{32}$. Some of these Lagrangians, such as the hidden gauge form, have the physically appealing property of explicitly incorporating $\rho$ meson dominance, and therefore being valid also at high momentum transfer. Holstein $^{25}$ shows that the physical picture underlying their approach is that $A_1(1260)$ meson exchange via an axial meson pole diagram provides the main contribution to the polarizability. Can such a high virtual excitation process be well described by a low excitation effective Lagrangian; or does it perhaps require an effective Lagrangian valid to higher momentum transfers? It would be interesting to see to what extent the pion polarizability results of Holstein and Donoghue are reproduced by other choices of the effective Lagrangian.

The next higher order Compton scattering amplitude (with more pion loops) can be calculated precisely with chiral perturbation theory. This correction has so far been estimated $^{25}$ to be at most 25% for $\tilde{\alpha}, \tilde{\beta}$, which should therefore be considered the present uncertainty on the theoretical value of Holstein and Donoghue. The overall experimental uncertainty for the pion polarizabilities should be evaluated from the relatively large statistical and systematic error bars shown in eqns. 15,16 from the data $^{17}$ of Antipov et al.. The null results of Ref. 18 offer no assistance until those authors provide experimental upper limits. Overall, Holstein and Donoghue's theoretical results $\tilde{\alpha}_\pi = -\tilde{\beta}_\pi = 2.8 \pm 0.7$ are therefore still at the limit of consistency with the data. Holstein $^{25}$ compared his calculation to the initial experimental results of Ref. 16 as given in eqn. 14; and discussed the striking differences (2.8 theory versus 6.8 experiment) in terms of a three standard deviation discrepancy, and therefore a surprising contradiction with a precise chiral symmetry prediction. The later analysis including the systematic uncertainties does not allow such a clear cut conclusion. One certainly expects the chiral symmetry prediction to be good to something like 25%, so that a large discrepancy if true would be very surprising indeed. Clearly required are polarizability experiments that give polarizabilities
with less than 10% overall uncertainties; and calculations to higher order via chiral
perturbation theory.

The promise of a high intensity, high quality, high energy KAON facility at
TRIUMF opens the possibility of Hadron Compton scattering for different particle
types, such as $\pi^+$, $K^+$, n, p, and others. Starkov, Fil'kov, and Tsarev have
given detailed cross section predictions for the radiative scattering with pions,
kaons, and nucleons, showing clearly the sensitivity of the cross sections to the
polarizabilities. The high intensities for pions will allow greatly improved statistics
compared to the 7000 small $t$ events of the radiative pion experiments. An
improved pion experiment is needed to establish whether or not chiral symmetry
(at the heart of QCD models) is violated for the pion polarizabilities. The kaon
polarizabilities are also of importance for complementary tests away from the chiral
limit. For kaon polarizabilities, KAON high intensity beams are particularly valu-
able, as kinematics reduces the polarizabilities compared to the pion by a factor
$m_K F_K^2 / m_\pi F_\pi^2 = 5.35$, where the F terms are the pion and kaon decay constants.

The improved duty factors and also higher purity beams at KAON allow much
improved experiments. Until now, only an upper limit at 90% confidence was mea-
sured (via energy shifts in heavy Z kaonic atoms) for the $K^-$, with $\alpha_{K^-} \leq (200.0)$. The 20 - 30 GeV beam energies at KAON are important to get a good yield for low $t$
events in the radiative scattering, and also to reduce backgrounds from the decay
of unstable hadrons by significantly boosting their lifetime. The easiest experiments
are those with charged hadrons, but those with uncharged hadrons should also be
possible. For unstable hadrons, the easiest experiments are those where the labora-
tory decay length ($L = \gamma \beta c r$) is at least ten meters (as for charged pions and kaons),
in order to allow easy detection of the scattered hadron. Otherwise, one would need
to measure the energy of the scattered hadron by observing its decay products.
The radiative scattering experiment with proton and neutron beams gives the same
physics in principle as with real gammas on proton and deuteron targets, with the
advantage that the neutron radiative cross section for a Pb target is enhanced by
about a factor of 75 compared to the $\gamma$-n Compton scattering cross section. Also,
the $\gamma$-n scattering is then cleaner without the complications of the deuteron. With
better statistics for $\gamma$-n scattering therefore, some more exotic effects (more than
polarizabilities) can also be studied; such as an experimental confirmation that the
sign of the vertex function describing the decay of the $\pi^0$ to two gammas is negative.
The figures of merit for the technique are to compare the polarizabilities of the
proton as deduced by radiative proton scattering and classic $\gamma$-p Compton scatter-
ing; and to measure $\gamma$-e and $\gamma$-\mu Compton scattering where the cross sections can be
calculated precisely with QED. Theoretical calculations for hadron polarizabilities
are needed for the large number of particle types that can be studied. A variety of
different high resolution gamma and hadron spectrometers must also be designed
and constructed. The gamma spectrometers must distinguish between $\pi^0$ or double
bremsstrahlung events and the single gamma events of Compton scattering. They
also should be designed with low energy thresholds that yield data for equivalent
gamma ray energies over a large energy range, such as 60 - 600 MeV in the hadron
rest frame. Experimental simulations are in progress to determine the cross section
precision required for sensitive determinations of both $\bar{\alpha}$ and $\bar{\beta}$ separately, and of
the combination $\bar{\alpha} + \bar{\beta}$. It should be possible for the theoretical and experimental
preparations for this exciting experimental program to start soon (with test runs
at low intensity accelerators) and then to match an early turn-on date for KAON.
An important requirement is for interested experimentalists and theoreticians
to come forward and join this effort.

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