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Proceedings of Two-Day Meeting on Roles of Pauli Principle in Few-Body Problems

—Discussions with Prof. V. I. Kukulin—

Institute for Nuclear Study (INS)
University of Tokyo
October 30-31, 1991

Edited by M. Kamimura

Report of an INS Two-Day Meeting on

Roles of Pauli Principle in Few-Body Problems

— Discussions with Prof. V.I. Kukulin —

PREPACE

This small INS meeting on "Roles of Pauli Principle in Few-Body Systems" was held on Oct. 30-31, 1991, taking the occasion that Prof. V.I. Kukulin (Moscow State University) was staying at Research Institute of Fundamental Physics, Kyoto University, in the autumn of 1991. Prof. Kukulin has made very important contributions to the study of nuclear cluster structure and nuclear few-body problems. These study fields have extensively been studied in many authors in Japan, too. Therefore, we considered the meeting was a very good chance for us to have stimulating discussions with him. A lecture was given by him on new physics with the quark-based Moscow N-N potential for few-nucleon systems and on microscopic studies of multi-cluster systems. Seven other speakers gave talks on various roles of the Pauli principle in few-body systems, in multi-cluster systems and in heavy-ion reactions. Since some of the talks have already been published, only abstract of them are given here. The other talks are reported here not in the form of formal paper, since they are expected to be published soon.

We would like to thank INS for the financial support on this meeting.

Organizers:

M. Kamimura, S. Oryu, Y. Koike and Y. Fujiwara

CONTENTS

A New Physics in the Problem of Nucleon-Nucleon Interaction and Implications for Few-Nucleon Systems.

(Extended version of lecture delivered at the meeting "Pauli Principle in Few-Body Systems" organized by the Tokyo Institute of Nuclear Study, Oct. 30 - 31, 1991)

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ABSTRACT

An attempt to understand better the modern situation with short range baryon-baryon (by example of nucleon-nucleon) interaction is undertaken. The talk covers four main ingredients of the whole problem : (i) visual demonstration of the inadequacy of modern models for short-range $N-N$ interaction, (ii) the general property of symmetry, i.e. $SU(4)$-splitting of interaction potential for composite particles, (iii) a short outline of the main results of realistic six-quark microscopic calculations for nucleon-nucleon interaction and based on this calculations the Moscow model for $N-N$ interaction and (iv) some possible interesting consequences for nuclear physics at whole. In particular, basing on the picture we suggest and substantiate microscopically the formation of specific nuclear bonds rather similar to the quantum chemistry case. In this pattern e.g. the structure of the ^{4}He-nucleus is similar to the structure of the CH_4-methane molecule having tetrahedral form due to the formation of the π-electron bonds, the each consisted from a pair of p-electrons with the opposite spin directions. It is shown also that the Moscow model of $N-N$ interaction is an alone model comprising a wide repulsive core of ~ 1 fm size dictated by modern electron data for the lightest nuclei ^{3}He and ^{4}He. Some arguments are given showing that the model considered can lead to the solving of many long standing puzzles in nuclear physics, in particular to the avoiding the nucleus collapse predicted many years ago when using realistic (e.g. RHC) $N-N$ force and ensure the saturation property of nuclear forces.

* On the leave of Inst. Nucl. Phys. Moscow State University

[This work by V.I. Kukulin was already published in the paper:

V.I. Kukulin and V.N. Pomerantsev, Prog. Theor. Phys. 88 (1992) 159]

Roles of Pauli Principle in the Quark Cluster Model Applied to NN Interaction

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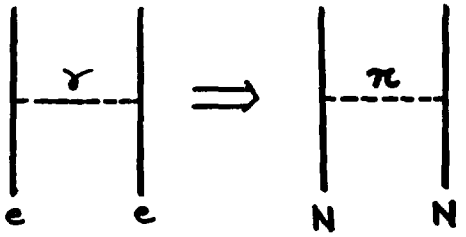
ABSTRACT

One of the essential features of QCD is the existence of the color degree of freedom carried by quarks and gluons. This attribute has made quarks fermions, thus solved the longstanding dilemma between spin and statistics in the Gell-Mann quark model with respect to three-quark baryons. The existence of the color degree of freedom causes rather drastic effects in quark cluster models where the effect of antisymmetrization is exactly taken into account. Namely, the Fermi-Breit interaction derived from the one gluon exchange diagram in dominant nonrelativistic approximation contains the $(\lambda_1\lambda_2)$ factor, which yields no contribution to the interaction between two color-singlet objects. Thus, all the quark model interaction originates from the quark exchange diagrams. This is a big contrast to the characteristics of the composite-particle interaction in the nuclear physics, where a dominant direct potential is slightly modified by the contributions from exchange terms. This feature is also preserved in an extended quark model where the effects of $(q\bar{q})$ and $(q\bar{q})^2$ excitations inherent in the quark-gluon interaction Lagrangian have been explicitly incorporated into the model space. The study of NN interaction in such a quark model is reviewed with focusing on this very unique way of manifestation of the Pauli principle.

§1. Introduction

Coulomb force



NN force

Mesons: finite mass

→ short range force

Isospin degree of freedom

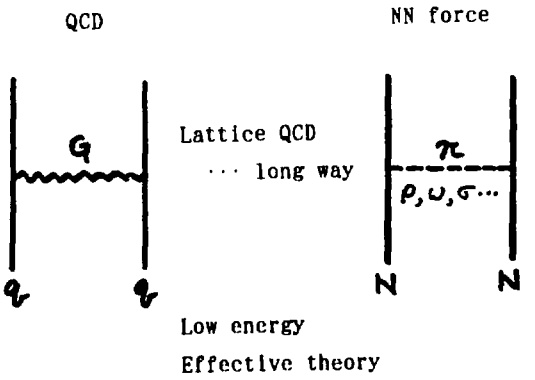
Characteristics of NN force

1. One Pion Exchange Tail OPEP
2. Intermediate Range Attraction TPEP $2\pi, \pi\rho$ or σ, $\delta\cdots$
   OBEP $\rho, \omega\cdots$
3. Strong Repulsion Core ω 4th component
   phenomenological repulsive core
4. Complex State Dependence
   Non-central force { LS force — Vector mesons $\omega, \rho, (\sigma)$
   Tensor force — Isovector mesons π-ρ
   Quadrutic LS }
   Central force : Weak momentum dependence
5. Rich Flavor Dependence Flavor SU_3

Phenomenological Potentials; H-J, Reid, G3RS etc.

OBEP : Bonn pot. Neijmehen pot.
Paris pot.

Moscow pot.



QCD characteristics: Modelling in the quark cluster model

1. Color degree of freedom
   G: Color octed : G exchange effect → quark-quark interaction
   q: Color triplet (Fermion) antisymmetrization

2. Confinement
   (realistic only with singlet)
   ·phenomenological r^n potential
   ·flip-flop model

3. Asymptotic freedom OGEP → color version of the Fermi-Breit interaction
   (perturbative in short range region)

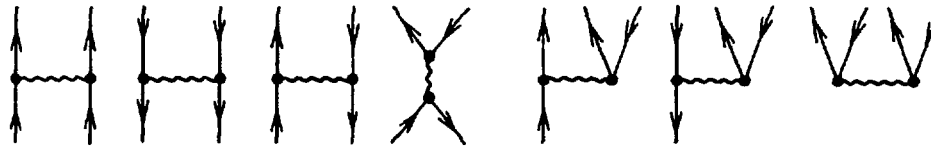
QCD-inspired (motivated) quark models

Purpose : to correlate structure of hadrons with hadron-hadron interaction
Keep essantial points: symmetry, antisymmetrization, ···
Dynamics: difficult → quark parameters b, m, α_s, a_c ···
For example: 1. N → NN, 2. NN → YN etc.

Model Hamiltonian in the Quark Cluster Model

$H = K.E. + U^{conf.} + H_{int}$

$U^{conf.} = -a\, r^2(\lambda_1\lambda_2)$ $(r=|\vec{x}_1-\vec{x}_2|)$: purely phenomenological



(1) (2) (3a) (3b) (4a) (4b) (5)

$$H_{int} = H_{qq} + H_{\bar{q}\bar{q}} + (H_{q\bar{q}} + H_{q\bar{q}}^{(ann)}) + \left(H_{q\to qq\bar{q}} + \text{c.c.}\right) + \left(H_{\bar{q}\to\bar{q}q\bar{q}} + \text{c.c.}\right) + \left(H_{\to(q\bar{q})^2} + \text{c.c.}\right)$$

$$H_{q\bar{q}} = \frac{\alpha_s}{4}(\lambda_1\lambda_2)\left[\frac{1}{r} - \frac{\pi}{m^2}\left\{1 + \frac{2}{3}(\vec{\sigma}_1\cdot\vec{\sigma}_2)\right\}\delta(\vec{r})\right] + LS + Tensor + \cdots$$

$\frac{1}{r}$ ⇓ Color-Coulombic

$\frac{\pi}{m^2}\left\{1 + \frac{2}{3}(\vec{\sigma}_1\cdot\vec{\sigma}_2)\right\}\delta(\vec{r})$ ⇓ Color-Magnetic

N-Δ mass difference
(Rujula, Georg and Glashow,
Phys. Rev. D12 (1975) 147)

Advantage and Weak Points of the Quark Cluster Model

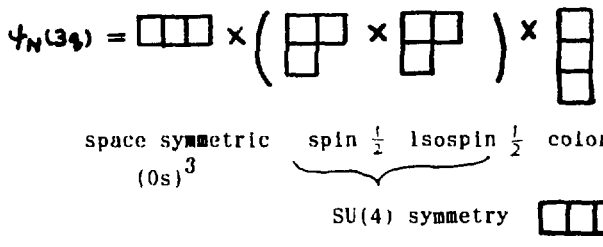
Advantage

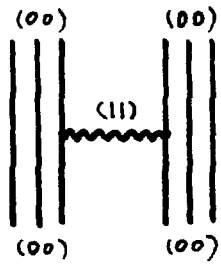
1. Exact antisymmetrization of quarks.
   (color degree of freedom is exactly treated.)
2. Single hadron structure is roughly reproduced.
   (exceptions: π, η' $\cdots$)
3. Center-of-mass motion is exactly treated.
   (indispensable for the study of hadron-hadron interactions)

Weak Points

1. Quark-quark interaction?
   confinement pot. + OGEP (Fermi-Breit int.)
   →too much simplification of the Gluon effect?
2. Non-relativistic
   ·N-N relative motion is O.K., but the quark motion inside of hadrons can not be nonrelativistic.
   ·Can one deal with virtual meson exchange effects in the non-relativistic framework
3. Maximal violation of the Chiral symmetry.
   Cf. π and OPEP

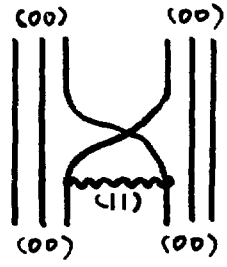
Importance and Danger in the Treatment of Color Degree of Freedom





forbidden by color degree of freedom

No direct term!



quark exchange term (nonlocal)

↓ localize

short range force with energy dep.

(confinement pot. does not contribute!)

Problem of the color van-der-Waals Force

Long range force of $(1/R)^{4-n}$ → no such force in NN solutions:

- flip-flop model (← string picture)
- truncation of the model space

«Proper Model Space and Proper Effective Interaction»

§2. (3q)+(3q) Resonating Group Method (RGM)

By Oka and Yazaki (1980) and many others

$$\Psi = \mathcal{A}\{\phi_N(3_1)\phi_N(3_2)\chi(\vec{R})\}$$

$$(E - T_R)\chi(\vec{R}) = \int d\vec{R}' G(\vec{R}\,\vec{R}')\chi(\vec{R}')$$

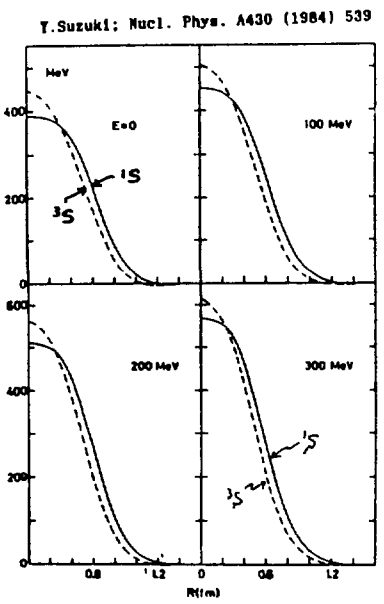
$$G_W(\vec{R},\vec{P}) = \int d\vec{s}\, \exp[\tfrac{i}{\hbar}(\vec{s}\cdot\vec{P})]\, G(\vec{R}-\tfrac{\vec{s}}{2}\;\vec{R}+\tfrac{\vec{s}}{2})$$

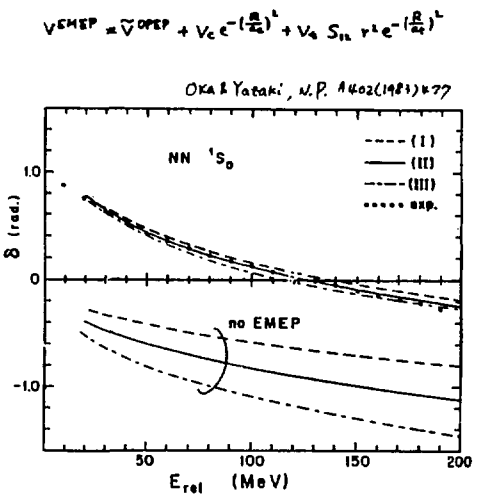
: Wigner transform

$$+\text{WKB} \rightarrow U^{\text{eff}}(R) : \text{Effective Local Potential}$$

(summary)

- Short range repulsion: mainly the Color-Magnetic term in the Fermi-Breit interaction contributions.
- No intermediate range attraction.
- Effects of Δ-Δ and C-C channel are small.





§3. Pair Creation Interaction and the Single Baryon Problem

$$\phi(N) = c_0\phi_0(3q) + \sum_{\alpha=1}^{1+f} c_\alpha\phi_\alpha((3q)(q\bar{q})) + \sum_{\beta=1}^{6} c_\beta\phi_\beta((3q)(q\bar{q})^2)$$

Relative $1\hbar\omega$ H.O. (P wave), $0\hbar\omega$ H.O. (S wave)

(3q): N, Δ, C	(3q): N
$(q\text{-}\bar{q})$: π, ρ, ω ···	$(q\text{-}\bar{q})$: σ, δ
(PS, V)	(S)

$\delta\langle\phi(N)|H|\phi(N)\rangle=0$ with $\langle\phi(N)|H|\phi(N)\rangle=1$

$$\Longrightarrow \quad \sum (H_{\gamma\gamma'} - E N_{\gamma\gamma'})C_{\gamma'}=0$$

Quark parameters

			Values
1. $M_\Delta - M_N$ =293 MeV	⟶	α_S	2.97
2. $m_\rho/(2M_N)\cdot(g_\rho+f_\rho)$=1.97-2.49	⟶	b	0.52 fm
3. $\epsilon(\mu_p-\mu_n)$=2.353	⟶	m	471 MeV
4. M_N=939 MeV	- - →	a	336 MeV·fm^{-2}

(for convenience)

Magnitude of c_α and c_β

c_α <0.25 for color singlet (3q) and $(q\bar{q})$

〃 ≲0.05 for hidden color $[(3q)_{(11)_c}(q\bar{q})_{(11)_c}]_{(00)_c}$

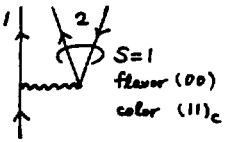
c_β ≲0.20 for $(3q)(q\bar{q})^2$

$1-c_0^2$=0.28 : $(q\bar{q})$ admixture ~30%

$(q\bar{q})$-Pair Creation Interaction (Off-Shell FB Int.)

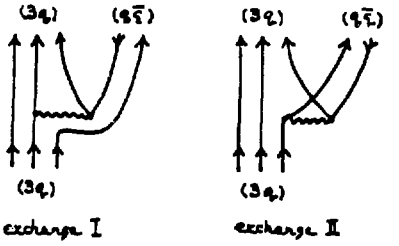
$$H_{q\to qqq} = \sum_{\alpha\beta\gamma\delta} \langle \psi_\gamma(1)\psi_\delta(2)| V_{q\to qqq}(12) |\psi_\alpha(1)\psi_\beta(2)\rangle\, a^\dagger_\gamma a^\dagger_\delta b^\dagger_\beta a_\alpha$$

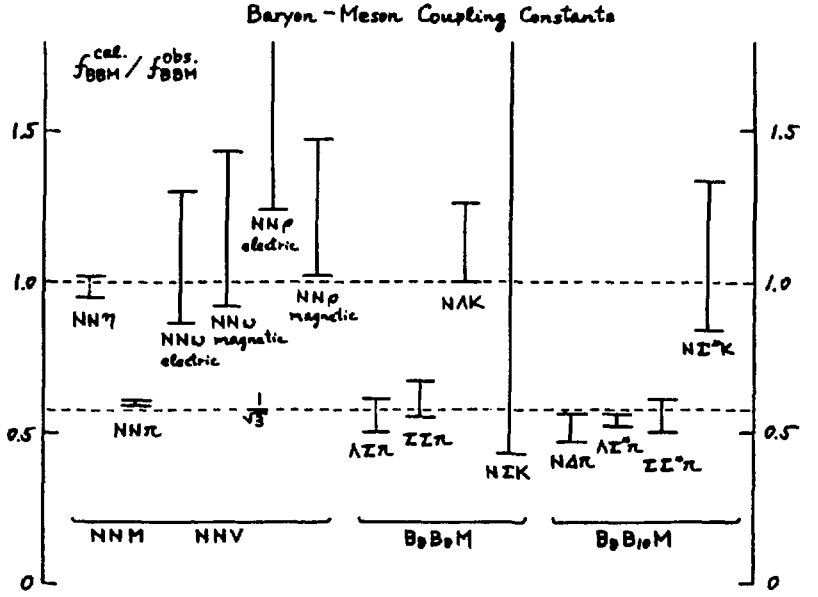
$$V_{q\to qqq}(12) = \frac{i}{2\pi}\alpha_s \hbar c \frac{1}{4}(\lambda_1\lambda_2) \sum_{O=M,D} (\mathbf{F}^O(\mathbf{x}_1,\mathbf{x}_2)\cdot \mathbf{O}^O(1,2))$$



M-type	D-type
$\mathbf{O}^M = 2\sigma_2$	$\mathbf{O}^D = 2\sigma_2 - i[\sigma_1 \times \sigma_2]$
$\mathbf{F}^M(\mathbf{x}_1,\mathbf{x}_2) = F(r)(\partial/\partial \mathbf{x}_1)$	$\mathbf{F}^D(\mathbf{x}_1,\mathbf{x}_2) = (\partial/\partial \mathbf{r})F(r)$

where $F(r) = \dfrac{\pi\hbar}{2mc}\dfrac{1}{r}$, $\mathbf{r} = \mathbf{x}_1 - \mathbf{x}_2$





§4. Analysis of the NN Interaction

$$G(\vec{R},\vec{R}') = G_0^\dagger G_0(\vec{R},\vec{R}') + \sum_{\alpha=1}^{24} G_0^\dagger G_\alpha G_\alpha(\vec{R},\vec{R}') + \sum_{\beta=1}^{6} G_0^\dagger G_\beta G_\beta(\vec{R},\vec{R}') + \cdots$$

	$(3q)+(3q)$	$(3q)(q\bar{q})$ cross terms	$(3q)(q\bar{q})^2$ cross terms
		$c_\alpha<0.25$	$c_\beta<0.20$
⇓	C,LS,T	C,LS,T	C only

$$G_W = G_W^C + G_W^{LS}(\vec{L}\cdot\vec{S}) + G_W^T S_{12} + \cdots$$

Simple $(q\bar{q})$ exchange potential ⟶ Yu-Zhang diagrams

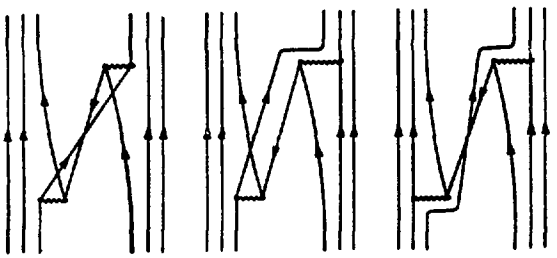
Antisymmetrization between only 2 clusters.
Good approximation to the full potentials In the long range part ($\gtrsim$1.2 fm) of the non-central force.

The effective local potentials have

1. longest range — similar to OBEP
2. weak energy dependence — correct spin-isospin dependence!

Central Force

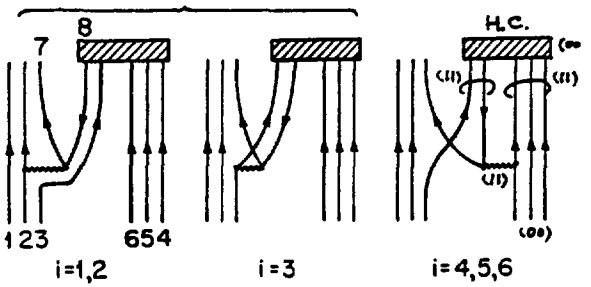
3-body exchange terms contribute
⟶ strong energy dependence from $(3q)(q\bar{q})$ cross terms.



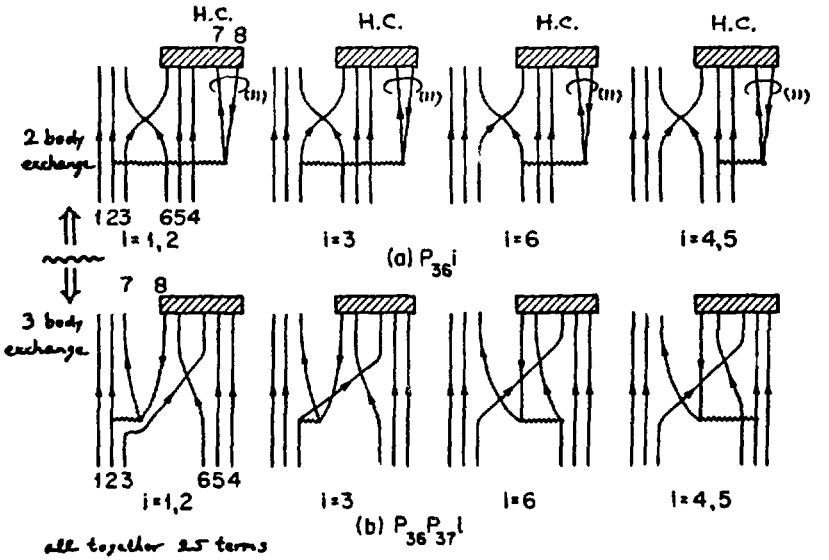
(a)

Yu-Zhang Diagrams
(N.P. A426(1984)557)

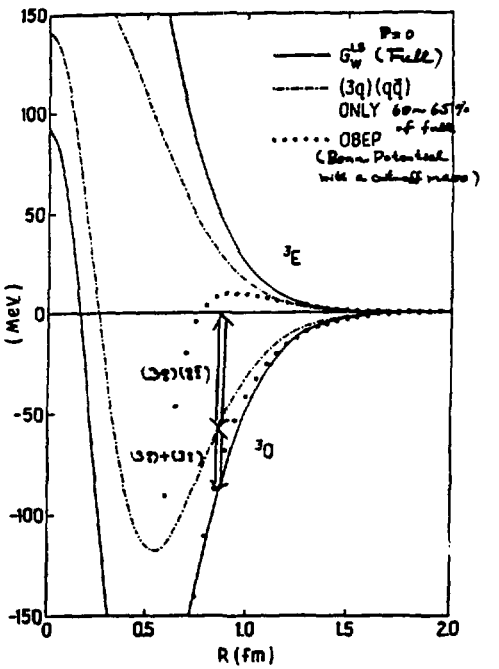
Simple $(q\bar{q})$ Exchange Potentials



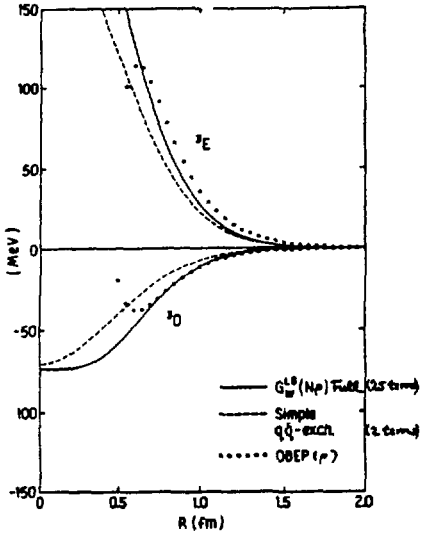
(b) $P_{37}i$



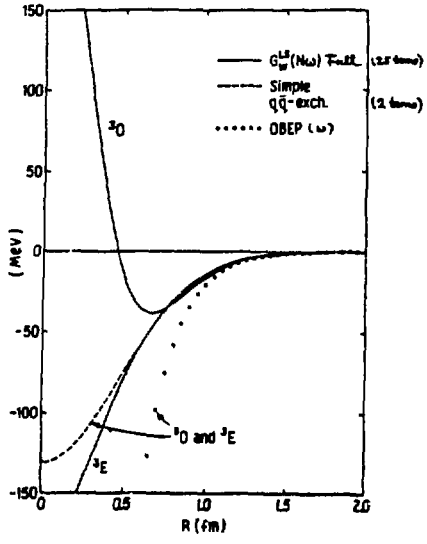
(3q) and ($q\bar{q}$) Contributions in the LS force



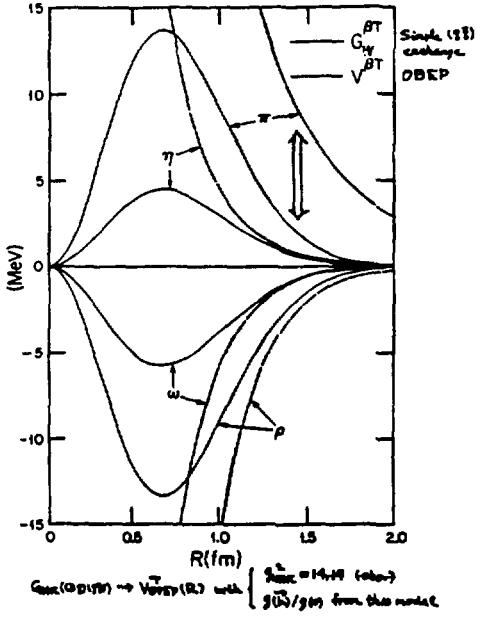
NN LS force from ρ-meson type $(q\bar{q})$-exchange

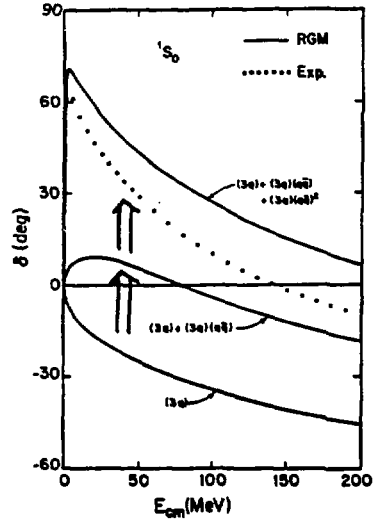


NN LS force from ω-meson type $(q\bar{q})$-exchange

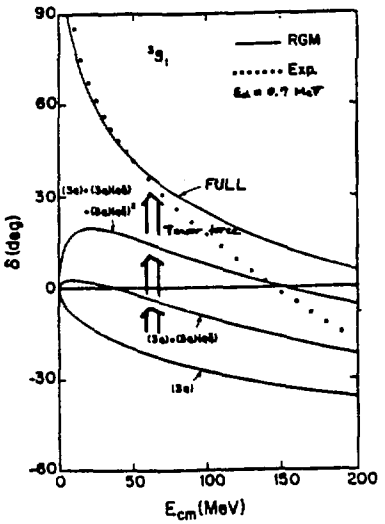


NN Tensor force from simple $(q\bar{q})$-exchange potential





Energy Dependence of the equivalent Local Potentials (Central only)



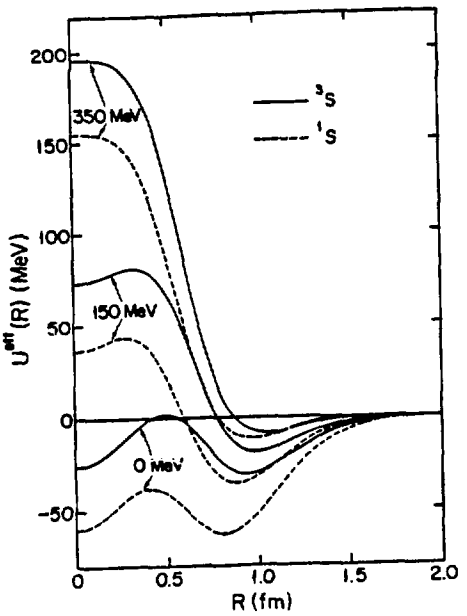


Table Characteristics of the Quark Model NN Interaction

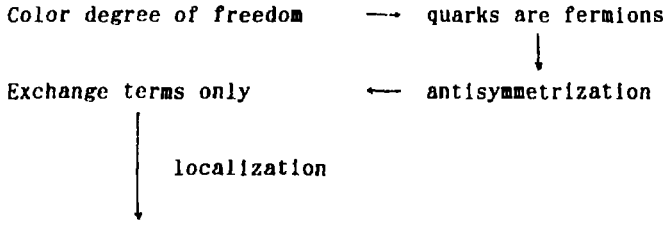
Components		Even	Odd
	C	Repulsive (~600-700 MeV, predominantly from color-magnetic contact term)	Repulsive
(3q) G_0	LS	Repulsive	Attractive (~70% symm. LS ~30% antis. LS)
	T	~0	~0
	C	Attractive (~-300 to -100 MeV) Strongly energy-dependent	Repulsive
(3q)(qq̄) G_α	LS	Repulsive	Attractive (~65% of total LS) Dominated by (qq̄)-exchange of N_P, N_M type
	T	Attractive Dominated by (qq̄)-exchange of N_T, N_P type	Repulsive
$(3q)(q\bar{q})^2$ G_β	C	Attractive Many characteristics of σ meson exchange	Attractive

Main features of the NN interaction is reproduced. Exception: OPEP.

§5. Summary

No direct term is a concequence of the Pauli principle in the present quark cluster model.

A essential feature of QCD·



Effective local potentials:

(3q)+(3q) contribution: on-shell Fermi-Breit interaction

($q\bar{q}$) exchange contribution : off-shell

Simple ($q\bar{q}$)-exchange potential · OBEP

«direct theory»

Is this feature universal in any quark models?

Three-body model of the d $-\alpha$ and the d $-^{12}$C Scatterings*

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Ridai-cho, Okayama 700, Japan

I report our three-body model based on the Faddeev theory which has been applied to two different types of scatterings, the d-α and the d-^{12}C scatterings.

The Faddeev theory distinguishes itself from conventional reaction theories by completely retaining three-body aspects of reactions. We introduce into the theory a two-body channel-coupling interaction which reproduces elastic scattering data, and at the same time bound states. In the present work, we eliminate inelastic components of the channel coupling interaction and obtain a kind of two-body optical potential designed in the positive and the negative energy regions[1),2)].

Using interactions thus obtained, we perform the three-body calculations for the the d-α[1)] and the d-^{12}C elastic scatterings at E_d=56 MeV. In the d-α analysis, a Pauli-forbidden state in the N-α S1/2 partial wave is excluded with the orthogonal projecting method of Kukulin et al.[3)]. The effect of this exclusion is clearly seen in the d-α scattering observables at 56 MeV. The fit to the data of the d-^{12}C scattering cross section is satisfactory, which shows that our model is promising in describing a wide class of three-body scatterings.

1) K.Miyagawa, Y.Koike, T.Ueda, T.Sawada and S.Takagi, Prog. of Theor. Phys. **74**(1985), 1264.
2) K.Miyagawa and Y.Koike, Prog. of Theor. Phys. **82**(1989), 329.
3) V.I.Kukulin, V.G.Neudatchin, I.T.Obukhovski and Yu.F.Simonov, *Clusters as Subsystems in Light Nuclei* (Vieweg & Sohn, 1983).

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*) This work is in collaboration with Y.Koike, T.Ueda and T.Sawada.

§1. Introduction

Our aim is to make a widely-used three-body model.

·Three-body Faddeev approach to d-α, d-^{12}C scattering.

n+p+α
n+p+^{12}C

·Inclusion of channel couplings into two-body subsystems

N-α N-α^*, d-^{3}He, ·····
N-^{12}C N-^{12}C*, ·····

·Exclusion of a Pauli forbidden state

N-α $s_{1/2}$
Orthogonal Proecting Method (Kukulin et al.)

·Energy-dependence effects of a N-^{12}C optical potential

§2. Nucleon-α Interaction

$$\tilde{V} = |\tilde{g}\rangle \lambda \langle \tilde{g}|$$

$$|\tilde{g}\rangle = \begin{pmatrix} |g_1\rangle \\ \vdots \\ |g_n\rangle \end{pmatrix} \quad , \quad \langle\tilde{g}| = \begin{pmatrix} \langle g_1| \cdots \langle g_n| \end{pmatrix}$$

$$\tilde{V} = \lambda \begin{pmatrix} |g_1\rangle\langle g_1| & \cdots & |g_1\rangle\langle g_n| \\ \vdots & & \vdots \\ |g_n\rangle\langle g_1| & \cdots & |g_n\rangle\langle g_n| \end{pmatrix}$$

Solving $\tilde{t} = \tilde{V} + \tilde{V} + \tilde{V}\tilde{G}_0\tilde{t}$,

$$t_{ij} = |g_i\rangle\tau(E)\langle g_j| \qquad (i,j=1,2,\cdots\cdots n)$$

$$\tau(E) = \frac{1}{\lambda^{-1} - \Sigma_i \langle g_i | G_0^{(i)} | g_j \rangle}$$

$$G_0^{(i)}(E) = (E - E_i - K_i)^{-1}$$

(E_i : threthold energy
K_i : Kinetic energy operator of channel i)

§3. Channels

In the present work, only 2 channels:

{ 1st channel ···· elastic N-α
2nd channel ···· A dummy which represents all inelastic channels, N-α^*, ····

$$\tilde{V} = \lambda \begin{pmatrix} |g_1\rangle\langle g_1| & |g_1\rangle\langle g_2| \\ |g_2\rangle\langle g_1| & |g_2\rangle\langle g_2| \end{pmatrix}$$

$$t_{ij} = |g_i\rangle\tau(E)\langle g_j| \qquad (i,j=1,2,\cdots\cdots n)$$

As inputs to three-body equations, we use only

$$t_{11} = |g_1\rangle\tau(E)\langle g_1|$$

⟶ absorption in 3-body system.

$$\tau(E) = \frac{1}{\lambda^{-1} - \langle g_1 | G_0^{(1)} | g_1 \rangle - \langle g_2 | G_0^{(2)} | g_2 \rangle}$$

where t_{11} is adjusted to elastic data.

This is exactly the same as the single-channel problem with the optical potential:

$$V_{opt}(e) = |g_1\rangle\lambda_{opt}(e)\langle g_1|$$

$$\lambda_{opt}^{-1}(e) = \lambda^{-1} - \langle g_2 | G_0^{(2)}(e) | g_2 \rangle$$

Fitting to p-α elastic scattering data

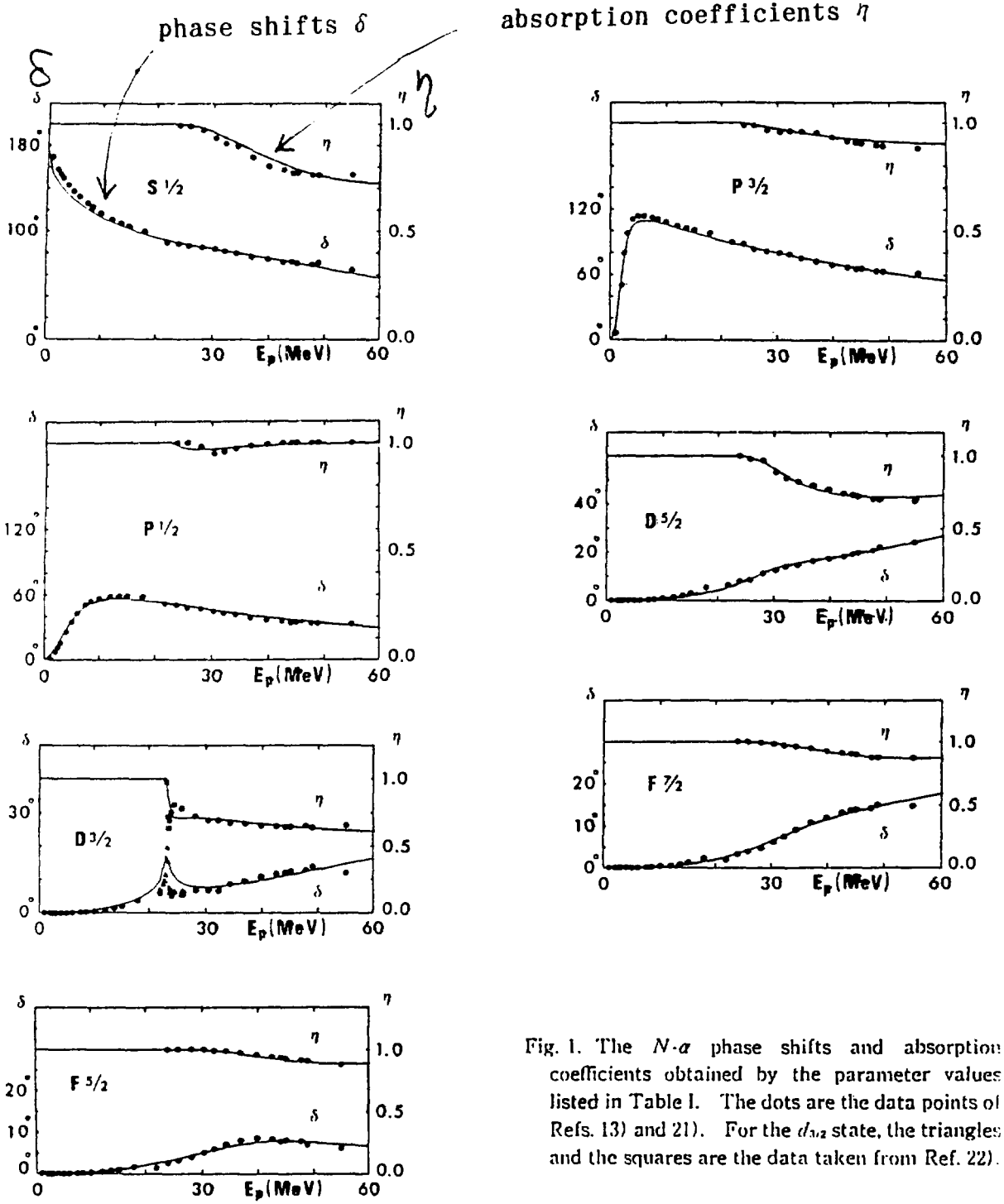


Fig. 1. The N-α phase shifts and absorption coefficients obtained by the parameter values listed in Table I. The dots are the data points of Refs. 13) and 21). For the $d_{3/2}$ state, the triangles and the squares are the data taken from Ref. 22).

§4. Exclusion of Pauli forbidden states (N-α $s_{1/2}$)

Orthogonal projecting method (Kukulin et al.)

H: original Hamiltonian

$$H\,\phi_0 = E_0\phi_0 \qquad \text{forbidden } (E_0 \simeq -13 \text{ MeV})$$

$$H\,\phi_k = E_k\phi_k$$

$$\bar{H} = H + k|\phi_0\rangle\langle\phi_0| \quad : \text{pseudo Hamiltonian}$$

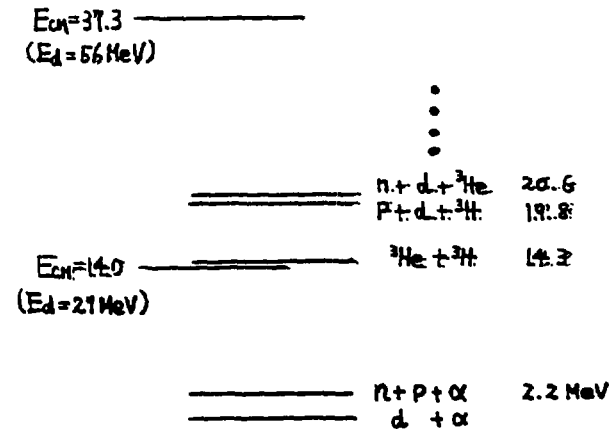
$$(\bar{H} - \bar{E})\,\bar{\psi} = 0$$

$$\tilde{\psi} = \begin{cases} \phi_0 & \bar{E}_0 = E_0 + k \\ \phi_k & \bar{E}_k = E_k \end{cases}$$

As $k \to \infty$, $\bar{E}_0 \to \infty$, forbidden state is moved out.

Practically

$$\bar{V} = |g\rangle\lambda\langle g| + k|\phi_0\rangle\langle\phi_0| \qquad \text{rank 2 separable potential}$$



Various channels in the d-α system

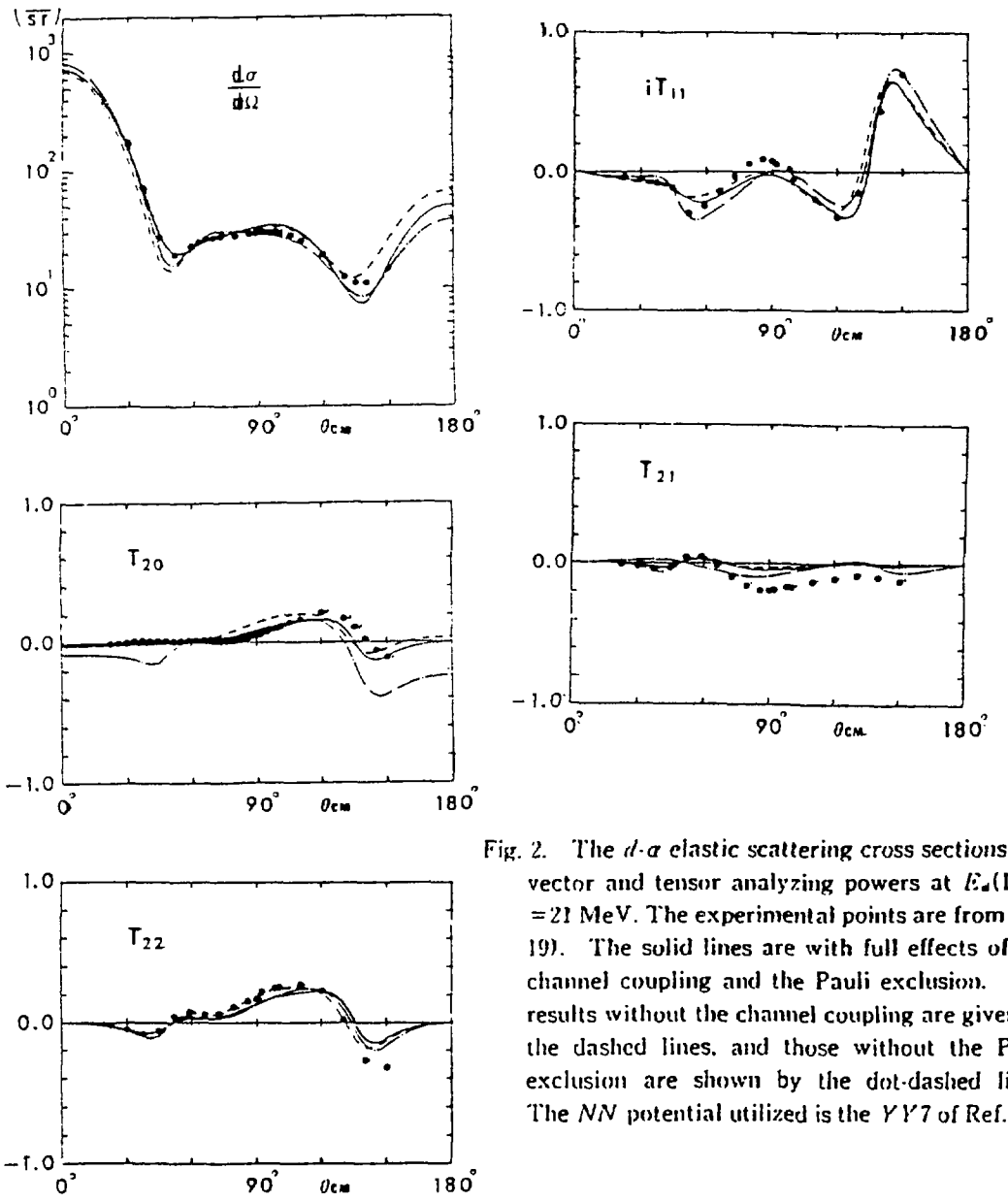


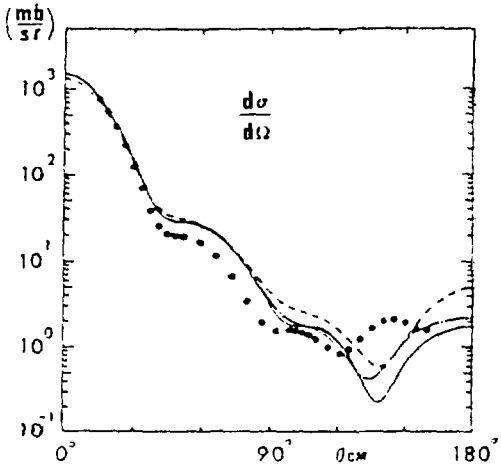
Fig. 2. The d-α elastic scattering cross sections and vector and tensor analyzing powers at E_d(Lab) =21 MeV. The experimental points are from Ref. 19). The solid lines are with full effects of the channel coupling and the Pauli exclusion. The results without the channel coupling are given by the dashed lines, and those without the Pauli exclusion are shown by the dot-dashed lines. The NN potential utilized is the $YY7$ of Ref. 17).

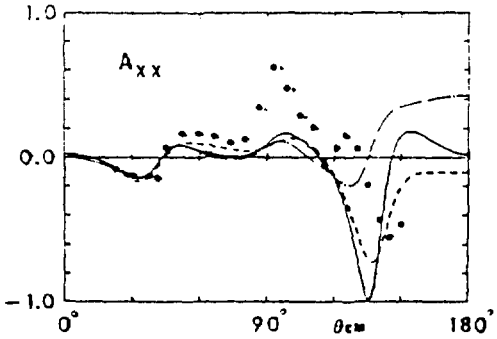
Faddeev calculation $\alpha(\vec{d},d)\alpha$ $E_d = 56$ MeV

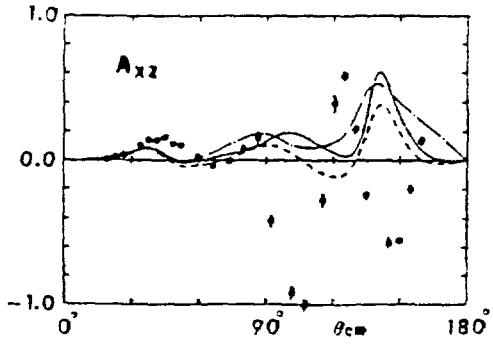
——— full

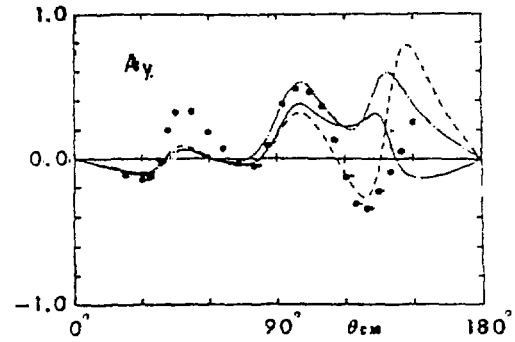
– – – – – without channel coupling

—.—.— without Pauli exclusion









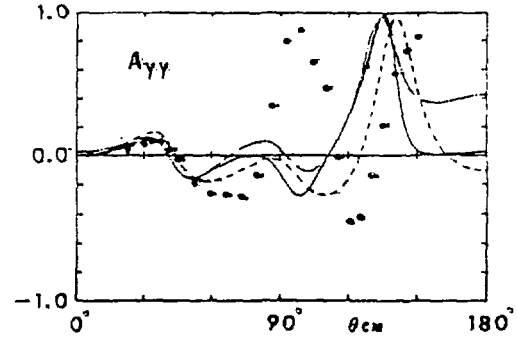
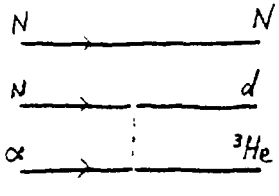
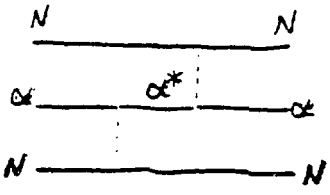


Fig. 3. The results for the d-α elastic scattering at E_d(Lab) = 56 MeV. The data are from Ref. 20. See the caption to Fig. 2 for other details.

Candidates for improving the fit at 56 MeV

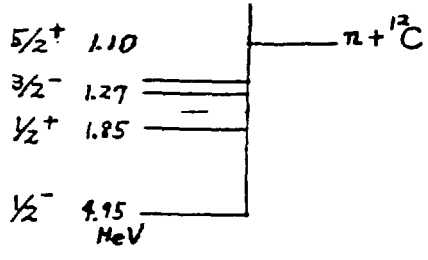




The problem is that we do not have reliable coupling interactions

Data to be adjusted

e<0 ^{13}C bound state energies



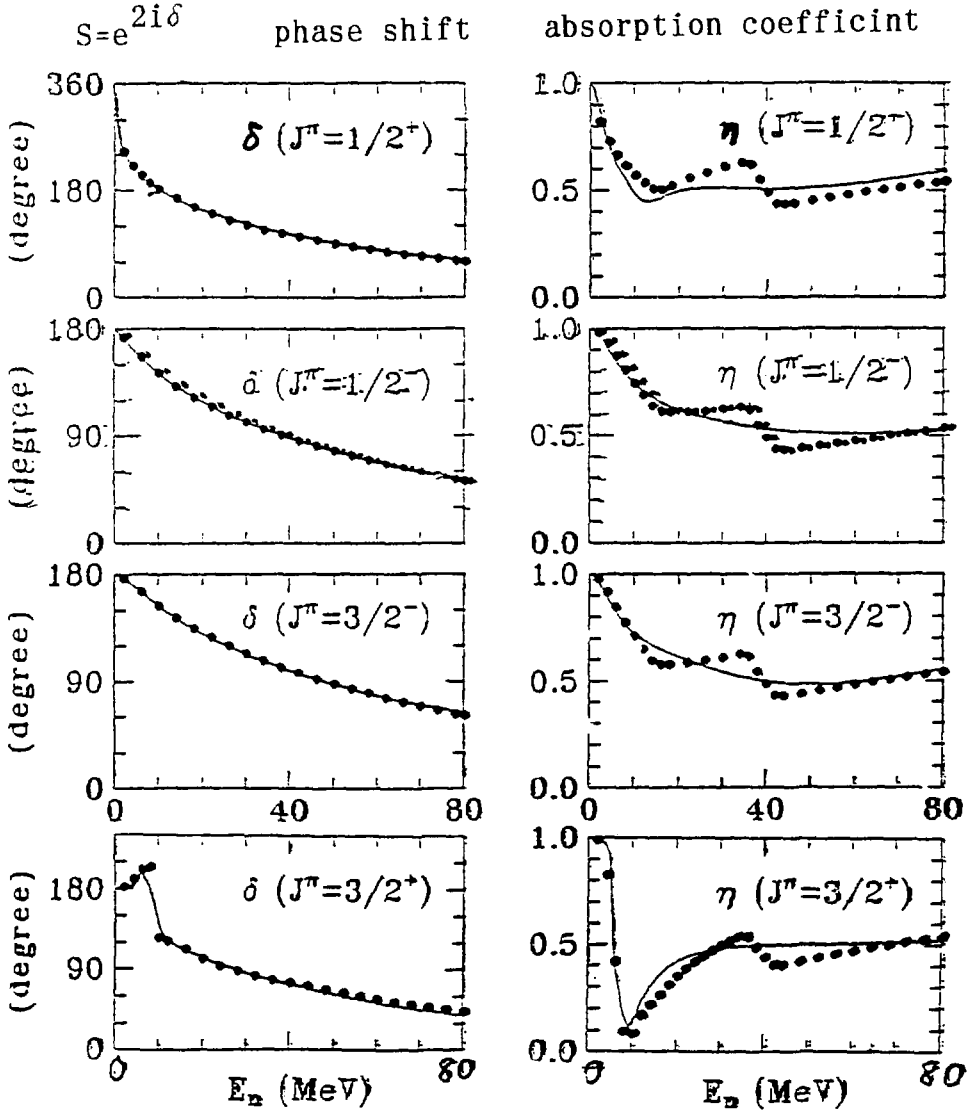
e>0 Watson's optical potential

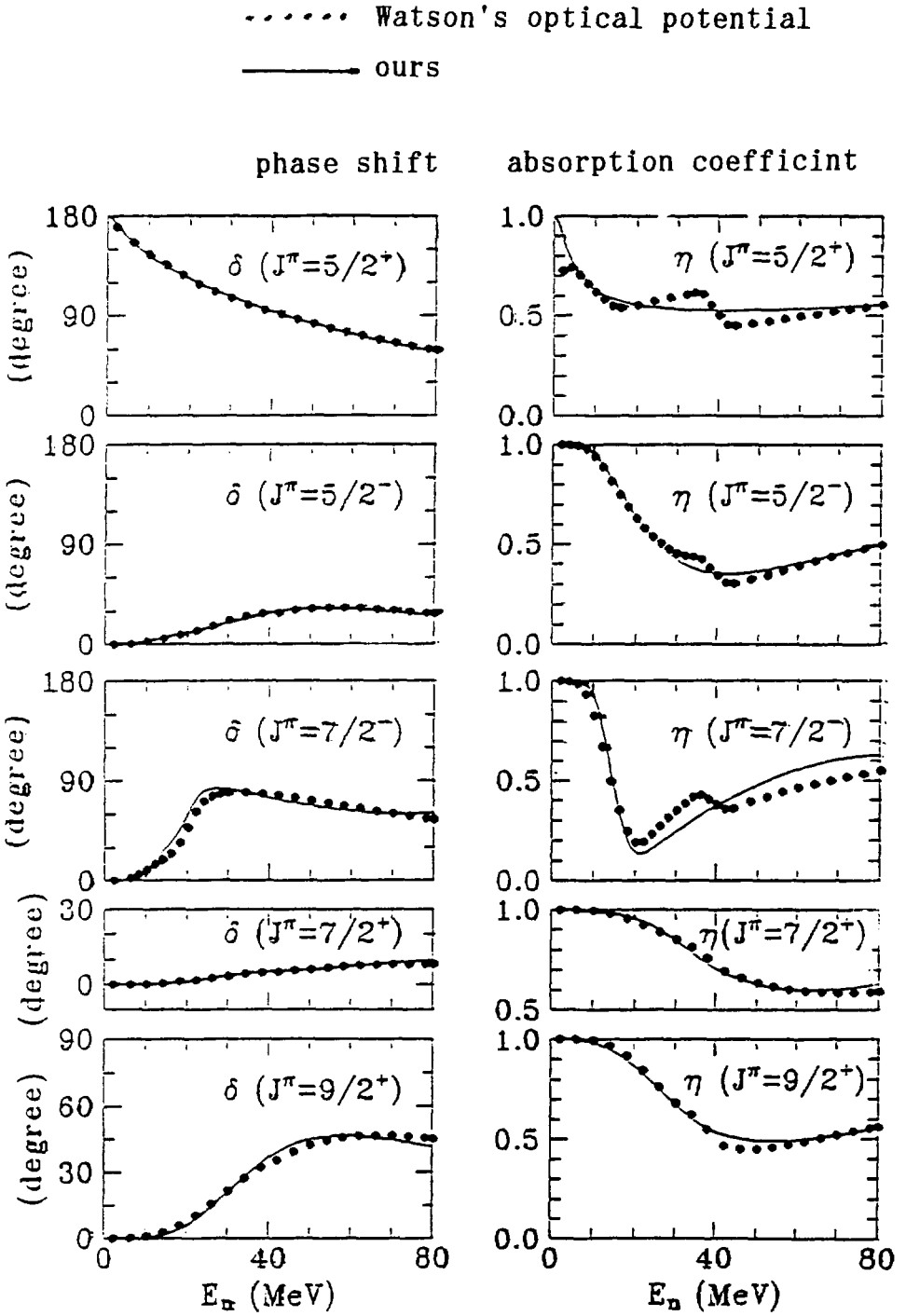
$$U = V_0 + i(W_v + W_s) + V_{so}\, \vec{L}\cdot\vec{S}$$

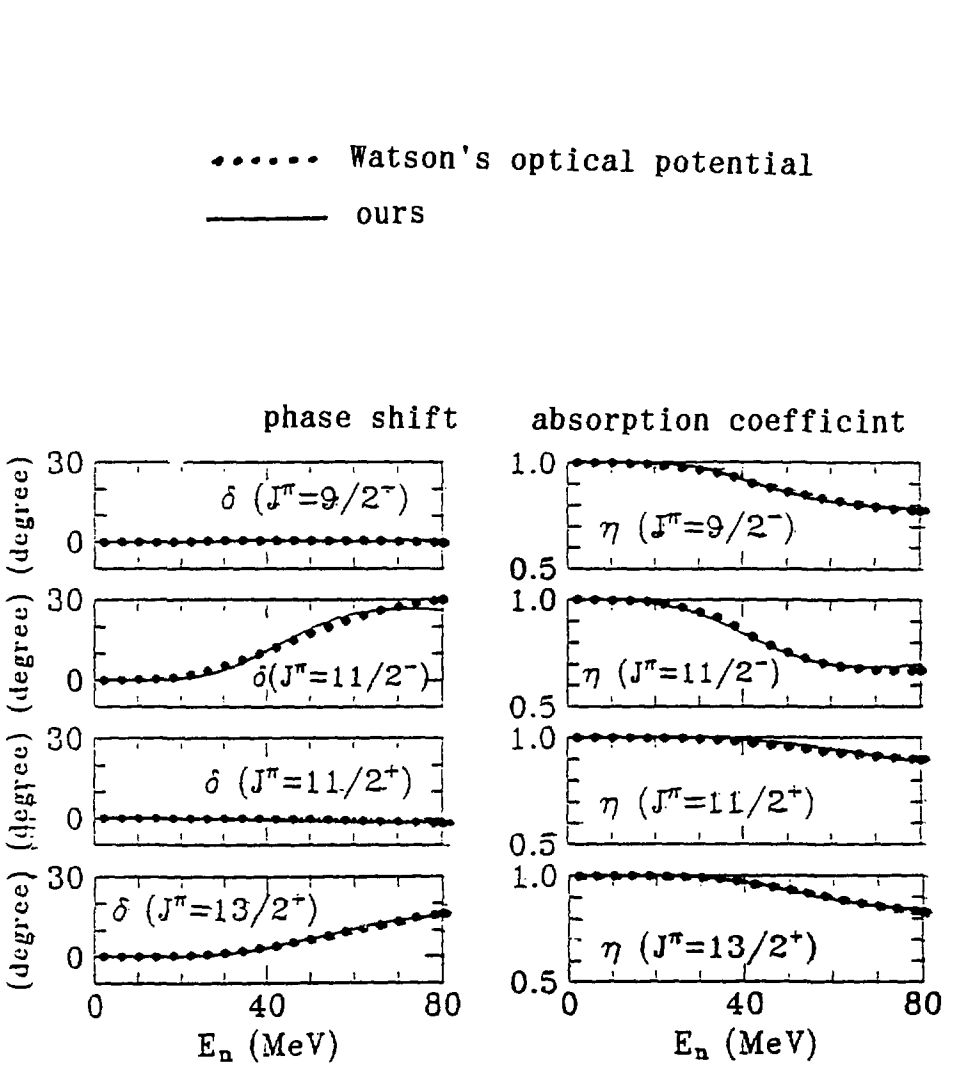
n + ^{12}C elastic scattering (E_n=0~80 MeV)

..... Watson's optical potential

——— ours




Watson's optical potential
ours
phase shift
absorption coefficint
δ (J^π=5/2^+)
η (J^π=5/2^+)
δ (J^π=5/2^-)
η (J^π=5/2^-)
δ (J^π=7/2^-)
η (J^π=7/2^-)
δ (J^π=7/2^+)
η(J^π=7/2^+)
δ (J^π=9/2^+)
η (J^π=9/2^+)
(degree)
180
90
0
30
60
1.0
0.5
0.0
40
80
E_n (MeV)

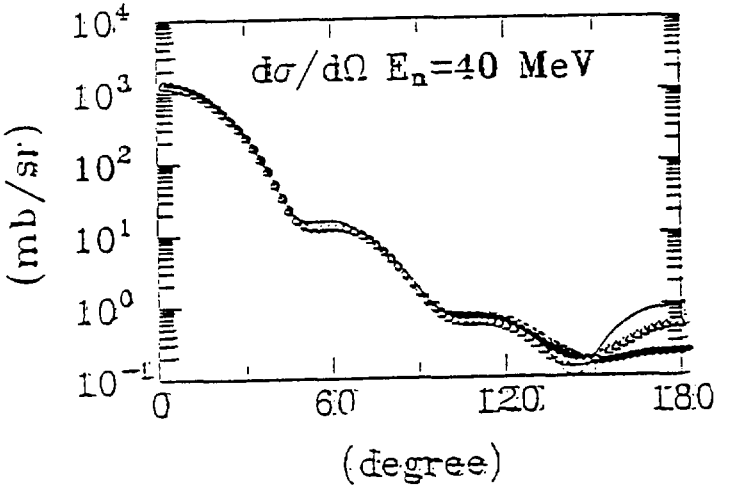

Watson's optical potential
ours
phase shift
absorption coefficint
(degree)
δ (J^π=9/2^-)
δ(J^π=11/2^-)
δ (J^π=11/2^+)
δ (J^π=13/2^+)
η (J^π=9/2^-)
η (J^π=11/2^-)
η (J^π=11/2^+)
η (J^π=13/2^+)
30
0
1.0
0.5
0
40
80
E_n (MeV)

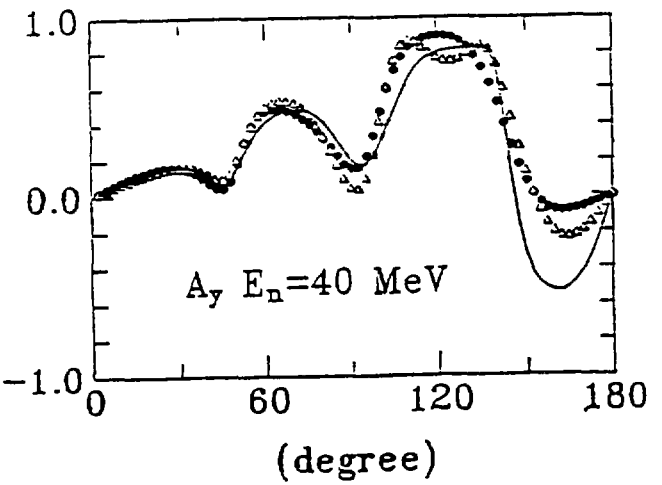
n + ^{12}C elastic scattering

•••••• Watson's

ΔΔΔΔΔ Watson's ($J^{\pi} \leq 13/2^{+}$)

——— ours ($J^{\pi} \leq 13/2^{+}$)





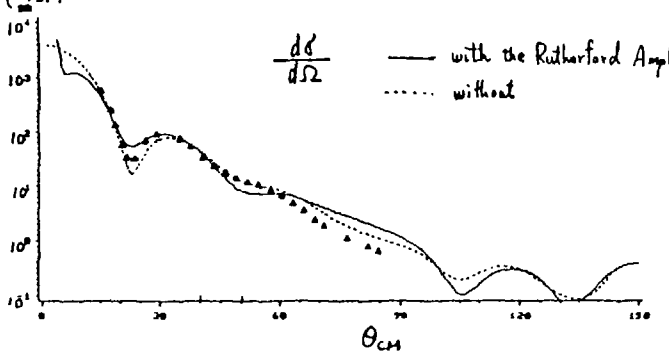
Faddeev calculation

$d + {}^{12}C$ elastic scattering

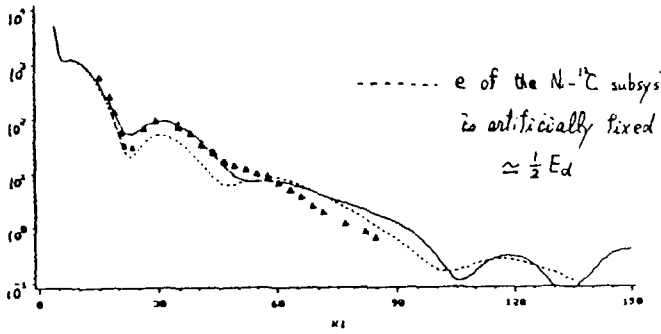
$\frac{d\sigma}{d\Omega}$ (mb/sr)

—— with the Rutherford Amplitude

····· without

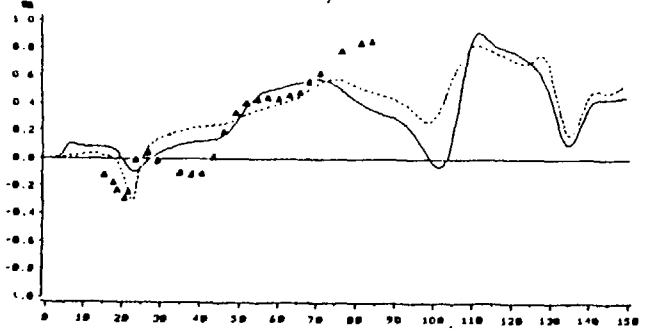


θ_{CM}

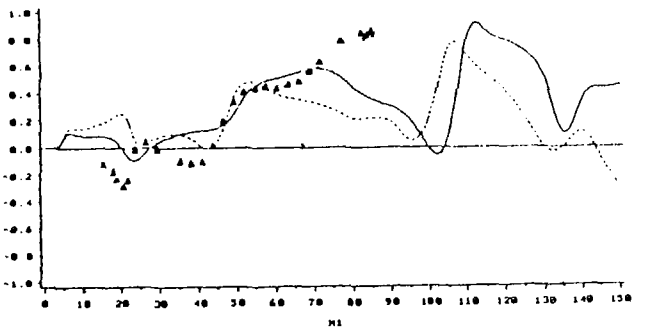


------ e of the N-^{12}C subsystems is artificially fixed to $\simeq \frac{1}{2} E_d$

A_y



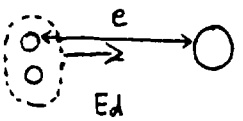
θ_{CM}



The absorption coefficients of Watson's optical potential fairly differ from those of Menet et al.

Differences between the imaginary potentials may influence on the d-^{12}C observables.

The two-body energy e has been fixed to about $1/2 \cdot E_d$ in all three-body models.



This approach is not valid here!!

§5. Summary

<u>$\alpha(\vec{d},d)\alpha$</u>

- At E_d=21 MeV, this reaction is well reproduced by our model.
- At E_d=56 MeV, the optical-model-like treatment is not enough.
- The Pauli exclusion effects is seen at 56 MeV.

<u>$^{12}C(d,d)^{12}C$ at 56 MeV</u>

- Promissing.
- How good is the optical potential used?
- The approximation $e_{N-^{12}C} \simeq 1/2 \cdot E_d$ is not valid.

Pauli Correlated Inter-Cluster Effective Potentials

- α+α, α+^{3}H, α+d, α+n -

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Abstract

The resonating-group method (R.G.M.) has been extensively applied to investigate, for instance, the α+α, α+^{3}H, α+d and α+n inter-cluster systems. By using above results, we can derive so called the R.G.M. potentials. Let us consider not only the Pauli forbidden states but also the partly Pauli forbidden states. Subtracting both of the Pauli forbidden states carefully, we can introduce the effective energy-independent potentials which may be called "the Pauli corrected potentials". In order to confirm whether these effective potentials behave properly or not, the phase-shifts are examined comparing with the R.G.M.'s results. It is found that these effective potentials can yield the same phase-shifts as the R.G.M. calculations have done. In addition, we illustrate the off-shell T-matrices which are based on those different sorts of potentials with the contour plots. It is expected that these potentials are useful for the straightforward application to the three-cluster Faddeev calculations on the realistic light nucleus.

I. INTRODUCTION

Motivation

In order to solve the 3-body Faddeev equation for the light nucleus as the 3-body model, we derive the 2-body, effective, potentials.

The derivation method

STEP I.

The 2-body potential between the clusters are derived with the resonating-group method(R.G.M.).

——— R.G.M. Potential

STEP II.

We take the forbidden states out of the above R.G.M. potential. We have the effective, energy-independent potentials.

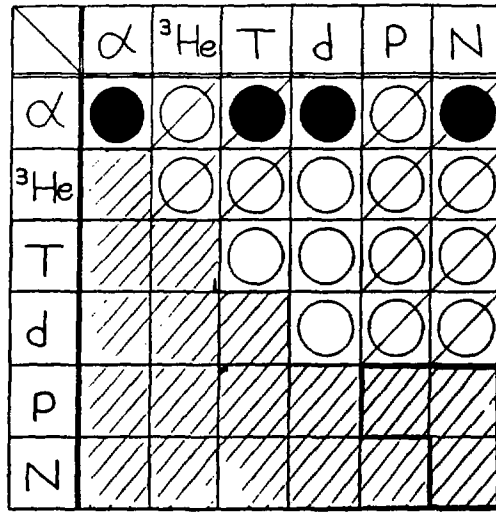
——— Pauli corrected potential

STEP III.

In order to confirm whether the derived potentials are true or not, we compare the results with the experimental data.

——— Phase shift

For the Coulomb correction, we use a screened Coulomb potential method with the proper renormalization.



[In p+p, p+N, N+N cases,

a nucleon-nucleon potential]

II. FORMURATION

The N-nucleon system:

Cluster a ···· n

Cluster b ····N-n

The total wave function is assumed to be

$$\Psi_{ab} = A[\phi_a(\xi_a)\phi_b(\xi_b)\chi(R)] \qquad (1)$$

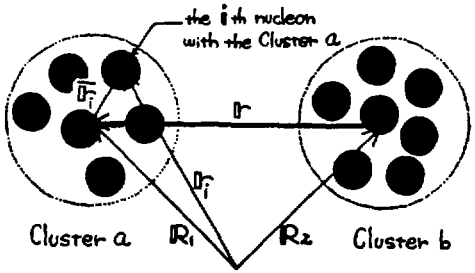
A: an antisymmetrization operator

The function ϕ_a and ϕ_b are given by

$$\phi_a = \exp\left[-\tfrac{1}{2}\alpha\sum_{i=1}^{n}(r_i - R_1)^2\right] \qquad (2)$$

and

$$\phi_b = \exp\left[-\tfrac{1}{2}\alpha\sum_{i=1}^{N-n}(r_i - R_2)^2\right] \qquad (3)$$



The Hamiltonian of the system

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{N}\nabla_i^2 + \sum_{i>j}^{N} V_{ij}(\vec{r}_i - \vec{r}_j) \tag{4}$$

The nucleon-nucleon potential is chosen as

$$V_{ij} = -V_0 \exp(-\kappa r_{ij}^2)(\omega + m P_{ij}^{r} + b P_{ij}^{\sigma} - h P_{ij}^{\tau}) + (e^2/4r_{ij})(1+\tau_{iz})(1+\tau_{jz}) \tag{5}$$

The relative wavefunction $\chi(r)$ is determined by

$$\delta\langle\Psi|(H-E')|\Psi\rangle = 0 \tag{6}$$

$$\int d\xi_a\, d\xi_b\, \phi_a^*\phi_b^*(H-E')\Psi = 0 \tag{7}$$

$$\left(-\frac{\hbar^2}{2\mu}\nabla_r^2 + V_D(r) - E\right)\chi(\vec{r}) + \int F(\vec{r},\vec{r}';E)\chi(\vec{r}')\,d\vec{r}' = 0 \tag{8}$$

——— R.G.M. equation

$$V_D = \int d\xi_a\, d\xi_b\, \phi_a^*\phi_b^*\, V_{ab}\, \phi_a\phi_b \tag{9}$$

$$F(\vec{r},\vec{r}';E) = K_T(\vec{r},\vec{r}') + K_V(\vec{r},\vec{r}') + E'\cdot N(\vec{r},\vec{r}') \tag{10}$$

Norm kernel

where, $E' = E + E_{int}$ (11)

$$V_{(\vec{r},\vec{r}')}^{RGM} = V_D(r)\cdot\delta(\vec{r},\vec{r}') + F(\vec{r},\vec{r}';E) = V_D + K_T + K_V + E'\cdot N \tag{12}$$

——— R.G.M. Potential

The Orthogonality Condition Model

The norm of the total wave function Ψ is given as follows:

$$
\begin{aligned}
1 &= \langle \Psi | \Psi \rangle = \langle \phi_a \phi_b | A[\phi_a \phi_b \chi] \rangle \\
&= \int d\xi_a \, d\xi_b \, d\mathbf{r} \, d\mathbf{r}' \, [\phi_a^* \phi_b^* \chi^*] A[\phi_a \phi_b \chi] \\
&= \int d\mathbf{r} \, d\mathbf{r}' \, \chi^* (1 - N) \chi \\
&= \underline{\langle \chi | (1 - N) | \chi \rangle} \qquad (13) \\
&\quad \hookrightarrow \therefore \langle \chi | \chi \rangle \neq 1
\end{aligned}
$$

It means that the norm of the relative wave function χ is not unity. Therefore, the function χ does not give the physical probability.

We should introduce the renormalization to χ by the following method.

$$
\downarrow
$$

$$
\sqrt{1 - N}\, \chi \equiv \varphi^{phy.} \qquad (14)
$$

$\varphi^{phy.}$ denotes the physical relative wavefunction.

Next, we consider the following eigenvalue equation for norm.

$$
\underline{\underline{N | U_i \rangle = \gamma_i | U_i \rangle}} \qquad (15)
$$

γ_i: eigenvalue

U_i: the normalized eigenfunction belong to γ_i.

Using eq.(15), norm kernel is written as

$$
N = \sum_{i=1}^{N} | U_i \rangle \gamma_i \langle U_i | \qquad (16)
$$

Here, we define the state U_{i1} as

$$N|U_{i1}\rangle = 1\cdot|U_{i1}\rangle \tag{17}$$

$$\langle\varphi^{Phys.}|U_{i1}\rangle = \langle\chi|\sqrt{1-N}\,|U_{i1}\rangle$$

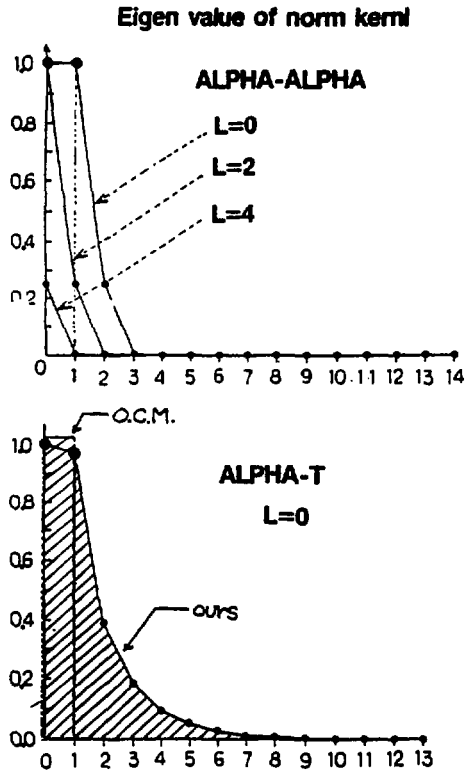
$$= \langle\chi|\sqrt{1-1}\,|U_{i1}\rangle = 0 \tag{18}$$

$$\therefore\ \varphi^{Phys.} \perp U_{i1}$$

Therefore, eigenfunction U_{i1} is the unphysical state.

↓

The forbidden state.



R.G.M. Equation

R.G.M. equation is written as (here, the integration sign is omitted)

$$(H_0 + V_D + K_T + K_V + E'N)\chi = E\cdot\chi \qquad (22)$$

the total energy $E' = E + E_{int}$ (11)

$$(H_0 + V_D + K_T + K_V + E_{int}\cdot N)\chi = E(1-N)\chi \qquad (23)$$

by using the renormalization wavefunction

$$\frac{1}{\sqrt{1-N}}(H_0 + V_D + K_T + K_V + E_{int}\cdot N)\frac{1}{\sqrt{1-N}}\varphi = E\cdot\varphi \qquad (24)$$

$$w \overset{d}{\equiv} V_D + K_T + K_V + E_{int}\cdot N \qquad (25)$$

$$\therefore \frac{1}{\sqrt{1-N}}(H_0 + w)\frac{1}{\sqrt{1-N}}\varphi = E\cdot\varphi \qquad (26)$$

Furthermore,

$$\left[H_0 + \frac{1}{\sqrt{1-N}}(H_0 + w)\frac{1}{\sqrt{1-N}} - H_0\right]\varphi = E\cdot\varphi \qquad (27)$$

Comparing eq.(27) with Schroedinger eq., we have the effective potential operator as

$$V^{esf} \overset{d}{\equiv} \frac{1}{\sqrt{1-N}}(H_0 + w)\frac{1}{\sqrt{1-N}} - H_0 \qquad (28)$$

The case of the α+α system.

It is known that the eigenfunction of the norm kernel is the harmonic oscillator function.

We rewrite eq.(28) using eq.(16) as

$$\therefore V^{esf} = \sum_{i,j}^{N_s} |v_i\rangle \left[\frac{1}{\sqrt{1-\gamma_i}}(H_{0ij} + w_{ij})\frac{1}{\sqrt{1-\gamma_j}} - H_{0ij}\right]\langle v_j| \qquad (29)$$

where, $H_{oij} \stackrel{d}{=} \langle v_i | H_o | v_j \rangle$

$w_{ij} \stackrel{d}{=} \langle v_i | w_j | v_j \rangle$

$$V^{eff} = \sum_{i,j}^{Ns} |v_i\rangle \Lambda_{ij} \langle v_j|$$

$$\Lambda_{ij} = \frac{1}{\sqrt{1-\gamma_i}} (H_{oij} + w_{ij}) \frac{1}{\sqrt{1-\gamma_j}} - H_{oij}$$

(30)

D.R. Thompson, I. Reichstein, W. McClure and Y. C. Tang
Phys. Rev. 185 (1969), p 1351.

The case of the α+T system.

The eigenfunction of the norm kernel is not given by the simple harmonic oscillator functions. So, we derive the eigenfunction with the superposition of the H.O.F.

$$N = \sum_{ij}^{Ns} |v_i\rangle G_{ij} \langle v_j| \qquad (31)$$

where, $G_{ij} \equiv \langle U_i | N | U_j \rangle$ which calculate the matrix elements analytically.

Next, the matrix G is numerically diagonalyzed with the Jacobi method.

$$N = \sum_{ijk}^{Ns} |v_i\rangle a_{ik} J_k a^{+}_{kj} \langle v_j|$$

$$= \sum_{k}^{Ns} |\tilde{v}_k\rangle J_k \langle \tilde{v}_k| \qquad (32)$$

J_k: eigenvalue of G

$\tilde{U}_k$: eigenfunction of G

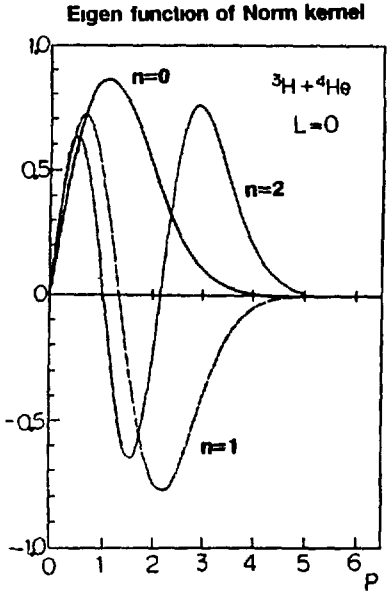
where,

$$|\tilde{v}_k\rangle \equiv \sum_{i}^{Ns} |v_i\rangle a_{ik} \qquad (33)$$

[Y.C. Tang, E. Schmid and K. Wildermuth, Phys. Rev. 131 (1963), 2632.

D.R. Thompson and Y.C. Tang, Phys. Rev. 179 (1969), 971.

R.E. Brown, and Y.C. Tang, Phys. Rev. 176 (1968), 1235]



Using eq.(32)

$$\frac{1}{\sqrt{1-N}} = \sum_{k}^{N_s'} |\tilde{v}_k\rangle \frac{1}{\sqrt{1-J_k}} \langle \tilde{v}_k|$$

$$= \sum_{ij(k)}^{N_s'} |\tilde{v}_i\rangle \underbrace{a_{ik} \frac{1}{\sqrt{1-J_k}} a^{+}_{kj}}_{\gamma'_{ij}} \langle \tilde{v}_j|$$

$$= \sum_{ij}^{N_s} |\tilde{v}_i\rangle \gamma'_{ij} \langle \tilde{v}_j| \qquad (34)$$

eq (28) $\Rightarrow$

$$V^{eff} = \frac{1}{\sqrt{1-N}} (H_0 + W) \frac{1}{\sqrt{1-N}} - H_0$$

$$\therefore V^{eff} = \sum_{ij}^{N_s} |\tilde{v}_i\rangle \left[\sum_{kl}^{N_s} \gamma'_{ik} (H_{0kl} + W_{kl}) \gamma'_{lj} - H_{0ij} \right] \langle \tilde{v}_j| \qquad (35)$$

$$V^{eff} = \sum_{ij}^{N_s} |\tilde{v}_i\rangle \tilde{\Lambda}_{ij} \langle \tilde{v}_j|$$

$$\tilde{\Lambda}_{ij} = \sum_{kl}^{N_s} \gamma'_{ik} (H_{0kl} + W_{kl}) \gamma'_{lj} - H_{0ij} \qquad (36)$$

The momentum transformation of eq.(30) gives the following form.

$$V^{eff}_{\ell}(p,p') = \sum_{i,j}^{Ns} \langle p|v_i\rangle \Lambda_{ij} \langle v_j|p'\rangle$$

We must solve the Lippman-Schwinger equation

$$T_\ell(p,p') = V_\ell(p,p') + \frac{1}{2\pi^2}\int V_\ell(p,p'') G_0(p'') T_\ell(p'',p') p''^2 dp'' \quad (37)$$

[rank-1 Generalized Separable Expansion:

S. Oryu, M. Araki and S. Satoh, Prog. Theor. Phys. (1977)]

With the G.S.E. method, the phase shift is given by

$$\delta_\ell = \arg T_\ell(k,k) \quad (38)$$

where, k is the initial momentum: $E=k^2/2\mu$ (μ: reduced mass) and also the scattering amplitude is given by

$$f_\ell(E) = -\frac{\mu}{2\pi} T_\ell(k,k) \quad (39)$$

The Coulomb correction for the phase shift

The screened Coulomb potential:

$$V^C(r) = \frac{ZZ'e^2}{r} \longrightarrow V^R(r) = \frac{ZZ'e^2}{r} e^{-r/R} \quad (40)$$

R: the screened range

The momentum representation of $V^R_\ell(p,p')$:

$$V^R_\ell(p,p') = \frac{ZZ'e^2}{2pp'} Q_\ell\left(\frac{p^2+p'^2+1/R^2}{2pp'}\right) \quad (41)$$

The total interaction:

$$V_\ell(p,p') = V^{Nucl.}_\ell(p,p') + V^R_\ell(p,p') \quad (42)$$

The corection in ALT's way is given by following:

$$\delta_\ell = \overbrace{\delta_\ell^{Nucl.}}^{\text{the Phase shift}} + \underbrace{\eta\cdot(\log 2kR - \gamma) - \sigma_\ell}_{\text{the Coulomb renormalized phase shift}} \qquad (43)$$

The sommerfeld parameter:

$$\eta = \frac{\mu z z' e^2}{k} \qquad (44)$$

The Euler constant:

$$\gamma = 0.5772156$$

Coulomb phase shift $\cdots$ σ_ℓ:

$$\sigma_\ell = \arg[\Gamma(\ell+1+i\eta)] \qquad (45)$$

The R.G.M. potential and the effective potential are given as follows:

$$V^{RGM} = V_D + K_T + K_V + (E+E_{int})\cdot N \qquad (12)$$

$$V^{eff} = \sum_{ij}^{N_S} |v_i\rangle \left[\frac{1}{\sqrt{1-\gamma_i}}(H_{0ij}+W_{ij})\frac{1}{\sqrt{1-\gamma_j}} - H_{0ij}\right]\langle v_j| \qquad (30)$$

$$\xrightarrow[\gamma_i\,\gamma_j \to 0]{i,j \text{ higher order}} \sum_{ij}^{N_S} |v_i\rangle W_{ij}\langle v_j|$$

$$\Longrightarrow V_D + K_T + K_V + E_{int}\cdot N$$

It is found that

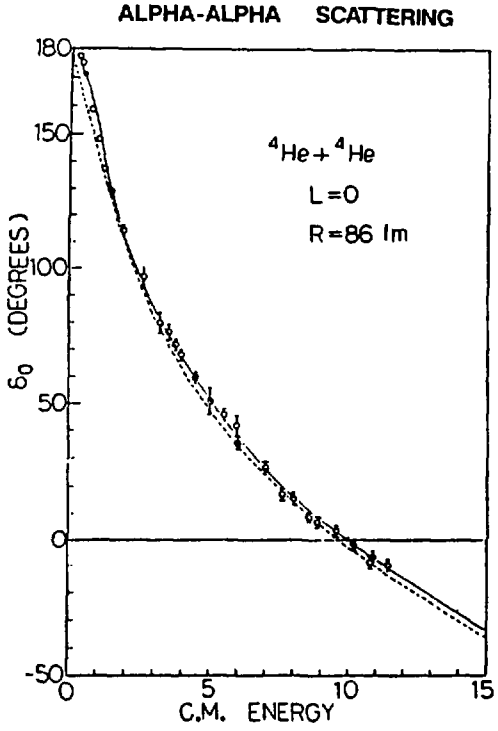
$$V^{eff} \xrightarrow{i,j \text{ higher order}} V^{RGM}_{E=0}$$

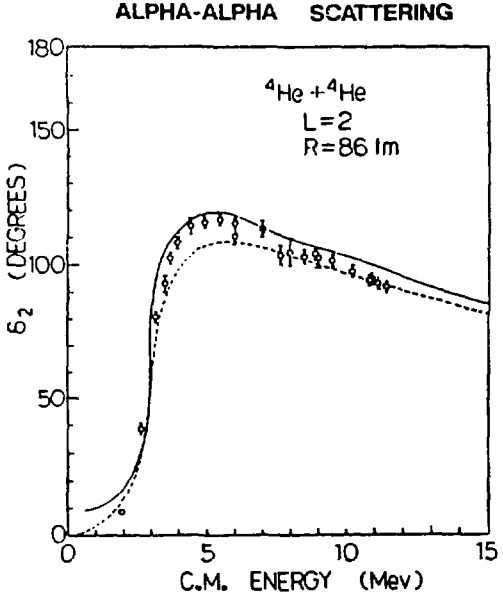
Therefore, we derive the effective potential with the base of the R.G.M. potential by the following method:

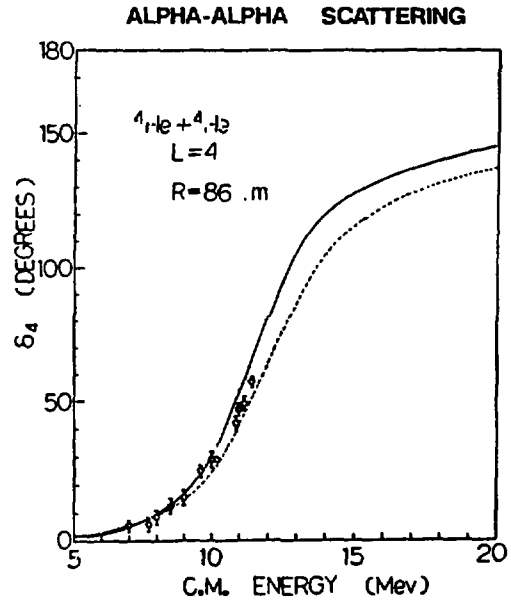
$$\tilde{V}^{eff}_{(p,p')} = V^{RGM}_{(p,p';E=0)} - \sum_{ij}^{(N_R)} \langle p|v_i\rangle \lambda_{ij} \langle v_j|p'\rangle$$

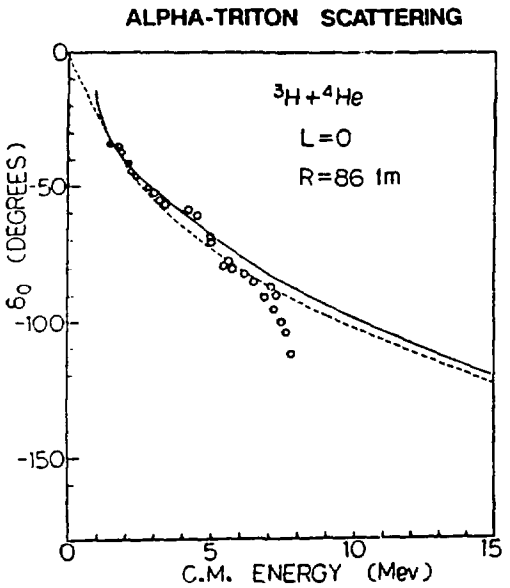
$$+ \sum_{ij}^{(N_R)} \langle p|v_i\rangle \left[\frac{1}{\sqrt{1-\gamma_i}}(H_{0ij}+W_{ij})\frac{1}{\sqrt{1-\gamma_j}} - H_{0ij}\right]\langle v_j|p'\rangle$$

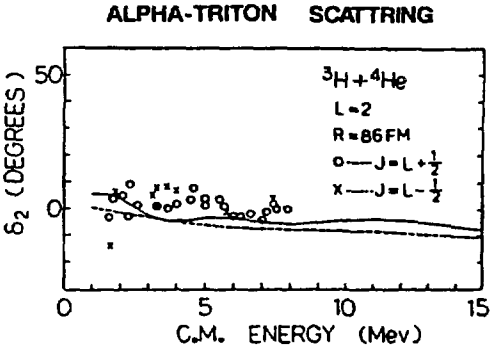
[The number of the rank N_R is 10 or so(this is confirmed numerically]

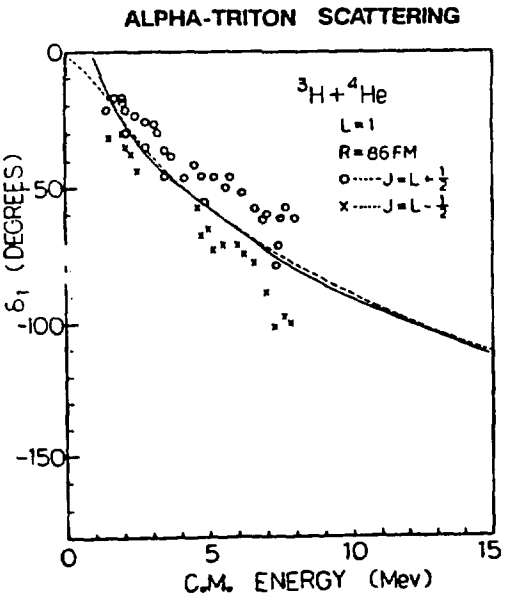

ALPHA-ALPHA SCATTERING
$^4He + ^4He$
L=0
R=86 fm
δ_0 (DEGREES)
C.M. ENERGY
180
150
100
50
0
-50
0
5
10
15

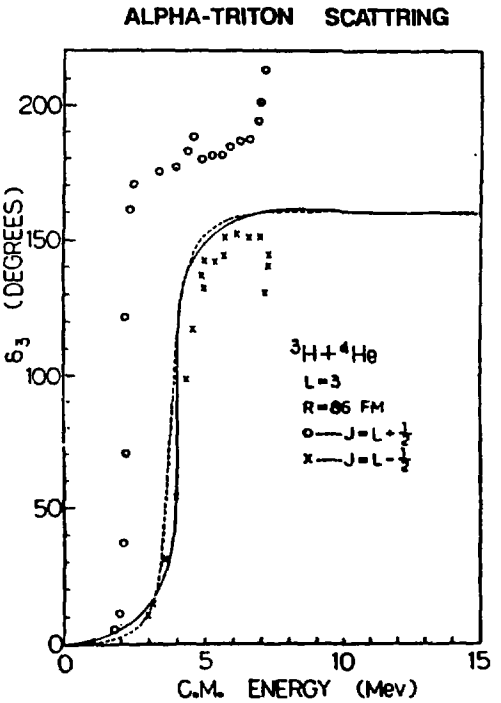

ALPHA-ALPHA SCATTERING
$^4He + ^4He$
L=2
R=86 fm
δ_2 (DEGREES)
C.M. ENERGY (Mev)
180
150
100
50
0
0
5
10
15


ALPHA-ALPHA SCATTERING
$^4He + ^4He$
L=4
R=86 fm
δ_4 (DEGREES)
C.M. ENERGY (Mev)
180
150
100
50
0
5
10
15
20

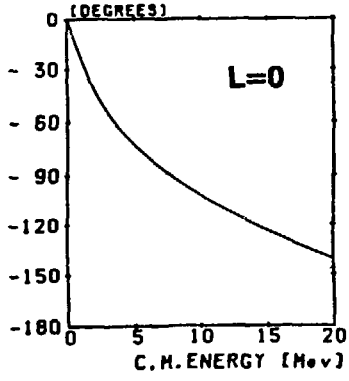

ALPHA-TRITON SCATTERING
$^3H+^4He$
L=0
R=86 fm
δ_0 (DEGREES)
C.M. ENERGY (Mev)

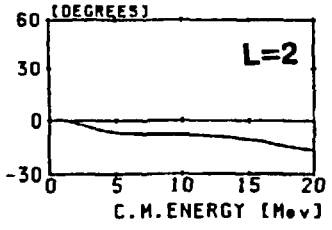

ALPHA-TRITON SCATTRING
$^3H+^4He$
L=2
R=86FM
o—J=L+½
x—J=L-½
δ_2 (DEGREES)
C.M. ENERGY (Mev)

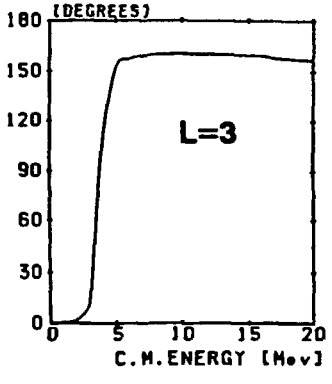

ALPHA-TRITON SCATTERING
$^3H+^4He$
L=1
R=86FM
o·····J=L+½
x·····J=L-½
δ_1 (DEGREES)
C.M. ENERGY (Mev)

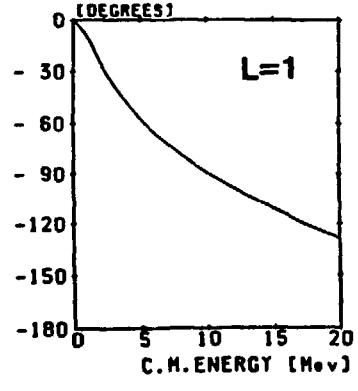

ALPHA-TRITON SCATTRING
$^3H+^4He$
L=3
R=86 FM
o—J=L+½
x—J=L-½
δ_3 (DEGREES)
C.M. ENERGY (Mev)

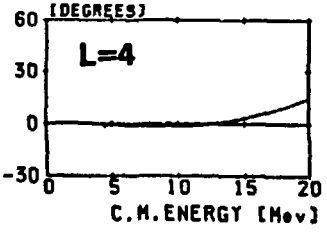
ALPHA-T PHASE SHIFT



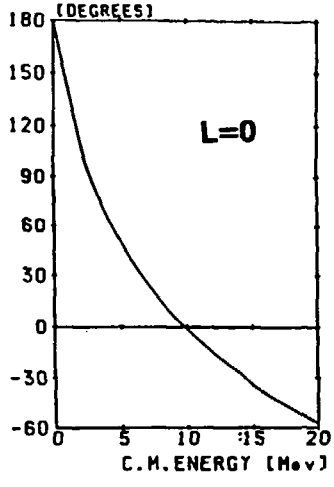


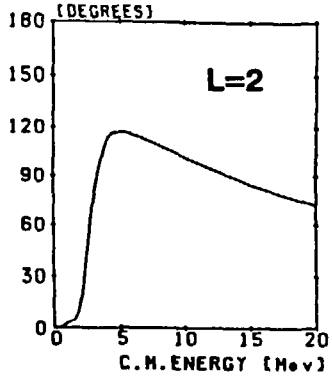


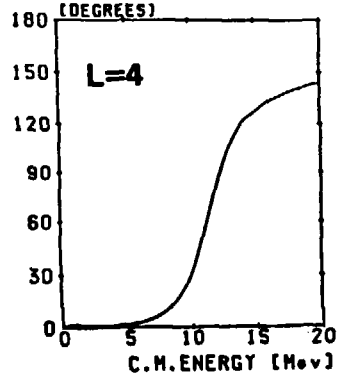




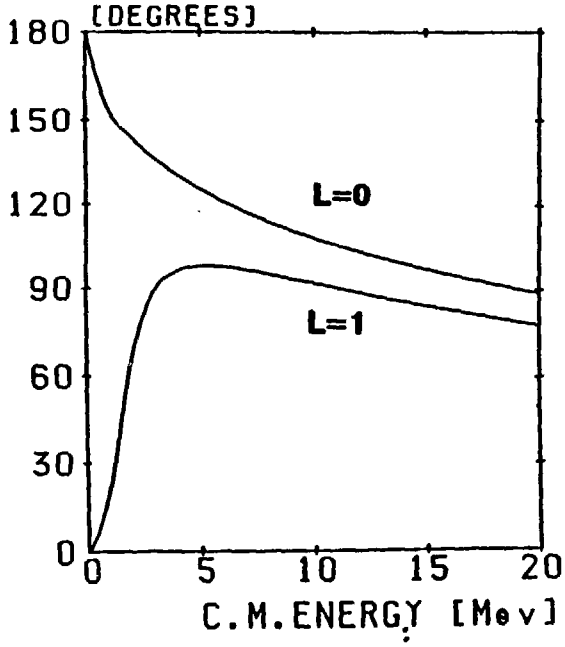
ALPHA-ALPHA PHASE SHIFT



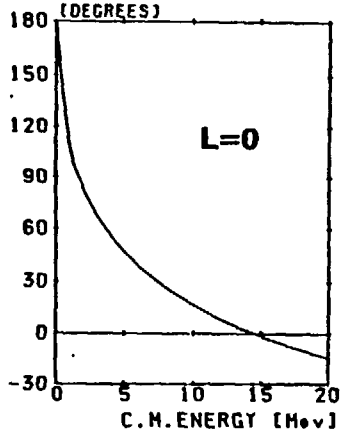


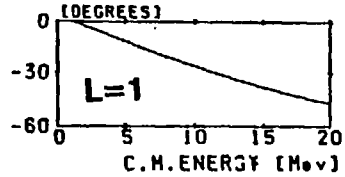


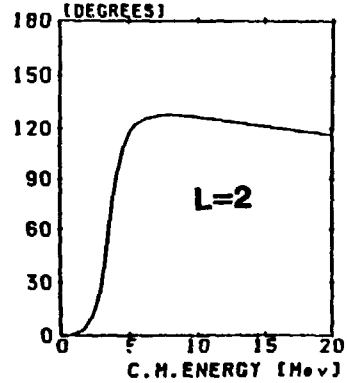
ALPHA-N PHASE SHIFT

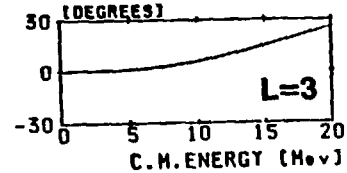


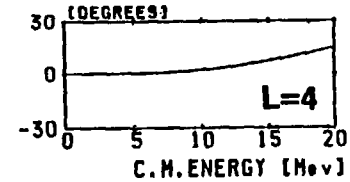
ALPHA-D PHASE SHIFT

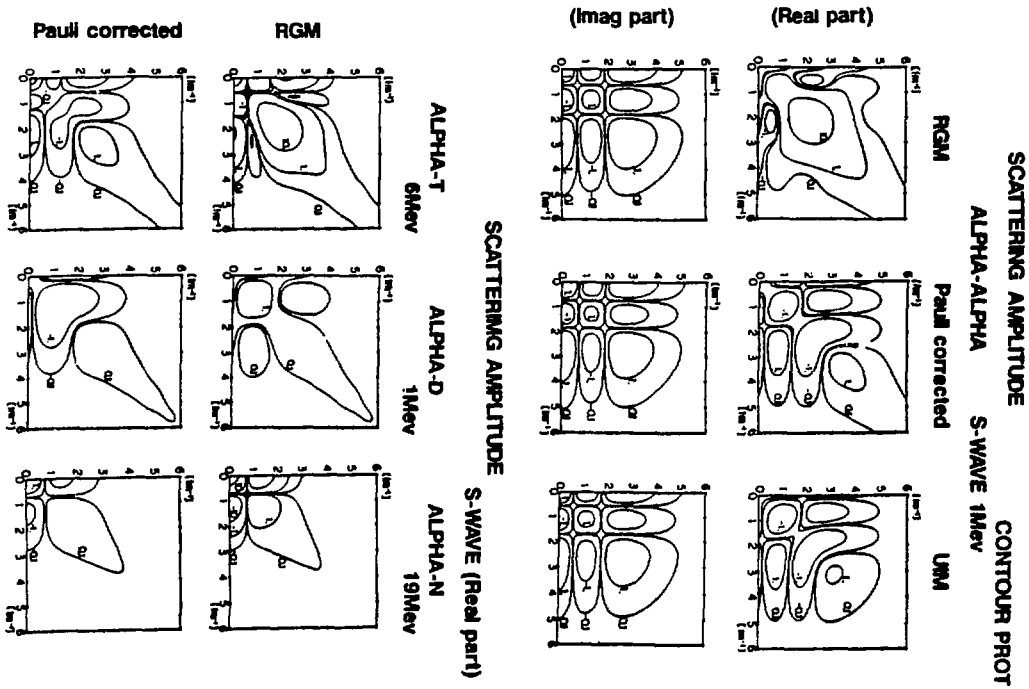












IV. CONCLUSION

We could derive the effective, energy-independent potentials between the Clusters.

It is found that these potentials can yield the same phase shift as the R.G.M. calculation do. And the numerical errors are excluded.

Finally, these potentials are useful in the application of the 3-cluster Faddeev calculations, I think.

Two- and Three-Body Problems with EWIPR method

Y. Koike

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Abstract: We propose a simple but rigorous formalism to solve the Lippmann-Schwinger equation, expanding the wave function inside the potential range (EWIPR). The solution gives us a separable t-matrix. The formalism can be applied to any finite range two-body interaction and is useful to solve few-body equations:

[This work was already published in the following two papers:

1) Y. Koike, Prog. Theor. Phys. **87** (1992) 775.

2) J.L. Friar, Y. Koike et al., Phys. Rev. **C42** (1990) 1838.

The first one is for the formalism and the second one is for part of numerical applications.]

Three-Nucleon Bound States

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Kasuga 1-2, Tsukuba 305
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Abstract: We have finished calculations of various physical quantities both for triton and ^{3}He. I have shown the results extensively at the meeting. However, since almost all of the results are yet to be published, I will write here only the outline of the results.

This is a report of the works of the Sendai group. People who are participating presently are S. Ishikawa, T-Y. Saito, Y. Wu, and T. Sasakawa.

[I] Our calculations have been performed by solving the 52 channel (Coulomb modified) Faddeev equation with realistic two nucleon potential, Tucson-Melbourne or pi-rho exchange three-nucleon force, charge-independent- and charge-symmetry-breaking forces, without or with Coulomb interactions. Therefore, the results for the triton and ^{3}He are obtained by switching-off and -on the Coulomb interaction of the same code.

[II] The tensor analyzing power A_{yy} of the pd radiative capture can be obtained only by the Argonne potential (AV14)with a three-nucleon potential. This indicates the importance of a three-nucleon potential.

[III] The calculated physical quantities such as the charge radius, the D/S ratio of the asymptotic normalization constants, and the dipole photo-absorption cross section are linearly correlated to the calculated binding energy. This means that unless we obtain the correct binding energy, we can not obtain correct value of a physical quantity.

By the way, this is the first calculation of the dipole photo-absorption in which the interaction and the wave function are consistent.

[IV] The charge form factor of both triton and ^{3}He are reproduced with the three-body force effect and the exchange current up to $q \lesssim 4$ fm$^{-1} \simeq 0.8$ GeV/c.

[V] The Coulomb force (0.648 MeV) itself renders nothing physically interesting. However, it is important for studying the charge-independence-breaking (CIB) and charge-symmetry-breaking (CSB) force effects. CIB and CSB yield repulsive effects of 0.232 MeV and 0.75 MeV, respectively. These forces act against the three-body force effect.

[VI] Since only AV14 can reproduce the tensor analyzing power of the pd radiative capture, we adopt this potential as the most realistic one. The AV14 yields the binding energy of 7.673 MeV in triton. If we take $\Lambda_{\pi NN}$ and $\Lambda_{\rho NN}$ to be 0.81GeV and 1.13 GeV, respectively, we obtain 1.044 MeV for the three-body force effect. Adding the CIB effect mentioned above, we get the triton binding energy of 8.485 MeV, which should be compared with the experimental value of 8.482 MeV. If we add the ^{3}H-^{3}He mass difference of - 0.760 MeV (Coulomb, -0.648;CSB, -0.075; others, -0.037) to 8.485 MeV, we obtain 7.725 MeV for ^{3}He binding energy, which should be compared with the experimental value of 7.718 MeV.

[VII] This set of two- and three-nucleon potentials reproduce the Gamow-Teller matrix element for the triton beta decay of $\sqrt{3}$*0.961 well giving the value of $\sqrt{3}$*0.955.

[VIII] To summarize, we have seen that there is no room to accommodate the quark effect in triton or ^{3}He. In the meeting, I discussed also (1) the antisymmetrization of nucleons may affect the study of the quark effect in an unfavorable manner.

(2) the analysis of the y-scaling does not show any suggestion about the quark effect, and (3) a potential derived from quark-motivated P-matrix theory also does not lead any indication of a quark effect in observables. Therefore, the distance between low energy physics and intermediate energy physics must be very remote.

Nucleus-Nucleus Collisions and Pauli Principle

Hisashi Horiuchi
Department of Physics, Kyoto University, Kyoto 606

Abstract: Treatments of the Pauli principle in nuclear collision problems are discussed. The concept of the Pauli-forbidden region in the phase space which has been developed in nucleus-nucleus interaction theories is shown to play a decisive role in a newly developed microscopic simulation framework of hevy-ion collisions, namely the antisymmetrized version of the molecular dynamics with inclusion of two-nucleon collisions.

[1] Introduction and summary

Recently we have developed a new microscopic simulation framework of heavy-ion collisions. It is an antisymmetrized version of molecular dynamics with inclusion of two-nucleon collisions. We give to it an abbreviated name AMD. The AMD framework without two-nucleon collisions has been already applied to several nuclear structure problems[1]-[3], and can be regarded to be a restricted version of Feldmeier's fermionic molecular dynamics[4] which treats even the time developments of spin functions and width parameters of nucleon wave packets. In order to treat heavy-ion collision problems, the inclusion of the two-nucleon collisions into AMD is indispensable and has been successfully formulated in collaboration with Akira Ono partially for the fulfillment of his master thesis work. The AMD with inclusion of two-nucleon collisions is just an antisymmetrized version of the quantum molecular dynamics of Aichelin and Stöcker[5],[6].

The inclusion of two-nucleon collisions into the AMD has been made possible firstly by the introduction of physical (approximately canonical) nucleon coordinates and secondly by the use of the practical treatment of the Pauli blocking. Under the antisymmetrization operation, the position and momentum parameters of nucleon wave packets are no more canonical coordinates. This situation is entirely the same as in the wave-packet theory of inter-cluster relative motion by Saraceno et al.[7]. Thus the introduction of the approximately canonical nucleon coordinates is made just by imitating the introduction of the canonical inter-cluster relative coordinates by Saraceno et al.. Our treatment of the Pauli blocking in terms of the newly

introduced physical (approximately canonical) nucleon coordinates relies decisively on the concept of the Pauli-forbidden region in the phase space. In the phase space description of the nucleus-nucleus relative motion, the Pauli-forbidden region in the phase space plays the same role as the Pauli-forbidden states and is one of the most important concepts of the nucleus-nucleus interaction theory[8].

As explained above, our formulation of the AMD with inclusion of two- nucleon collisions is strongly related to the microscopic cluster theory of the nucleus-nucleus interaction. Therefore in this talk I start with the brief recapitulation of the Pauli-forbidden states and its phase-space image, the Pauli-forbidden region in the phase space in microscopic inter-nucleus interaction theories. Then in order to clarify characteristic features of the AMD, I briefly explain the formulation of the quantum molecular dynamics putting emphasis on how the Pauli principle is treated in this microscopic simulation framework. After giving a rather detailed explanation of our AMD with inclusion of two-nucleo collisions, I will show one example of its application to heavy-ion collision problems made by Ono. It is the problem of the fragment mass distribution of the ^{14}N(35MeV/nucleon) +^{12} C reaction, and we will see that our approach is successful in reproducing the experimentally observed shell effect in fragment production. It is to be noted that the successful description of the shell effect in microscopic nucleus-nucleus collision simulation approaches has been realized for the first time by our present AMD calculation. It is because all the other approaches are of classical nature.

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[4] H.Feldmeier, Nucl.Phys.**A515**(1990),147.

[5] J.Aichelin and H.Stöcker, Phys.Letters **176B**(1986),14.

[6] J.Aichelin, Phys.Reports **202**(1991), 233.

[7] M.Saraceno,P.Kramer,and F.Fernandez, Nucl.Phys.**A405**(1983), 88.

[8] H.Horiuchi, Trends in Theoretical Physics, Vol.2, eds. J.P.Ellis and Y.C.Tang, (Addison-Wesley,1991), Chapter 13.

[2] Pauli-forbidden states

The inportance of the Pauli principle in nuclear physics is widespread and overwhelming.

In nucleus-nucleus collision problems, I have struggled with the Pauli principle in treating especially the following three subjects:

(i) Pauli-forbidden states of the nucleus-nucleus reactive motion including multi-cluster relative motion.

(ii) Momrntum-(or energy-)dependence of the nucleus-nucleus interaction (includind) nucleon-nucleus interaction, namely nucleon mean field).

(iii) Nucleon-nucleon Pauli principle in microscopic simulation of nucleus-nucleus collisions.

Here I discuss (i) and (iii) which are strongly related.

Two-body (cluster) systems

Usually, forbidden states = quantum states with $2n+\ell \leq N_F$
(in many cases, harmonic oscillator wavefunctions)

Many-body (cluster) systems

Any pair of clusters cannot occupy the forbidden states of their relative motion.

Two approachs:

(i) Explicit construction of many-body allowed states (for example, 3α OCM); Kyoto, Moscow, Sapporo, Nagoya, $\cdots$

(ii) Pauli potential; Moscow, Tubingen, Fukuoka, $\cdots$

$$\lambda \Sigma_{\alpha,\beta} \Sigma_F |\chi_F^{\alpha\beta}\rangle\langle\chi_F^{\alpha\beta}| \qquad (\alpha,\beta=\text{cluster pair})$$

[3] Pauli-forbidden region in phase space

Phase-space description of nuclear dynamics

Forbidden states with $2n+\ell \leq N_F$

$\updownarrow$

Forbidden region in phase space with $1/2\mu \cdot P^2 + \mu\omega^2/2 \cdot R^2 \leq \hbar\omega \cdot NF$

Two approaches for this correspondence

(i) WKB application to RGM (Kyoto); equivalent local potential for RGM

interaction.

(ii) Time-dependent wave-packet description of inter-cluster relative motion (Sapporo, Kramer, Fernandez)

$$\mathcal{A}\{\chi(\vec{r},\vec{z})\Phi_I\Phi_{II}\} = \Phi(\vec{z})$$

$$\chi(\vec{r},\vec{z}) \propto \exp\left[-\nu\mu_A\left(\vec{r}-\frac{\vec{z}}{\sqrt{\nu\mu_A}}\right)^2\right]$$

$$\propto \exp\left[-\nu\mu_A(\vec{r}-\vec{D})^2 + i\vec{K}\vec{r}/\hbar\right]$$

$$\vec{z} = \sqrt{\nu\mu_A}\left(\vec{D} + i\frac{\vec{K}}{2\nu\mu_A\hbar}\right)$$

Here, $\vec{Z}$ ($\vec{D}$ and $\vec{K}$) is not canonical coordinate. Canonical coordinate $\vec{\omega}$ ($\vec{R}$ and $\vec{P}$) is given by

$$\vec{\omega} = \vec{z}\left[\frac{\partial \ln N}{\partial(\vec{z}\cdot\vec{z}^*)}\right]^{1/2} \qquad \vec{\omega} = \sqrt{\nu\mu_A}\left(\vec{R} + i\frac{\vec{P}}{2\nu\mu_A\hbar}\right)$$

$$N(\vec{z}\cdot\vec{z}^*) = \langle\chi(\vec{r},\vec{z})|\, N^{RGM}\,|\chi(\vec{r},\vec{z})\rangle$$

$$= \langle\Phi(\vec{z})|\,\Phi(\vec{z})\rangle$$

$$N_F < \frac{\langle\Phi(\vec{z})|\,\overbrace{\vec{a}^{+}_{rel}\cdot\vec{a}_{rel}}\,|\,\Phi(\vec{z})\rangle}{\langle\Phi(\vec{z})|\,\Phi(\vec{z})\rangle}$$

(oscillator quanta of relative motion)

$$= \vec{z}\cdot\vec{z}^* \frac{\partial \ln N}{\partial(\vec{z}\cdot\vec{z}^*)} \qquad \frac{\sum_N N\mu_N(\vec{z}\cdot\vec{z}^*)^N/N!}{\sum_N \mu_N(\vec{z}\cdot\vec{z}^*)^N/N!}$$

$$= \vec{\omega}\cdot\vec{\omega}^*$$

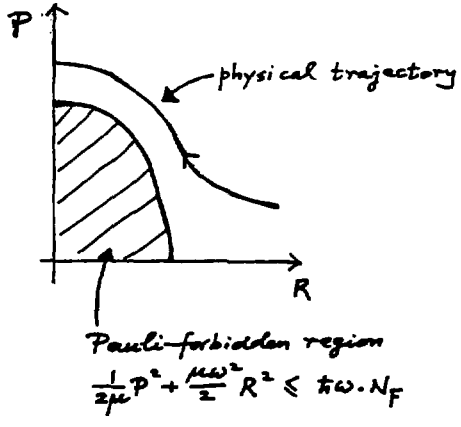
$$= \frac{1}{\hbar\omega}\left[\frac{1}{2\mu}P^2 + \frac{\mu\omega^2}{2}R^2\right]$$

Pauli-allowed $\vec{\omega}$ ($\vec{R}$ and $\vec{P}$) should satisfy

$$1/2\mu\cdot P^2 + \mu\omega^2/2\cdot R^2 > \hbar\omega\cdot NF.$$

Physical momentum P at R=0 is very large.

$$(\because)\left(\frac{1}{2\mu}P^2\right)_{R=0} > \hbar\omega\cdot N_F$$



Therefore the potential V(R) at R=0 is very deep:

$$V(R=0) = E - \left(\frac{1}{2\mu}P^2\right)_{R=0}$$

$$= \text{very deep}$$

[4] Quantum molecular dynamics and Pauli principle

$$\Phi = \prod_j \phi_j(\vec{r}_j)\,\chi_{\substack{\text{spin}\\\text{isospin}}}(j)$$

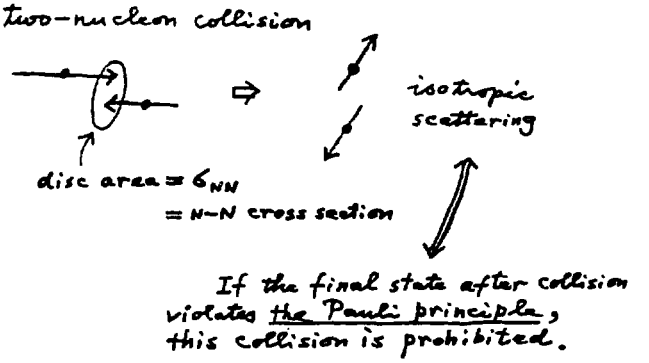
$$\phi_j(\vec{r}) = \left(\frac{2\nu}{\pi}\right)^{\frac{3}{4}} \exp\left[-\nu(\vec{r}-\vec{D}_j)^2 + i\vec{K}_j\cdot\vec{r}/\hbar\right]$$

$$\frac{d}{dt}\vec{D}_j = \frac{\partial H}{\partial \vec{K}_j}$$

$$\frac{d}{dt}\vec{K}_j = -\frac{\partial H}{\partial \vec{D}_j}$$

$$H = \sum_j \frac{\vec{K}_j^2}{2m} + \sum_{j>k}\langle \phi_j\chi_j\,\phi_k\chi_k \,|\, v \,|\, \phi_j\chi_j\phi_k\chi_k - \phi_k\chi_k\phi_j\chi_j\rangle$$

$$+ (\text{density-dep. force})$$



How is the Pauli principle treated?

In phase space, a unit volume h^3 should not contain more than one nucleon.

If this Pauli principle condition is satisfied at the initial time, then the Liouville theorem of the classical mechanics ensures the non-violation of the Pauli principle at later time.

Therefore, what is necessary is the construction of ground state nuclei without violation of the Pauli principle.

Two construction method:

(i) collisional cooling method (only by Kyoto),

(ii) frictional cooling method (by all groups including Kyoto, originally by Wilets et al.).

One needs to introduce Pauli potential for this method:

Pauli potential:

$$V_P = v_P \sum_{i,j} \exp\left[- \frac{(\vec{D}_i - \vec{D}_j)^2}{2(\Delta D)^2} - \frac{(\vec{K}_i - \vec{K}_j)^2}{2(\Delta K)^2} \right] \delta_{\tau_i \tau_j} \delta_{\sigma_i \sigma_j}$$

strong repulsion

$$\Delta D \cdot \Delta K / 2 \sim \hbar$$

This Pauli potential ensures the non-violation of the Pauli principle.

Equation of frictional cooling

$$\left.\begin{aligned} \frac{d}{dt}\vec{D}_j &= -\alpha^2 \frac{\partial}{\partial \vec{D}_j}(H + V_P) \\ \frac{d}{dt}\vec{K}_j &= -\beta^2 \frac{\partial}{\partial \vec{K}_j}(H + V_P) \end{aligned}\right\} \Rightarrow \frac{d}{dt}(H + V_P) < 0$$

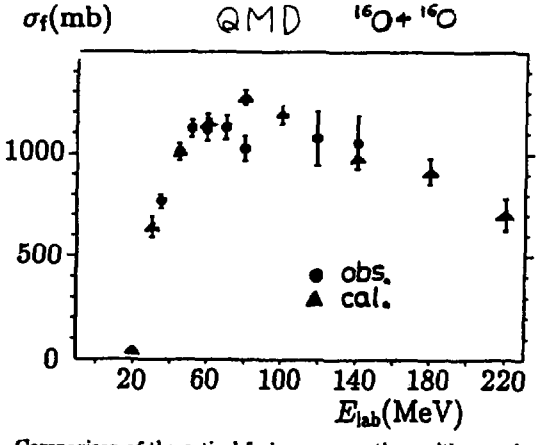
Note:

Wigner transform $\longrightarrow$ W

$$V_P = \left(\frac{v_P}{g} \sum_{i,j} |\chi_{\sigma s}(\vec{D}_i - \vec{D}_j)\rangle\langle\chi_{\sigma s}(\vec{D}_i - \vec{D}_j)| \right)$$

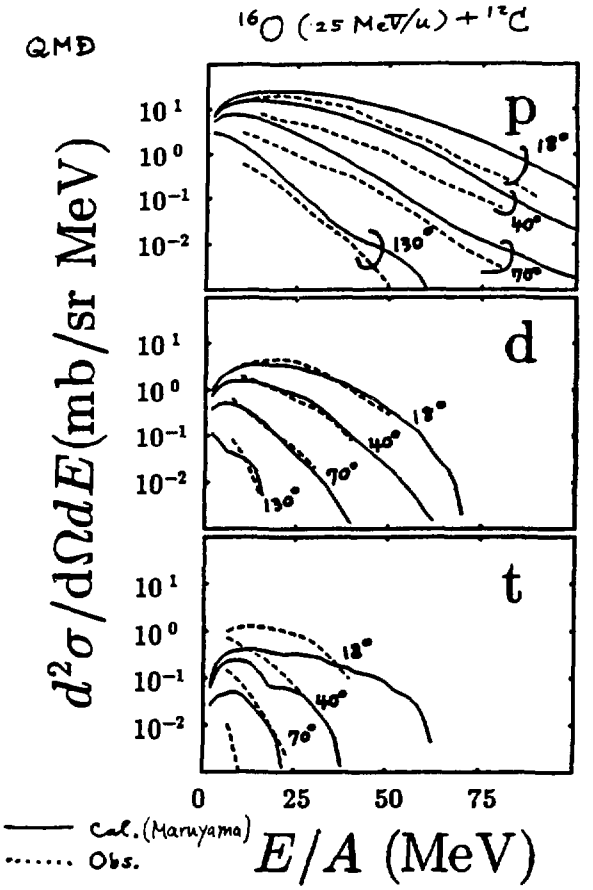
$$\chi_{\sigma s}(\vec{r}) \equiv \left(\frac{1}{2\pi(\Delta D)^2}\right) \exp\left(-\frac{r^2}{4(\Delta D)^2}\right)$$

Moscow-Pauli-Potential form



Comparison of theoretical fusion cross sections with experiments.

by Maruyama



[5] Antisymmetrized version of molecular dynamics with two-nucleon collisions

A restricted version of fermionic molecular dynamics (FMD) by H. Feldmeier (N.P. A515 (1990), 147) if two-nucleon collisions are absent.

Wave function

$$\Phi = \det\left[\prod_{i=1}^{A} \phi_i(\vec{r}_i)\,\chi^{(i)}_{\text{spin-isospin}}\right]$$

$$\phi_i(\vec{r}) = \left(\frac{2\nu}{\pi}\right)^{\frac{3}{4}} e^{-\nu(\vec{r}-\vec{B}_i)^2 + i\vec{K}_i\cdot\vec{r}/\hbar}$$

Time-dependent variables

nucleon positions $\vec{D}$ (i=1~A)

nucleon momenta $\vec{K}$ (i=1~A)

(In FMD, nucleon oscillator parameters ν_i

nucleon spin functions χ_i)

Equation of motion (derived by time-dependent variational principle)

$$\frac{d}{dt} Z_{i\alpha} = \sum_{j\beta} D_{i\alpha,j\beta} \frac{\partial}{\partial Z^*_{j\beta}} \left(\frac{\langle \Phi | H | \Phi \rangle}{\langle \Phi | \Phi \rangle} \right)$$

$$\left(\vec{Z}_i = \sqrt{\nu} \left(\vec{D}_i + \frac{i}{2\hbar\nu} \vec{K}_i \right) \right)$$

〈Application to structure problems〉

Construction of ground state wave function.

1) At first we choose $\vec{Z}$ randomly.

The AMD w.f. Φ with those $\{\vec{Z}_i\}$ represents a highly excited state.

2) We cool down this Φ into ground state by frictional cooling.

3) Equation of frictional cooling:

$$\frac{d}{dt} Z_{j\tau} = (\lambda + i\mu) \sum_{k\sigma} D_{j\tau,k\sigma} \frac{\partial}{\partial Z^*_{k\sigma}} \left(\frac{\langle \Phi | H | \Phi \rangle}{\langle \Phi | \Phi \rangle} \right)$$

↑ frictional parameter

One can easily show

$$\frac{d}{dt} \left(\frac{\langle \Phi | H | \Phi \rangle}{\langle \Phi | \Phi \rangle} \right) \begin{cases} < 0 & \text{for } \mu < 0 \\ = 0 & \text{for } \mu = 0 \\ > 0 & \text{for } \mu > 0 \end{cases} \Leftarrow \text{cooling}$$

(L. Wilet, E.M. Henly, M. Kraft, A.D. MacKeller, Nucl. Phys. A282 (1977), 341)

Contruction of ground states with frictional cooling method starting with gas configuration (random $\{\vec{Z}_i\}$) is without prejudice (no model assuption):

no assumption of axial symmetry

no assumption of clustering

.....

Self-conjugate 4N Nuclei

1) λ=1.0, μ=-0.7

2) Volkov force

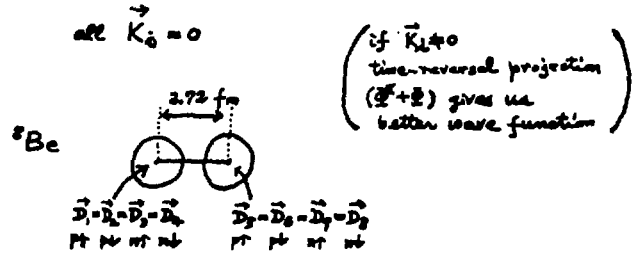
No.1 m=0.56, ν=0.268 fm^{-2} : ^{8}Be

No.1 m=0.60, ν=0.16 fm^{-2} : ^{12}C

No.1 m=0.60, ν=0.16 fm^{-2} : ^{16}O

No.2 m=0.60, ν=0.19 fm^{-2} : ^{20}Ne

Converged ground-state configuration



Brink's 2α wave function

$$N_{tot} = \langle \Phi | \sum_{i=1}^{A} \vec{a}_i^{+} \cdot \vec{a}_i | \Phi \rangle / \langle \Phi | \Phi \rangle$$

= total number of oscillator quanta

^{12}C : "coverged ground state"

= "$(0s)^4(0p)^8(\lambda\mu)=(0,4)$ intrinsic state"

N_{tot}=8.000

If Majorana parameter is increased: m=0.60→0.65

regular triangle of 3α

α-α distance = 1.43 fm

N_{tot}=8.26

^{16}O : "coverged ground state"

= "$(0s)^4(0p)^{12}$ double closed shell"

N_{tot}=12.000

Even if m=0.60→0.65, this result is unchanged.

Neutron-rich Be Isotopes

How does the α-α cluster structure of ^{8}Be change in going from ^{8}Be to neutron-rich side of Be isotopes?

1) Volkov force No.1 m=0.56

Spin-orbit force $u^{LS}(e^{-\alpha r^2}-e^{-\beta r^2})P(^3O)\vec{L}\cdot\vec{S}$

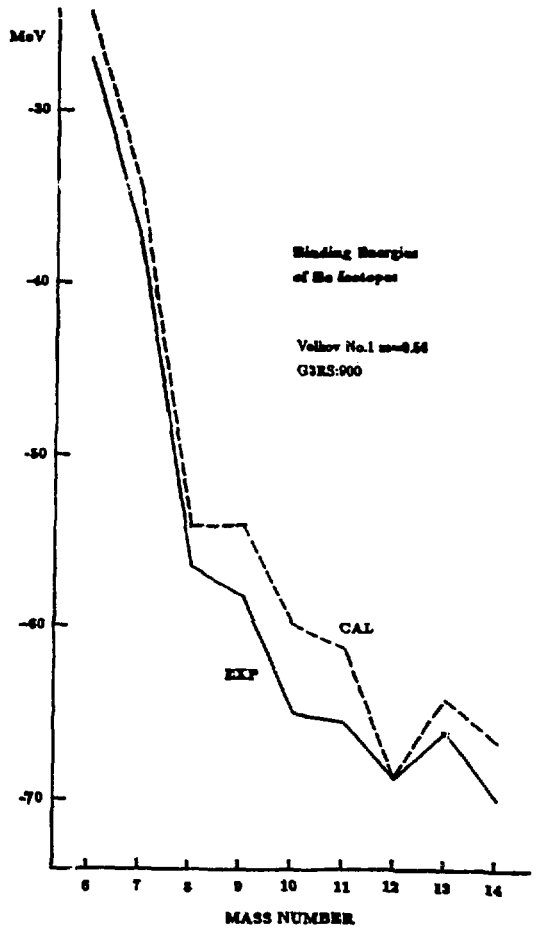
u^{LS}=900MeV, α=5.0fm^{-2}, β=2.778 fm^{-2}

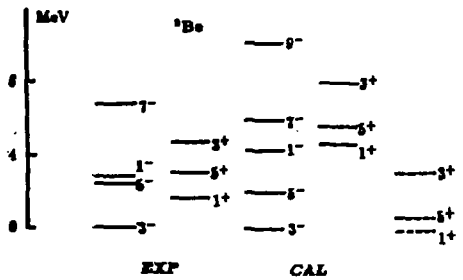
No Coulomb force

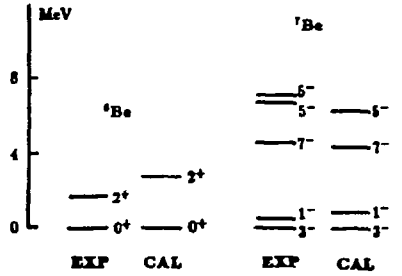
2) Oscillator parameter ν : optimum value is searched for.

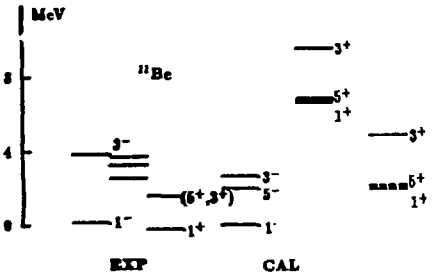
3) Frictional cooling for parity-projected $\Phi^{\pm}$

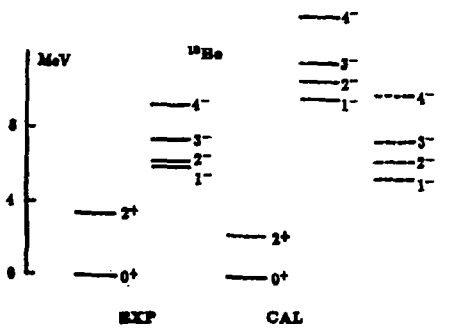
4) Angular-momentum projection without K-mixing

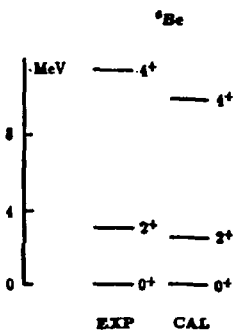


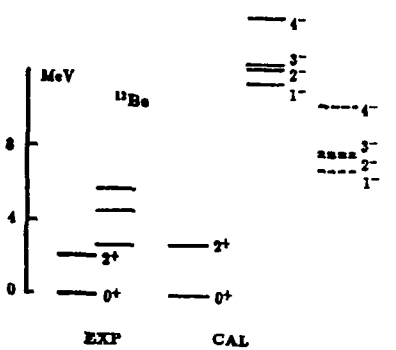

MeV
8
4
0
^{9}Be
7^-
1^-
6^-
3^-
9^-
7^-
1^-
5^-
3^+
5^+
1^+
3^+
5^+
1^+
EXP
CAL


MeV
8
4
0
^{7}Be
^{6}Be
2^+
0^+
2^+
0^+
5^-
5^-
7^-
1^-
3^-
5^-
7^-
1^-
3^-
EXP
CAL
EXP
CAL


MeV
8
4
0
^{11}Be
3^-
1^-
$(5^+,3^+)$
1^+
3^-
5^-
1^-
3^+
5^+
1^+
3^+
5^+
1^+
EXP
CAL


MeV
8
4
0
^{10}Be
2^+
0^+
4^-
3^-
2^-
1^-
2^+
0^+
4^-
3^-
2^-
1^-
EXP
CAL


^{8}Be
MeV
8
4
0
4^+
2^+
0^+
4^+
2^+
0^+
EXP
CAL


MeV
8
4
0
^{12}Be
4^-
3^-
2^-
1^-
2^+
0^+
2^+
0^+
4^-
3^-
2^-
1^-
EXP
CAL

Representation of the observed energy spectra is good except the energy gap between normal parity states.

Use of density-dependent effective nuclear force "MV1 force, case 3" (Tohsaki et al.)

"modified Volkov No.1 force + $t_3\delta(\vec{r}_1-\vec{r}_2)\delta(\vec{r}_2-\vec{r}_3)$" ($t_3$=4000MeV·fm^6)

(the strength of the short-range repulsive part of Volkov No.1 force is reduced)

Energy spectra shown by dotted lines are obtained by calculating the expectation values of "MV1-case 3" force with the AMD wave functions obtained with Volkov No.1.

Binding energies of normal-parity states with "MV1-case 3" force are close to those with Volkov No.1 force with the difference less than 1 MeV.

Converged values of $\{\vec{Z}_i\}$

- $\vec{K}_i$=very small
- Four protons are devided into two groups in each of which two protons with spin up and down locate at (almost) the same spatial point.

- Neutrons which form two α-clusters in ^{8}Be by locating in the vicinity of protons tend to separate from protons in going from ^{8}Be to ^{12}Be.
- Although the formation of α-cluster is not complete, we can define inter-α distance $R_{\alpha\alpha}$ as the distance between two groups of protons.
- We can also evaluate the survival probability of the α-α core, $P_{\alpha\alpha}$.

〈 $P_{\alpha\alpha}$; survival probability of the α-α core 〉

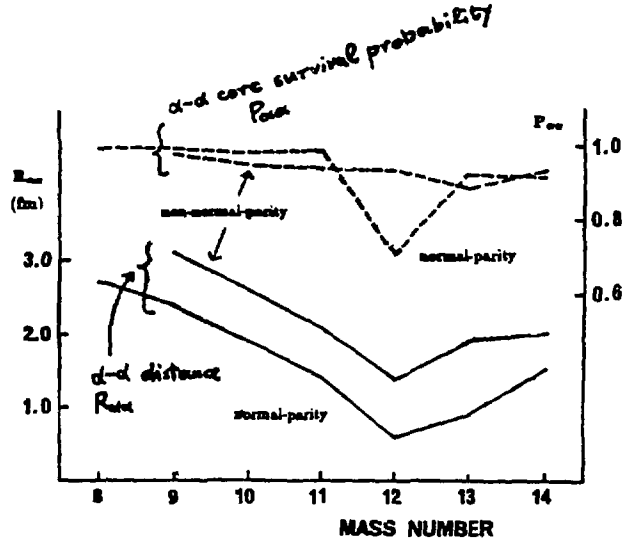
$$P_{\alpha\alpha} = \| \hat{\Phi}(\vec{r}_1, \cdots \vec{r}_A) \|^2$$

$$\hat{\Phi}(\vec{r}_1, \cdots \vec{r}_A) = \sqrt{\binom{A}{8}} \langle \psi^B_{\alpha\alpha}(\vec{r}_1, \cdots \vec{r}_8) | \hat{\Phi}(\vec{r}_1, \cdots \vec{r}_A) \rangle$$

$$\hat{\Phi} = \Phi / \| \Phi \|$$

$\psi^B_{\alpha\alpha}(\vec{r}_1 \cdots \vec{r}_8)$=normalized Brink's α-α wave function with α-α distance equal to $R_{\alpha\alpha}$ of $\hat{\Phi}$.

When the single-particle wave functions of 4 protons and 4 neutrons are identical with those in ψ^B, $P_{\alpha\alpha}$ is exactly equal ti unity, irrespectively of the single-particle wave functions of the remaining (A-8) neutrons.



Mass-number dependence of the $\alpha-\alpha$ distance $R_{\alpha\alpha}$ (solid lines) and the $\alpha-\alpha$ -core survival probability $P_{\alpha\alpha}$ (dotted lines) of the normal-parity and non-normal-parity intrinsic states of Be isotopes.

Non-normal parity states have low density.

Improvement of Wave function

Here we have discussed the α-clustering of neutron-rich Be isotopes. In order to study other properties of neutron-rich Be isotopes such as the neutron distribution, however, we need to make some improvements of the AMD wave function.

- Fixed and common value of n (oscillator parameter)
  ⟶ Variational and different ν for each nucleon.
- One wave packet for each nucleon, $A_\nu(\vec{r},\vec{z})$.
  ⟶ Sum of wave packets for each nucleon, $\Sigma_i C_i A_{\nu_i}(\vec{r},\vec{z})$.
- One Slator determinant Φ.
  ⟶ Sum of Slator determinants, $\Sigma_i C_i \Phi_i$.

〈Application to collision problems〉

We need to include "two-nucleon collision" process.

Since $\vec{D}_I$ and $\vec{K}_I$ are not canonical, we need to construct canonical (physical) coordinates $\vec{R}_I$ and $\vec{P}_I$: $\vec{D}_I$, $\vec{K}_I \longrightarrow \vec{R}_I$, $\vec{P}_I$.

Non-canonicity of $\vec{D}_I$ and $\vec{K}_I$ is entirely due to antisymmetrization.

Without antisymmetrization, the theoretical framework is jjust equivalent to the quantum molecular dynamics (QMD).

$$\begin{array}{ccc} (\vec{D}_I\ ,\ \vec{K}_I) & \xrightarrow{\text{transformation}} & (\vec{R}_I\ ,\ \vec{P}_I) \\ & & \Downarrow \quad \text{two-nucleon collision} \\ (\vec{D}'_I\ ,\ \vec{K}'_I) & \xleftarrow{\text{back-transformation}} & (\vec{R}'_I\ ,\ \vec{P}'_I) \end{array}$$

Physical (approximately canonical) coordinates $(\vec{R}'_I\ ,\ \vec{P}'_I)$

In two-nucleon system, $\vec{W}_I$ are exactly canonical.

⟵ Saraceno, Kramer, Fernandez.

$$\vec{W}_i \equiv \sum_j (\sqrt{Q})_{ij}\,\vec{Z}_j$$

$$\vec{W}_j = \sqrt{\nu}\,\vec{R}_j + \frac{i}{2\hbar\sqrt{\nu}}\,\vec{P}_j$$

$$\vec{Z}_j = \sqrt{\nu}\,\vec{D}_j + \frac{i}{2\hbar\sqrt{\nu}}\,\vec{K}_j$$

$$Q_{ij} \equiv B_{ij}(B^{-1})_{ji} = (UDU^{\dagger})_{ij}$$

$B_{ij} = \langle \phi_i | \phi_j \rangle$ = norm matrix

$$\sqrt{Q} = U\sqrt{D}\,U^{\dagger}$$

· No antisymmetrization $\vec{W}_I = \vec{Z}_I$.

· Oscillator quanta

$$\langle \xi | \sum_i a^{\dagger}_{i\sigma} a_{i\tau} | \xi \rangle / \langle \xi | \xi \rangle = \sum_{ij} Z^{*}_{i\sigma} Q_{ij} Z_{j\tau}$$

$(\sigma, \tau = x, y, z)$

$$= \sum_i W^{*}_{i\sigma} W_{i\tau}$$

· Angular momentum

$$\langle \xi | \vec{L} | \xi \rangle / \langle \xi | \xi \rangle = (-i\hbar) \sum_{ij} (\vec{Z}^{*}_i \times \vec{Z}_j) Q_{ij}$$

$$= (-i\hbar) \sum_i \vec{W}^{*}_i \times \vec{W}_i$$

· Center of mass $\Sigma_I \vec{Z}_I = \Sigma_I \vec{W}_I$.

· Displacement

$$\vec{Z}_i \to \vec{Z}_i + \vec{a} \quad \Rightarrow \quad \vec{W}_i \to \vec{W}_i + \vec{a}$$

Pauli blocking

$$\begin{array}{ccc} \vec{Z}_1, \vec{Z}_2, \vec{Z}_3, \cdots & \xrightarrow{\text{transformation}} & \vec{W}_1, \vec{W}_2, \vec{W}_3, \cdots \\ & & \Downarrow \quad \text{two-nucleon collision} \\ \vec{Z}'_1, \vec{Z}'_2, \vec{Z}'_3, \cdots & \xleftarrow{\text{back-transformation}} & \vec{W}'_1, \vec{W}'_2, \vec{W}'_3, \cdots \end{array}$$

If this back-transformation does not exist, the two-nucleon collision is Pauli blocked.

Pauli-forbidden region in phase space $\longleftrightarrow$ In two-nucleon system if $|\vec{W}'_1 - \vec{W}'_2| < \sqrt{2}$, the back-transformation does not exist.

〈Simplified check of Pauli blocking〉

- If there exists k(≥3) such that $|\vec{W}'_1 - \vec{W}_k| < a$ or $|\vec{W}'_2 - \vec{W}_k| < a$, the collision is judged to be Pauli blocking.
- Otherwise, we actually try to find back-transformation.
  We choose $a = \sqrt[6]{6} \simeq 1.35$ (six-dimensional sphere of radius $\sqrt[6]{6}$ has the volume h^3 in $(\vec{R}, \vec{P})$.
- 2-body system (the same spin and isospin)

$$1 \le N_{\substack{\text{oscil.} \\ \text{quanta}}} = \sum_j \vec{W}_j^* \cdot \vec{W}_j$$
$$= \frac{1}{2}|\vec{W}_1 - \vec{W}_2|^2 + \frac{1}{2}\underbrace{|\vec{W}_1 + \vec{W}_2|^2}_{=0}$$
center of mass

$\therefore$ $|\vec{W}_1 - \vec{W}_2| < \sqrt{2}$ is the Pauli-forbidden region in the phase space.

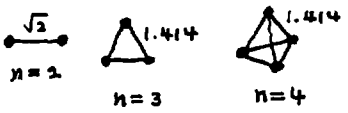
o 3-body 2
  4-body 3
  5-body 5
  ↑ n, ↑ note!

$$\le N_{\substack{\text{oscil.} \\ \text{quanta}}} = \frac{1}{n}\sum_{i<j} |\vec{W}_i - \vec{W}_j|^2$$
$$= \frac{n-1}{2}\overline{|\vec{W}_i - \vec{W}_j|^2}$$

for $n = 2 \sim 4$ $\quad \overline{|\vec{W}_i - \vec{W}_j|} \ge \sqrt{2}$

for $n \ge 5$ $\quad \overline{|\vec{W}_i - \vec{W}_j|} > \sqrt{2}$ (↑ no equality)

When all $\vec{z}_i = 0$



in 6-dimensional phase space.

one's prescription

$$\mathcal{H} = \frac{\langle \Phi | H | \Phi \rangle}{\langle \Phi | \Phi \rangle} - \frac{3\hbar^2\nu}{2M} A + T_0 (A - N_{Frag}) \qquad N_{Frag} \equiv \sum_j \frac{1}{n_j m_j} \approx \text{Number of fragments}$$

$$n_j \equiv \sum_k \hat{f}_{jk}, \quad m_j \equiv \sum_k \frac{f_{jk}}{n_k}$$

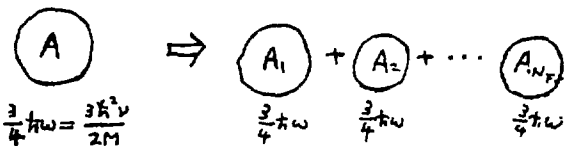
$$\hat{f}_{jk} = e^{-2\nu(\vec{D}_j - \vec{D}_k)^2} = (f_{jk})^2$$

Volkov No.1 $m = 0.575$

$\nu = 0.16\ fm^{-2}$

$T_0 = 7.7$ MeV

〈Zero point energy〉 〈Any wave packet theory has this problem〉.



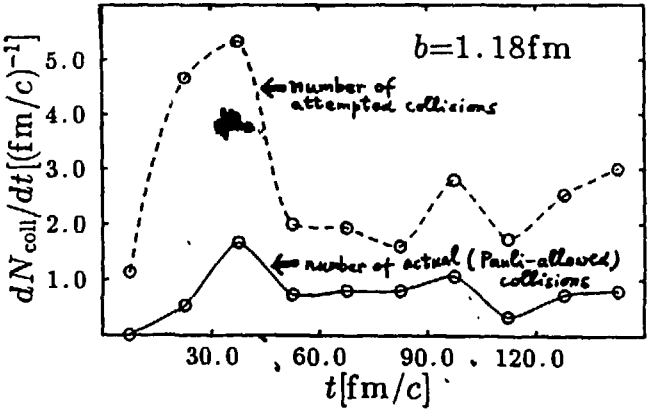
Kinetic energy of fragments are greatly reduced spuriously!

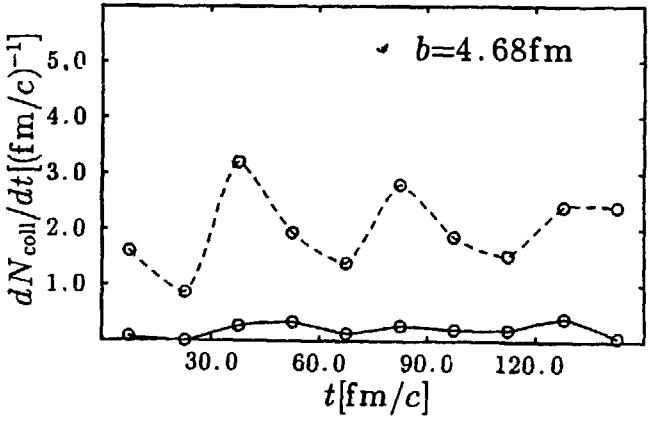
[6] Concluding remarks

- Since the Pauli principle is very important, we have developed a new microscopic simulation framework, the antisymmetrized version of molecular dynamics with two-nucleon collisions (AMD).

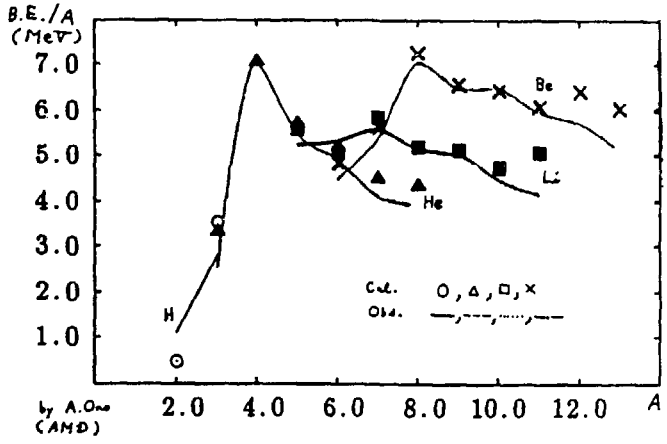
- The concept of the "Pauli-forbidden region in the phase space" plays a decisive role in this new framework of AMD. This concept has been developed in nucleus-nucleus interaction theory as phase-space description of the Pauli-forbidden states.

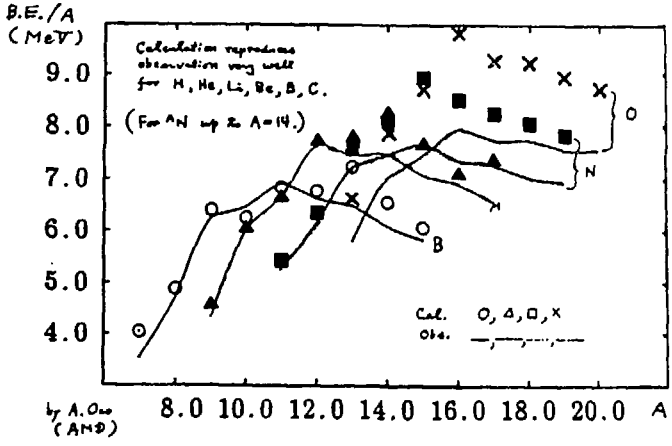
- The explicit treatment of the antizymmetrization in the AMD has enabled us to reproduce the shell effect in the fragment (cluster) production for the first time in simulation approaches to heavy-ion collisions.

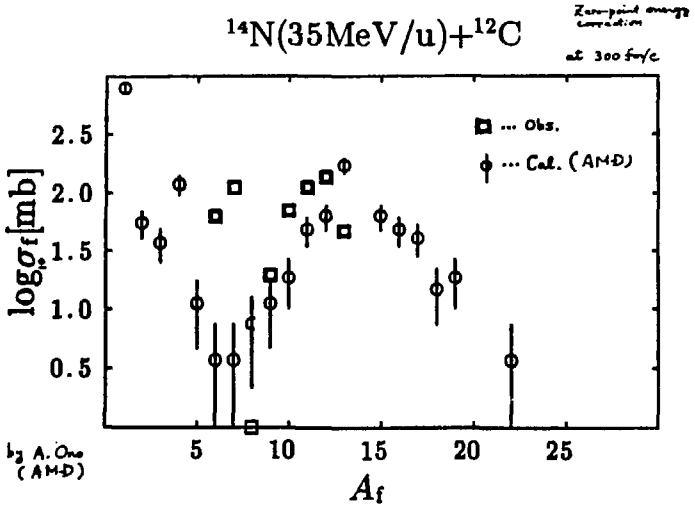


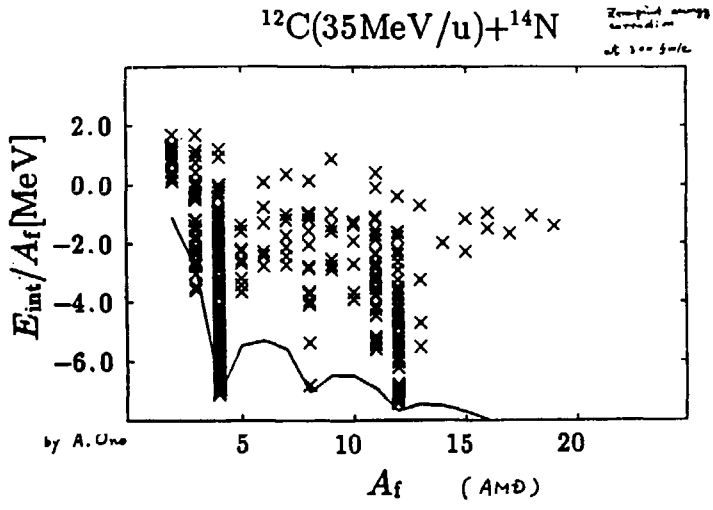


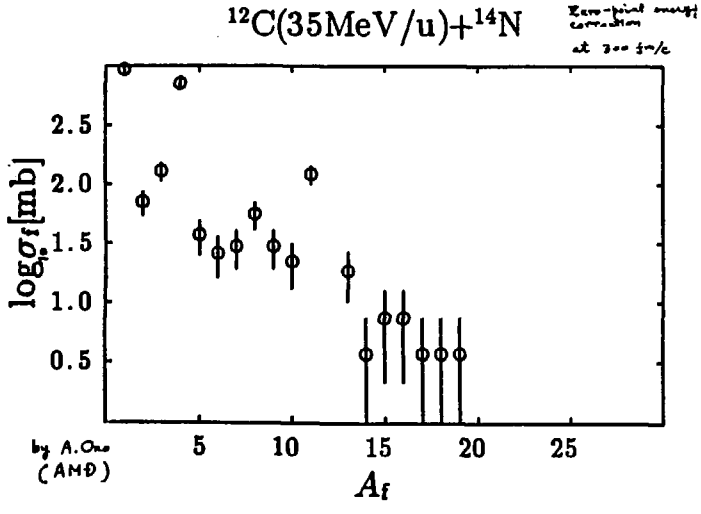
^{14}N (35 MeV/u) + ^{12}C (AMD)

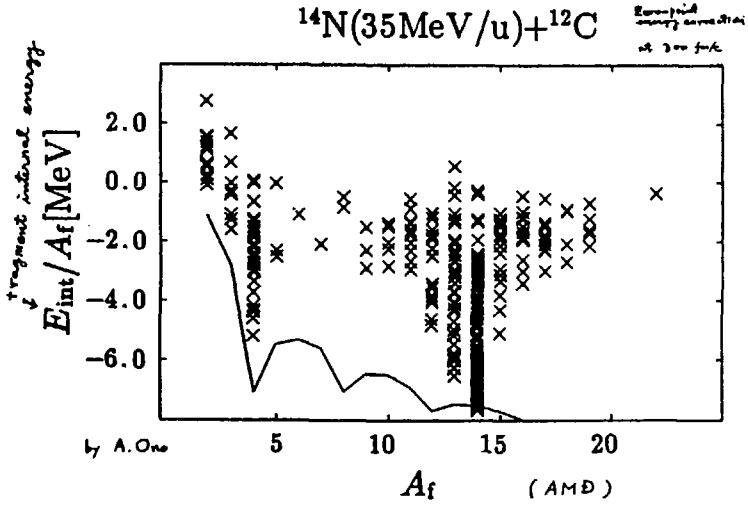


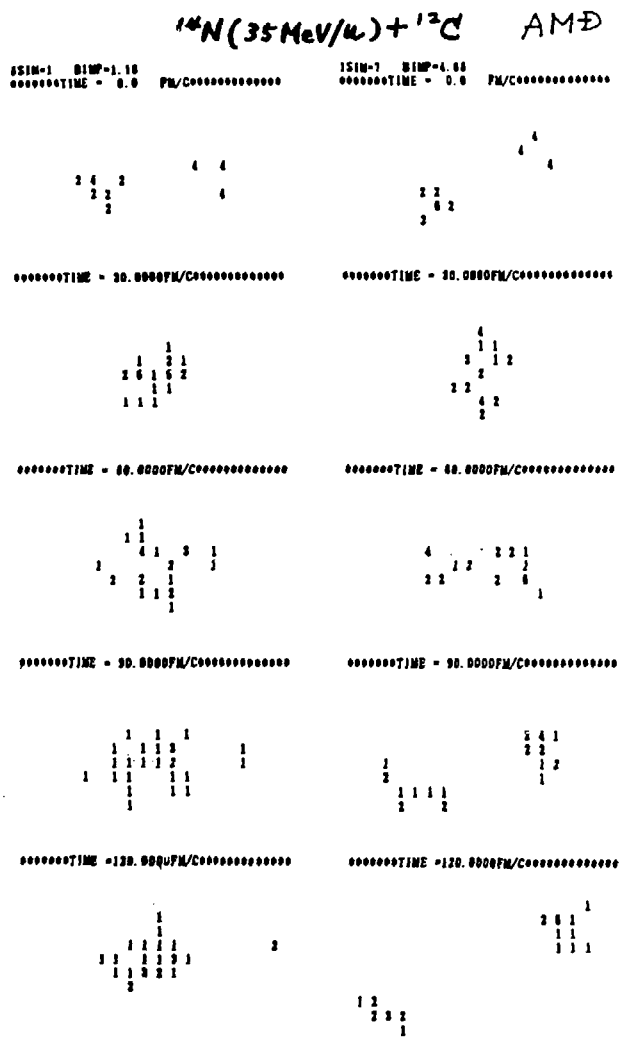



^{14}N(35MeV/u)+^{12}C
... Obs.
... Cal. (AMD)
$\log\sigma_f$[mb]
A_f
by A.Ono (AMD)


^{12}C(35MeV/u)+^{14}N
E_{int}/A_f[MeV]
A_f (AMD)
by A.Ono


^{12}C(35MeV/u)+^{14}N
$\log\sigma_f$[mb]
A_f
by A.Ono (AMD)


^{14}N(35MeV/u)+^{12}C
fragment internal energy
E_{int}/A_f[MeV]
A_f (AMD)
by A.Ono



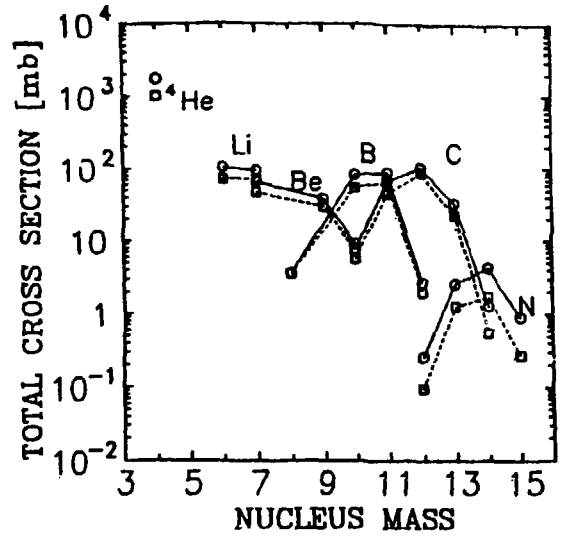


FIG. 6. Energy- and angle-integrated yields for all measured isotopes for ^{12}C (circles) and ^{27}Al (squares) targets. Peaks corresponding to transitions to discrete states were excluded. The solid (^{12}C) and dashed (^{27}Al) lines are drawn to guide the eye.

(28.7 MeV/u) (exp.)

Three-body OCM calculation with the pseudo-potential method

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Abstract: The orthogonality condition model with the Moscow-type pseudo potential is combined with the coupled-rearrangement-channel three-body method, and is applied to $^{12}C=3\alpha$ and $^{11}Li=^{9}Li+n+n$. In the meeting, also discussed is an application of the coupled-channel method to the study of decay-mechanism of muonic molecule $(d^3He\mu)$ and $(d^4He\mu)$ together with a discussion about the possibility of muon catalyzed d-^{3}He fusion in which the huge ^{3}He source on the moon's surface.

§1. Pseudo-potential OCM

The three-body OCM equation is given by

$$[\ T + \sum_{i<j} V_{ij}(r_{ij}) - E\ ]\ \Phi = 0 \qquad (1)$$

with

$$\langle\ \xi_\alpha(r_{ij})\ |\ \Phi\ \rangle = 0 \quad , \quad \text{(for any } \alpha) \qquad (2)$$

where $\xi_\alpha(r_{ij})$ is two-cluster forbidden states between any two constituent clusters. Ususally, Eqs.(1) and (2) are solved, for bound states, by diagonalizing the Hamiltonian of (1) in a space spanned by the three-cluster basis functions which are orthogonal to all ξ_α or in another space spanned by the three-cluster allowed states.

Kukulin et al.[1] proposed an alternate method to solve Eqs.(1) and (2) which does not need to construct the three-body allowed states. They introduced pseudo potentials $\lambda\Sigma_\alpha|\xi_\alpha\rangle\langle\xi_\alpha|$ and solve

$$[\ T + \sum_{i<j} \{\ V_{ij}(r_{ij}) + \lambda\Sigma_\alpha|\xi^\alpha_{ij}(r_{ij})\rangle\langle\xi^\alpha_{ij}(r_{ij})|\} - E\ ]\ \Phi = 0 \quad . \qquad (3)$$

The stable finite solutions in letting $\lambda \to \infty$ are the very solution that are surely orthogonal to all the two-cluster forbidden states ξ_α. The method

seems more straightforward and tracable than the method to construct the three-cluster allowed states first. But, the stability for $\lambda \to \infty$ has not been clarified in actual calculations.

The aim of this report is to test this stability with the use of our coupled-rearrangement-channel Gaussian basis variational method for three-body systems which has been very successful for muonic molecules[2] and trinucleon bound states[3]. As an example, we take the $C^{12}=3\alpha$ system. The total wave is expanded in term of Gaussian-tail basis functions of the three-rearrangement channels c=1~3:

$$\Phi_{JM}(3\alpha) = \sum_{i\ell kL} A_{i\ell kL} \sum_{c=1}^{3} [\ \phi_{i\ell}(\mathbf{r}_c)\chi_{kL}(\mathbf{R}_c)\ ] \quad , \tag{5}$$

$$\phi_{i\ell m}(\mathbf{r}) = r^{\ell}\exp(-\nu_i r^2)Y_{\ell m}(\hat{\mathbf{r}}) \quad , \quad \nu_i=\nu_1 a^{i-1} \ (i=1\sim i_{max}) \tag{6}$$

$$\chi_{kLM}(\mathbf{R}) = R^{L}\exp(-\lambda_k R^2)Y_{LM}(\hat{\mathbf{R}}) \quad , \quad \lambda_k=\lambda_1 a^{k-1} \ (k=1\sim k_{max}) \tag{7}$$

where the Gaussian ranges are taken as geometrical progressions. Usefulness of this type of basis functions can be seen in Refs. 4 and 5.

We take ℓ,L≤4 and i_{max} and k_{max} ≃15. The α-α forbidden states are

$$\xi^{\alpha} = \text{0S , 1S and 0D states for any pair of } \alpha\text{-}\alpha \quad . \tag{8}$$

Harmonic oscillator potential

First, we examine the case of H.O. potential for $V_{ij}(r_{ij})$:

$$V_{ij}(r_{ij}) = \frac{1}{2} m\omega^2 r_{ij}^2 \quad . \tag{9}$$

The result is shown in Fig 1. If the orthogonality condition is not imposed (namely, λ=0), we have the solution with $0\hbar\omega$, $2\hbar\omega$, $4\hbar\omega$, ····· for J=0. As the magnitude of λ increases, enegy of the forbidden states, especially those with $6\hbar\omega$ excitation or less, increases quickly. When λ is greater than several handred MeV, the enegy of all the forbidden states go up and disapear from the physically important region. The solution is stable up to $\lambda \simeq 10^{10}$ MeV, and it becomes unstable for larger λ but this is simply due to numerical round-off error in the double-precision (~15 decimal digits) calculation. I say I was rather surprized and satisfied to

see this stability of the solution. In acutal calculation, $\lambda=10^5$~10^6 MeV is recommended from our experience.

Schmid-Wildermuth N-N force

The next test is made by taking the Schmid-Wildermuth (SW) force as the two-nucleon force (Fig. 2). The α-α potewntial V_{ij} in Eq.(3) is constructed by folding the SW force and $\tau\eta\varepsilon$ Coulomb force into the alpha-particle density.

The 3α OCM with the same interactions was made by Horiuchi[4] and Kato et al.[5]. Their results of energy spectrum are shown in Fig. 3; difference between the two solutions comes from the number of the three-body basis functions of SU(3) representation. Our results are shown in Fig. 3, too. As for the shell-like states (0^+_1, 2^+_1, 4^+_1, 3^-_1) , our results agree very well with those of Refs. 4 and 5. But, for the clustering states, our calculation gives better solution (lower energy). This is because our Gaussian basis functions are more suited to states with long-range tail (weakly bound states and quasi-bound states).

§2. Application to $^{11}Li=^{9}Li+n+n$

This part was already published in Ref. 6 and therefoe is skipped here.

§3. Decay mechanism of ($d^3He\mu$) and ($d^4He\mu$) molecules

This part is going to be published in Ref. 7, and therefore is skipped here.

References

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5) K. Kato et al., Prog. Theor. Phys. **76** (1986) 75.
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7) Y. KIno and M. Kamimura, Muon Catalyzed Fusion, **9** (1993), in press.

Fig. 1. Test of stability of the solution (J=0) of the 3α OCM with respect to increasing λ. Harmonic oscillator potential is assumed between two α's. Three-body configurations with ℓ=L=0 are taken.

Fig. 2. Energy spectra of ^{12}C by the 3α OCM calculations.

Fig. 3. Stability of the solution of 0_1^+ and 0_2^+ states with respect to increasing λ.

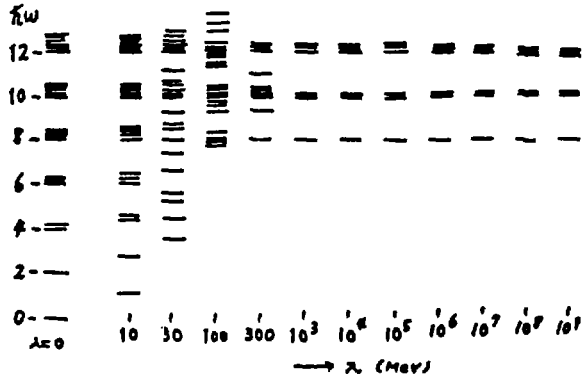

Fig. 1

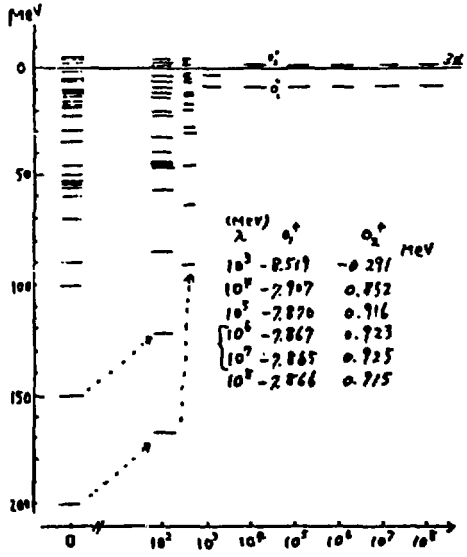

Fig. 3

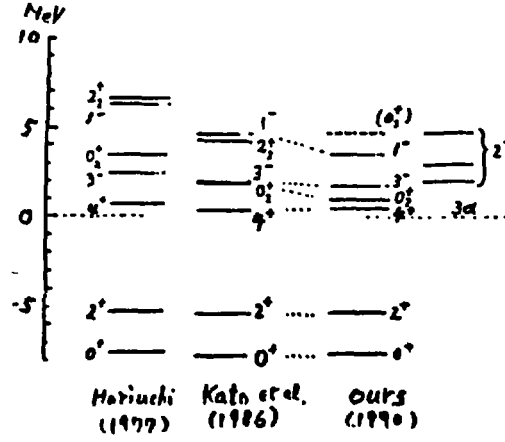

Fig. 2