Paper presented at Lattice '90 - International Conference of Lattice Field Theory, Tallahassee, October 8 - 12 CA9350039

TRI-PP-90-83
UK/90-11
Nov 1990
(UK-PP--90-11)

ON THE ELECTROMAGNETIC PROPERTIES OF THE BARYON OCTET *

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A numerical simulation of quenched QCD on a $24 \times 12 \times 12 \times 24$ lattice at $\beta = 5.9$ is used to calculate the electric and magnetic form factors of the baryon octet. Magnetic moments, electric radii, magnetic radii, and magnetic transition moments are extracted from the form factors.

1. INTRODUCTION

Here we report the results of a calculation of the electric and magnetic form factors for the octet baryons in lattice QCD. From these quantities we extract magnetic moments, electric radii, magnetic radii, and magnetic transition moments. A systematic examination of all the octet baryons reveals the interplay of quark mass effects and spin dependent forces which are expected in an underlying quark description of baryons. To put our results into perspective, we compare our calculations with experimental measurements where available, with recent quark and Skyrme model calculations, and with QCD sum rule calculations.

2. THEORETICAL FORMALISM

The electromagnetic form factors may be extracted from the following ratio of 2 and 3 point Green functions.

$$R(t_2,t_1;\vec{p}',\vec{p};\Gamma,\Gamma';\mu) = \sqrt{\frac{\langle G^{Bj^\mu B}(t_2,t_1;\vec{p}',\vec{p};\Gamma)\rangle\langle G^{Bj^\mu B}(t_2,t_1;-\vec{p},-\vec{p}';\Gamma)\rangle}{\langle G^{BB}(t_2;\vec{p}';\Gamma')\rangle\langle G^{BB}(t_2;-\vec{p};\Gamma')\rangle}}$$

$$\to \sqrt{\frac{E_p+M}{2E_p}}\sqrt{\frac{E_{p'}+M}{2E_{p'}}}\,\overline{R}(\vec{p}',\vec{p};\Gamma,\Gamma';\mu).$$

where

$$\langle G^{BB}(t;\vec{p},\Gamma)\rangle = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \Gamma^{\beta\alpha}\,\langle 0\,|\,T\left(\chi^\alpha(x)\overline{\chi}^\beta(0)\right)|\,0\,\rangle,$$

and

$$\langle G^{Bj^\mu B}(t_2,t_1;\vec{p}',\vec{p};\Gamma)\rangle = \sum_{\vec{x}_2,\vec{x}_1} e^{-i\vec{p}'\cdot\vec{x}_2} e^{+i(\vec{p}'-\vec{p})\cdot\vec{x}_1} \Gamma^{\beta\alpha}\,\langle 0\,|\,T\left(\chi^\alpha(x_2)j^\mu(x_1)\overline{\chi}^\beta(0)\right)|\,0\,\rangle.$$

χ denotes standard baryon interpolating fields. The Sachs form factors may be isolated with appropriate choices for Γ and the component of the current.

$$\overline{R}(\vec{q},\vec{0};\Gamma_4,\Gamma_4,4) = \mathcal{G}_E(q^2),$$
$$\overline{R}(\vec{q},\vec{0};\Gamma_j,\Gamma_4,k) = \frac{\mathcal{G}_M(q^2)|\epsilon_{ijk}q^i|}{(E_q+M)}.$$

where

$$\Gamma_j = \frac{1}{2}\begin{pmatrix}\sigma_j & 0\\ 0 & 0\end{pmatrix} \quad ; \quad \Gamma_4 = \frac{1}{2}\begin{pmatrix} I & 0\\ 0 & 0\end{pmatrix}.$$

The form factors are calculated at the smallest finite q^2 available on our lattice, $\vec{q} = \frac{2\pi}{24}\hat{x}$.

We employ the Wilson action for both the Gauge and Fermion actions, and calculate at three values of $\kappa = 0.152,\ 0.154,\ 0.156$. κ_s is fixed at 0.152, and $\kappa_u = \kappa_d$ are extrapolated to $\kappa_{cr} = 0.1598(2)$. We use a symmetric combination of the conserved electromagnetic current derived from the fermionic action via the Noether procedure. The lattice Ward identity responsible for charge conservation guarantees that at $q^2 = 0$ the electric 3-point function measures the total baryon charge. Twenty-eight gauge configurations are analysed. A third order single elimination jackknife procedure is used to estimate the statistical uncertainties.

*Presented by Derek B. Leinweber

3. ELECTRIC PROPERTIES

The electric charge radius of a baryon may be extracted from the electric form factor with the standard small q^2 expansion of the Fourier transform of a spherical charge distribution by

$$\langle r^2 \rangle = -6\frac{d}{dq^2}\mathcal{G}_E(q^2)\mid_{q^2=0} .$$

To calculate the derivative we exploit a dipole form

$$\mathcal{G}_E(q^2) = \frac{\mathcal{G}_E(0)}{(1+q^2/m^2)^2}; \quad q^2 \geq 0,$$

which is known to fit the experimental data well. This yields a dipole result,

$$\frac{\langle r^2 \rangle}{\mathcal{G}_E(0)} = \frac{12}{q^2}\left(\sqrt{\frac{\mathcal{G}_E(0)}{\mathcal{G}_E(q^2)}} - 1\right).$$

The difference in radii extracted using a monopole form is small relative to the statistical uncertainties.

Fixing the lattice spacing to reproduce the nucleon mass gives $r_P \equiv \sqrt{\langle r_P^2 \rangle} = 0.65(8)$ fm. Experimentally $r_P = 0.862(12)$ fm. This underestimation of the charge radius is most likely due to finite volume effects. For example, the lattice proton diameter is approximately 9 lattice units (lu) at the lightest quark masses and therefore largely fills the lattice in the y-z directions of length 12 lu. The periodic boundary conditions in the spatial directions allow overlap of the wavefunctions of surrounding baryons which may cause the baryon size to be restricted. Similarly, magnetic radii are underestimated in our lattice calculations compared to the experimental radii.

Figure 1 displays the lattice predictions of the electric charge radii for the charged members of the baryon octet. We have included the results of two recent model calculations for comparison with the lattice results. The same pattern of relative sizes of the baryons is observed in each calculation. The only significant difference is the rather small Skyrme model radius for Ξ^-.

The lattice results suggest there are three effects responsible for the details of the distribution of electric charge within baryons. In the case of equal mass quarks the important effect may be described as a spin dependent force that acts repulsively between doubly represented quarks. These quarks

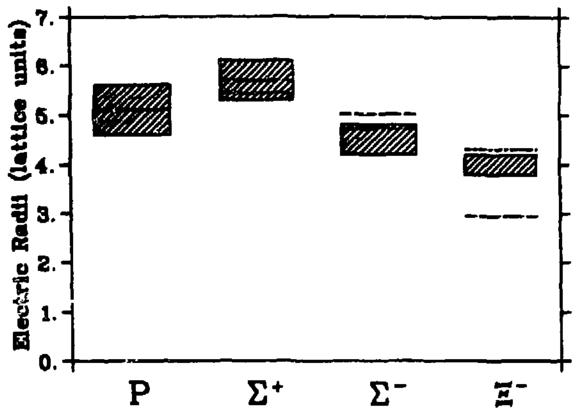

Figure 1: Electric charge radii of the charged octet baryons. The short dashed lines correspond to the quark model results [1] and the long dashed lines correspond to the Skyrme model results [2]. The model results are scaled to the proton lattice radius.

have larger electric charge distributions, which results in a negative squared charge radius for the neutron. The electric properties of baryons involving strange quarks are altered in two ways. The dominant effect is the standard reduction of the charge radius due to the relatively large mass of the strange quark. However there is a more interesting and subtle effect. As the u and d-quark masses become lighter the electric charge radius of the strange quark distribution is seen to decrease indicating a shifting of the centre-of-mass towards the strange quark. As a result, the electric radius of the light quark distribution is further increased. For example, consider the u quark radius in $\Xi^0(ssu)$ and N(ddu).

$$r_u^{\Xi^0} - r_u^N = 0.50^{+.65}_{-.15} \text{ lu}.$$

4. MAGNETIC PROPERTIES

Our calculation of magnetic form factors is done at the smallest finite value of q^2 available on our lattice. On the other hand, the magnetic moment is defined at $q^2 = 0$ as $\frac{\mu}{(e/2M_B)} = \mathcal{G}_M(0)$. Lattice extrapolations in q^2 to $q^2 = 0$ suffer from large statistical errors. To make contact with the experimental magnetic moments, we assume a scaling of electric and magnetic form factors in q^2. This is suggested

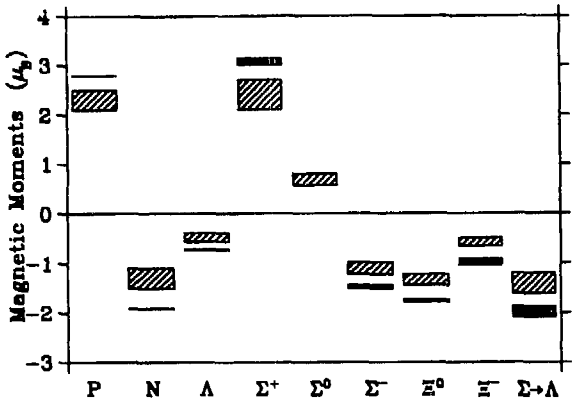

Figure 2: Magnetic moments of the baryon octet in natural magnetons. Experimental moments and uncertainties are indicated in solid black.

by the experimentally measured relation

$$\frac{\mathcal{G}_M(q^2)}{\mathcal{G}_M(0)} \simeq \frac{\mathcal{G}_E(q^2)}{\mathcal{G}_E(0)}.$$

For hyperons we scale the strange and light sectors separately

$$\frac{\mathcal{G}^s_M(q^2)}{\mathcal{G}^s_M(0)} = \frac{\mathcal{G}^s_E(q^2)}{\mathcal{G}^s_E(0)}, \text{ and } \frac{\mathcal{G}^l_M(q^2)}{\mathcal{G}^l_M(0)} = \frac{\mathcal{G}^l_E(q^2)}{\mathcal{G}^l_E(0)},$$

such that the total magnetic moment is given by $\mathcal{G}^B_M(0) = \mathcal{G}^l_M(0) + \mathcal{G}^s_M(0)$. In the proton and neutron the u and d-quark sectors are scaled separately.

Figure 2 displays the lattice predictions of magnetic moments in units of natural magnetons. While the signs of the moments are correctly determined, the moments appear to be underestimated by an amount that appears to be constant in magnitude for most baryons. Once again, finite volume effects are expected to play a leading role in the restriction of the magnetic moments.

Explaining the magnetic moments of baryons has been a long standing problem of hadronic physics. In Fig. 3 we have collected together recent results of the best known approaches to QCD including quark model [1], Skyrme model [2] and QCD sum rule [3] calculations. With the quark masses fixed, the lattice results are parameter free. The lattice calculations out perform the model calculations in predicting the experimental magnetic moment ratios.

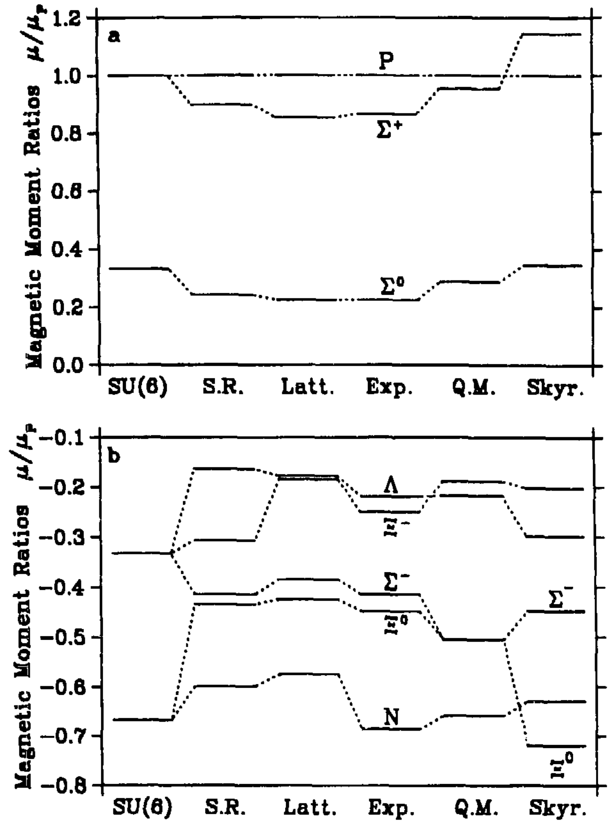

Figure 3: A comparison of positive (a) and negative (b) magnetic moment ratio calculations. The SU(6) symmetry, QCD sum rule (S.R.), Lattice (Latt.), quark model (Q.M.), and Skyrme model (Skyr.) calculations are compared with experimental measurements (Exp.). The experimental and sum rule result for Σ^0 are obtained using SU(2)-flavor symmetry.

To gain a deeper understanding of the underlying quark dynamics, it is useful to consider the individual quark sector contributions to the magnetic moments. In the simple quark model, the magnetic moment of the proton is given by

$$\mu^P = \frac{4}{3}\mu^u - \frac{1}{3}\mu^d.$$

In the SU(2) limit where $\mu^u = -2\mu^d$, the ratio of the quark sector contributions in the simple quark model is $\frac{4}{3}\mu^u / -\frac{1}{3}\mu^d = 8$. In contrast, the lattice results indicate this ratio is 10.3(7) at $\kappa = 0.152$ and increases as the quarks become lighter. A ratio of 8 may be recovered at heavy quark masses. These

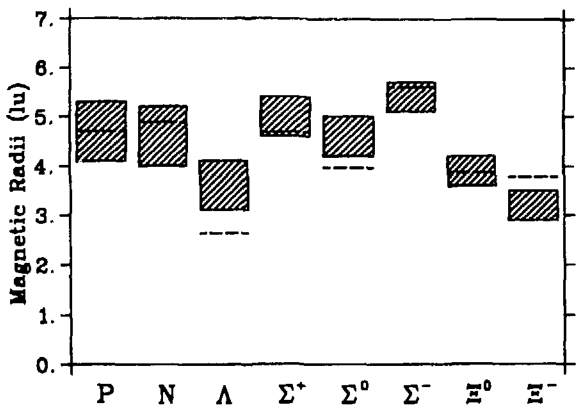

Figure 4: Magnetic radii of the baryon octet. Dashed lines indicate Skyrme model results which are scaled to agree with the proton magnetic lattice radius. The pattern of the radii appear quite similar in the two calculations with the exception of Ξ^- which has a negative squared magnetic radius in the Skyrme model.

results give strong evidence of relativistic and gluon effects which are not accounted for in conventional quark models.

Away from the SU(3) flavor limit, one can search for quark mass effects analogous to those seen in the electric properties. Of course, the effective moment of the strange quark is smaller than the light quarks as expected. However there are more subtle effects seen in the effective magnetic moments of the quarks which are due to the shifting of the center-of-mass. For example, consider the u-quark contribution to the magnetic moments of N(ddu) and $\Xi^0(ssu)$. In the SU(3) flavor limit, the effective moments of the u-quark in these two baryons are found to be the same. However, with the light quarks extrapolated to κ_{cr} we find the u-quark contribution to the neutron magnetic moment is -0.25(17) μ_N, while the u-quark contribution in Ξ^0 is larger at -0.36(5) μ_N. This effect, due to unequal s and d-quark masses, is not accounted for in a constituent quark picture.

In Fig. 4 the magnetic radii $\sqrt{|\langle r^2 \rangle / \mathcal{G}_M(0)|}$ for each baryon of the octet are plotted. In general, the two calculations reproduce the same pattern of magnetic radii, with the exception of Ξ^-.

5. SUMMARY

In our investigations of electric structure we have seen evidence of a spin dependent force that acts repulsively between doubly represented quarks. This accounts for a negative squared charge radius in the neutron. We have also seen center-of-mass effects which act to increase the distribution radius of light quarks and decrease the distribution radius of s-quarks. Investigations of the magnetic properties also reveal center-of mass and spin dependent effects in the quark sector contributions to the magnetic moments. The effective moment of a quark is dependent on the baryon in which it resides. These effects become larger as the quarks become lighter indicating relativistic motion and gluon dynamics which are not accounted for in constituent quark models.

Calculations of the electromagnetic properties of hadrons may ultimately provide one of the best quantitative tests of QCD. A method for extracting the four electromagnetic form factors of the spin $\frac{3}{2}$ system on the lattice has been established [4]. With recent attempts to measure the Ω^- magnetic moment, a lattice investigation of the electromagnetic properties of the low-lying spin $\frac{3}{2}$ baryons seems very timely, and such an analysis is currently in progress.

ACKNOWLEDGMENTS

We thank Keh-Fei Liu for his assistance in making computing arrangements. This work is supported in part by the Natural Sciences and Engineering Research Council of Canada and the U.S. Department of Energy under grant number DE-FG05-84ER40154.

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