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Phase Transition Dynamics in Ultra-Relativistic Heavy-Ion Collisions

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PHASE TRANSITION DYNAMICS IN ULTRARELATIVISTIC HEAVY ION COLLISIONS

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Abstract

We investigate various problems related to the dynamics of a first-order phase transition from quark-gluon plasma to hadronic matter in ultra-relativistic heavy ion collisions. These include nucleation, growth and fusion of hadronic bubbles in either the Bjorken longitudinal hydrodynamic expansion model or the Cooper-Frye-Schonberg spherical hydrodynamic expansion model. With reasonable input parameters the conversion of one phase into the other is relatively close to the idealized adiabatic Maxwell construction, although one can choose parameters such that the conversion is strongly out of equilibrium.

1 Introduction

A standard picture of a central collision between two massive nuclei at RHIC (Relativistic Heavy Ion Collider) or at LHC (Large Hadron Collider) is that the two nuclei pass through each other, creating a hot plasma of quarks and gluons in the region between the receding projectile and target fragments [1]. This plasma subsequently cools during hydrodynamic expansion. Eventually the energy density becomes low enough that the quarks and gluons hadronize. Often, an idealized adiabatic Maxwell construction for two-phase equilibrium is invoked as a model of the hadronization process. However, it is by no means clear that the nucleation rate is large enough, compared to the expansion rate, for this idealization to be a good representation of reality.

In a recent paper two of us analyzed in some detail the rate to nucleate hadronic bubbles in quark-gluon plasma, particularly the pre-exponential factor [2]. This rate was then used to study the time evolution of the system as it converted from one phase to the other during a central collision at RHIC [3]. It was found that the system supercooled about 20% before nucleation and bubble growth caused a reheating close to T_c, and that the transition generated about 30% extra entropy.

It is our purpose here to elaborate on this phase transition dynamics. Our plan is as follows: First we review the form of the nucleation rate and the dynamical evolution equations used in ref. [3]. Then we show that the kinetic equation for the fraction of the system which has been converted can be approximated by a simpler differential equation involving the average number and size of hadronic bubbles. Bubble fusion may become important during the late stages of the transition, and this is incorporated into the dynamics. The Bjorken longitudinal hydrodynamic model [4] does not take into account transverse expansion. Therefore we also consider the phase transition dynamics arising from a spherical expansion in the framework of the Cooper-Frye-Schonberg model [5]. An important parameter in the nucleation rate is the surface free energy; variation of this parameter is also considered.

2 Review of Earlier Work

The rate for the nucleation of the hadron phase out of the plasma phase can be written as [2]

$$I = I_0 e^{-\Delta F_*/T}, \tag{1}$$

where ΔF_* is the change in the free energy of the system with the formation of a critical size hadronic bubble and I_0 is the prefactor with dimensions of inverse volume inverse time. In general, statistical fluctuations at $T < T_c$ will produce bubbles with associated free energy

$$\Delta F = \frac{4\pi}{3}[p_q(T) - p_h(T)]R^3 + 4\pi R^2 \sigma. \tag{2}$$

Here p is the pressure of the quark or hadron phase at temperature T, and σ is the surface free energy of the quark-gluon/hadron interface. Since $p_q < p_h$ it follows that there is a bubble of critical radius

$$R_*(T) = 2\sigma/[p_h(T) - p_q(T)]. \tag{3}$$

Smaller bubbles tend to shrink because the surface energy is too large relative to volume energy, and larger bubbles tend to grow. The free energy of the critical size bubble is therefore

$$\Delta F_* = \frac{4}{3}\pi\sigma R_*^2. \tag{4}$$

The prefactor has very recently been computed in a coarse-grained effective field theory approximation to QCD [2]

$$I_0 = \frac{16}{3\pi}\left(\frac{\sigma}{3T}\right)^{3/2} \frac{\sigma \eta_q R_*}{\xi_q^4 (\Delta w)^2}, \tag{5}$$

where η_q is the shear viscosity in the plasma phase, ξ_q is a correlation length in the plasma phase, and Δw is the difference in the enthalpy densities of the two phases. At the critical temperature, $R_* \to \infty$, and the rate vanishes.

Given the nucleation rate one would like to know the (volume) fraction of space $h(t)$ which has been converted from QCD plasma to hadronic gas at the proper time t, which is the time as measured in the local comoving frame of an expanding system. This requires a kinetic equation which uses I as an input. If the system cools to T_c at time t_c then at some later time t the fraction of space which has been converted to hadronic gas is [3]

$$h(t) = \int_{t_c}^{t} dt' I(T(t'))[1 - h(t')] V_{bub}(t', t). \tag{6}$$

Here $V_{bub}(t', t)$ is the volume of a bubble at time t which had been nucleated at the earlier time t'.

The growth of bubbles has been studied numerically with relativistic hydrodynamics by Miller and Pantano [6]. The asymptotic radial growth velocity is consistent with the growth law

$$v(T) = v_0[1 - T/T_c]^{3/2}, \tag{7}$$

where v_0 is a model-dependent constant. As in ref. [3] we shall use $v_0 = 3c$, which corresponds to their parameter $\alpha = 1$. This expression is intended to apply only when $T > \frac{2}{3}T_c$ so that the growth velocity stays below the speed of sound of a massless gas, $c/\sqrt{3}$. The simple illustrative model for bubble growth is

$$V_{bub}(t', t) = \frac{4\pi}{3}\left(R_*(T(t')) + \int_{t'}^{t} dt'' v(T(t''))\right)^3. \tag{8}$$

This expression assumes that the interface between the inside and outside of the bubble is created at rest in the local comoving frame. A less likely, but plausible, alternative is that the bubble's surface is created at rest in the center-of-momentum frame of the bubble, even though the system as a whole is expanding. Then the right side of eq. (8) should be multiplied by the volume dilution factor $V_{tot}(t')/V_{tot}(t)$, where $V_{tot}(t)$ is the total volume of the system at time t. Equation (8) as it stands will be considered our default dynamics. We will also perform some calculations with the dilution factor to test its influence. In general this is a difficult and unsolved problem which may be answerable only with a detailed microscopic simulation. (See also the discussion of the early universe by Linde [7].)

A dynamical equation is needed to describe how the system expands. This was taken from Bjorken's longitudinal scaling hydrodynamics [4]. The derivative of the energy density is related to the enthalpy density as

$$\frac{de}{dt} = -\frac{w}{t}. \tag{9}$$

This assumes kinetic equilibrium among the particles but not phase equilibrium. It is a statement of energy conservation. The energy density is

$$e(T) = h(t)e_h(T) + [1 - h(t)]e_q(T), \tag{10}$$

where $e_h(T)$ and $e_q(T)$ are the energy densities in the two phases at temperature T, and similarly for w.

We model the hadronic phase by a massless gas of pions, and the plasma phase by a gas of gluons and massless quarks of two flavors, with a bag constant B to simulate confinement dynamics. We use the same parameters as in [2, 3], namely, $\sigma = 50$ MeV/fm^2, $B^{1/4} = 235$ MeV, $\xi_q = 0.7$ fm, and $\eta_q = 14.4T^3$. This gives $T_c = 169$ MeV.

3 Simplified Kinetic Equations

The integral equation for $h(t)$ requires considerable computational effort. If one divides the time interval of the phase transition into N equal steps then, because of the integrals over t' and t'', the number of computational steps grows like N^3. To avoid this much effort we propose a set of coupled differential equations for the number of bubbles, the total bubble surface area and the total bubble volume:

$$\begin{aligned}\frac{dN_{bub}}{dt} &= I(T(t))\,[1 - h(t)]\,V_{tot},\\ \frac{dS_{had}}{dt} &= \frac{dN_{bub}}{dt}\,4\pi R_*^2 + N_{bub}\,8\pi\overline{R}\,v(T(t)),\\ \frac{dV_{had}}{dt} &= \frac{dN_{bub}}{dt}\,\frac{4}{3}\pi R_*^3 + S_{had}\,v(T(t)),\end{aligned} \tag{11}$$

where $\overline{R} \equiv \frac{1}{2}[S_{had}/(4\pi N_{bub})]^{1/2} + \frac{1}{2}[3V_{had}/(4\pi N_{bub})]^{1/3}$ is defined to be the average bubble radius. If all bubbles are the same size then it makes no difference whether the average radius is defined by their surface area or their volume and, furthermore, the integral equation for $h(t)$ is identical to the above set of equations. It turns out that the dispersion in bubble radii is small because most bubbles are not nucleated until the matter supercools about 20%, at which point critical bubbles have a radius of about 1 fm independent of temperature. Numerical comparison between these equations and the integral equation will be shown later; generally the results are the same within a few percent.

Another advantage of using these coupled differential equations is that they can easily be modified to include other processes. They will shortly be modified to take into account the binary fusion of bubbles.

4 Bubble Fusion

When bubbles are sufficiently large and their density is suffieciently high, two bubbles may fuse to form a larger bubble, which will rapidly become spherical. Such fusion processes may be hindered by repulsive bubble-bubble interactions in dilute systems, where bubbles are small and move slowly. It is not clear if such a repulsive interaction exists in QCD between two hadronic bubbles. Based on low energy analogies we may, however, assume the existence of such repulsion. The simplest way to consider such a threshold is to introduce a lower critical hadron volume fraction, h_F, such that bubble fusion is allowed only above this value. Typical values for this threshold lie between $h_F = 0.68 - 0.81$ (c.f. Kirkwood and Born-Green-Yvon approximations [8]). However, rather than using a step function, $\Theta(h - h_F)$, which is somewhat unphysical, we use a smoother distribution, $\Phi(h) = \exp[-(1-h)^2/2\Delta h^2]$, with $\Delta h = 1/4$.

In a system with few and relatively small bubbles we can estimate the fusion rate, I_F, just as the nucleation rate, composed of a statistical and a dynamical factor.

The statistical factor can be estimated by assuming that the distribution of the actual bubble number, $N_{bub}^{(a)}$, follows a Poisson distribution,

$$P_{N_{bub}^{(a)}}(V_{tot}) = \frac{1}{N_{bub}^{(a)}!}(nV_{tot})^{N_{bub}^{(a)}}\, e^{-nV_{tot}}, \tag{12}$$

where n is the average bubble density, $n = \left\langle N_{bub}^{(a)} \right\rangle / V_{tot}$. If the average bubble radius is $\overline{R}$ two bubbles may fuse if their centers happen to be closer to each other than $2\overline{R}$. The probability to have one and *only one* bubble in some volume V is $P_1(V) = nVe^{-nV}$. For a small volume, such that $nV \ll 1$, the probability per unit volume to have one bubble in it is $\Omega_1 = n$. To have another bubble within the $2\overline{R}$

vicinity of the 1st bubble is

$$P_1(V_{2\overline{R}}) = nV_{2\overline{R}}e^{-nV_{2\overline{R}}}, \tag{13}$$

where $V_{2\overline{R}} = \frac{4\pi}{3}8\overline{R}^3$. Thus the probability per unit volume to have two bubbles, one of them in a small volume and the other in its vicinity (considering that they are indistinguishable) is

$$\Omega_F = \frac{1}{2}n^2\, V_{2\overline{R}}\, e^{-nV_{2\overline{R}}}. \tag{14}$$

If $nV_{2\overline{R}} \ll 1$ the exponential factor can be neglected so that

$$\Omega_F \approx \frac{1}{2}n^2\, V_{2\overline{R}}, \tag{15}$$

which is our estimated statistical prefactor.

Fusion does not happen instantly. The rearrangement of the surface takes some time. The physical processes which change the shape of the surface is similar to bubble growth (see eq. (7)) in as much as both are limited by the strength of transport processes. The simplest, order of magnitude estimate for the dynamical prefactor for bubble fusion is

$$\tau_F^{-1} \approx \frac{v(T(t))}{\overline{R}}. \tag{16}$$

Finally, we set the exponential rate factor, $e^{-\Delta F_{fusion}/T}$, to unity by assuming that the surface rearrangement energy due to fusion is much smaller than the thermal energy at T.

Therefore the total fusion rate (per unit volume and per unit time) is approximated to be

$$I_F = \Omega_F/\tau_F = C_F n^2 \tag{17}$$

where

$$C_F \approx \frac{16\pi}{3}\, v(T(t))\, \overline{R}^2\, \Phi(h(t))\;. \tag{18}$$

The probability of fusion, P_F, per bubble per unit time is $P_F \equiv I_F/n = C_F n$.

The average radius of a bubble fused from two separate bubbles is $2^{1/3}\overline{R}$. Consideration of bubble fusion leads to a modification of the dynamical equations (11):

$$\frac{dN_{bub}}{dt} \;=\; I(T(t))\,[1-h(t)]\,V_{tot} - \frac{1}{2}N_{bub}P_F,$$

$$
\begin{aligned}
\frac{dS_{had}}{dt} &= I(T(t))\,[1-h(t)]\,V_{tot}\,4\pi R_*^2 + N_{bub} 8\pi \overline{R}\, v(T(t)) \\
&\quad - N_{bub} P_F\,[2-2^{2/3}]\,2\pi \overline{R}^2, \\
\frac{dV_{had}}{dt} &= \frac{dN_{bub}}{dt}\,\frac{4}{3}\pi R_*^3 + S_{had}\, v(T(t)). \qquad (19)
\end{aligned}
$$

5 Longitudinal versus Spherical Expansion

In our first work we considered only the idealized Bjorken model which takes into account the inside-outside cascading of partons. The Bjorken model treats only the longitudinal dynamics of the nuclear collision, and supposes that transverse expansion is a negligible secondary effect. To incorporate transverse expansion in the Bjorken model is very computationally intensive [9]. We shall not attempt to incorporate transverse expansion in this way in this paper, since our studies are still somewhat in the exploratory stage. However, it is important to somehow include the effects of 3-dimensional expansion of the matter because it is expected that expansion in more dimensions will cause the system to cool at a higher rate, and this enhanced cooling may be so fast that the matter does not nucleate and follow the Maxwell idealization to any reasonable approximation. The partons may simply hadronize in a very nonequilibrium way. To get some handle on this possibility, we shall compare the 1-dimensional longitudinal expansion to a model of matter expanding spherically in 3-dimensions, following Cooper, Frye and Schonberg [5].

From now on we use the symbol τ for proper time and t for coordinate time.

5.1 Longitudinal Expansion

We briefly summarize the Bjorken longitudinal hydrodynamic scaling model for a central collision between two equal mass nuclei. The model assumes that there is no dependence on the transverse coordinates x and y. This can be interpretted either as a collision between two nuclear slabs which are infinite in transverse extent, or that the matter is somehow confined by an impenetrable cylindrical wall surrounding the two nuclei and of the same radius as them.

Local thermodynamic quantites are a function only of the proper time $\tau = \sqrt{t^2 - z^2}$ where t and z are the time and longitudinal coordinate as defined in the center-of-momentum frame of the colliding nuclei. A comoving volume increases with time because of the hydrodynamic expansion according to $V(\tau)/V(\tau_c) = \tau/\tau_c$ where τ_c is the proper time at which the temperature has fallen to its critical value T_c. (Of course any reference time may be used.) The local expansion velocity is $v = z/t = z/\sqrt{\tau^2 + z^2}$. Then a solution to the equations of motion of relativistic hydrodynamics is obtained if the energy density and enthalpy density are related by the differential equation

$$\frac{de}{d\tau} = -\frac{w}{\tau} . \tag{20}$$

This is the equation for energy conservation for matter undergoing fluid expansion. It is important to note that an equation of state has not been invoked yet. If one further uses an equation of state and applies the second law of thermodynamics, then it is easily shown that entropy is conserved. For example, if the pressure has the form $p = aT^4 + b$, where a and b are constants, then it follows that the temperature decreases according to the law $T(\tau) \propto \tau^{-1/3}$. In general nucleation of the phase transition changes this behavior.

5.2 Spherical Expansion

Complementary to the Bjorken model we consider a model for spherical hydrodynamic expansion. It is assumed in this model that the quark-gluon plasma is produced in a spherically symmetric region of space between the target and projectile rapidities. The model is essentially due to Cooper, Frye and Schonberg. It is analogous to the Bjorken model in that it is assumed that the local thermodynamic variables depend only on the proper time defined by $\tau = \sqrt{t^2 - r^2}$ where t is the time as measured in the center-of-momentum frame and r is the radial coordinate. The volume of a sphere bounded by a comoving radius increases with time because of the hydrodynamic expansion according to $V(\tau)/V(\tau_c) = (\tau/\tau_c)^3$ where τ_c is the proper time at which the temperature has fallen to its critical value T_c. The local expansion velocity is $v = r/t = r/\sqrt{\tau^2 + r^2}$. Then a solution to the equations of motion of relativistic hydrodynamics is obtained if the energy density and enthalpy density are related by the differential equation

$$\frac{d\epsilon}{d\tau} = -\frac{3w}{\tau} . \tag{21}$$

It is interesting to note the factor of 3 in this expression compared to the equation for energy conservation in the Bjorken model; it arises because of expansion in three, rather than one, dimension If one further uses an equation of state and applies the second law of thermodynamics, then it is easily shown that entropy is conserved. For example, if the pressure has the form $p = aT^4 + b$, where a and b are constants, then it follows that the temperature decreases according to the law $T(\tau) \propto \tau^{-1}$, faster than in the Bjorken model. In general nucleation of the phase transition changes this behavior.

6 Results

In this section we compare the results obtained from the integral equation for $h(t)$ versus the set of simplified differential equations, we compare the results obtained in the longitudinal scaling hydrodynamics versus spherical scaling hydrodynamics, and we investigate the influence of bubble fusion and the dependence on the numerical value of the surface free energy.

To proceed we must first specify the parameters to be used in the hydrodynamical expansion models. Based on the results of the parton cascade model [10] we suppose that a thermalized quark-gluon plasma is formed with a temperature of $T_i = 2T_c \approx 340$ MeV. General arguments based on the inside-outside cascade dynamics and the uncertainty principle suggest that this happens at the proper time 3/8 fm/c. This implies that the plasma has cooled to the critical temperature at the proper time $\tau_c = 3$ fm/c. This is the only parameter needed for our study of the longitudinal hydrodynamical model.

To determine τ_c in the spherical hydrodynamics requires different arguments because the inside-outside cascade analysis is inconsistent with the spherical expansion. (Which model is closer to the real physical situation at finite beam energy at RHIC or LHC is still unknown.) For comparison we still assume that plasma is formed with a temperature of $2T_c$. We assume that the plasma is formed within a sphere of radius equal to the radius of the colliding nuclei, R_{nuc}. The radius of this sphere increases with time proportional to τ because the volume is proportional to τ^3. Let us write $R(\tau) = u_s\tau$. The velocity of this surface is

$$v(R(\tau)) = \frac{R(\tau)}{\sqrt{\tau^2 + R^2(\tau)}} = \frac{u_s}{\sqrt{1 + u_s^2}} = v_s \, . \tag{22}$$

This is equivalent to saying that as a function of coordinate time $R(t) = v_s t$. Thus the matter is created with a nonzero radial expansion velocity, and it continues to expand indefinitely with time. A natural scale for the expansion velocity is the speed of sound, and this will be our choice. If we find that the phase transition remains close to the Maxwell idealization then a smaller expansion velocity would result in the transition being even closer to the Maxwell result. Thus we choose the initial time in the spherical model to be $\tau_i = R_{nuc}/u_s = \sqrt{2}R_{nuc}$ corresponding to $v_s = 1/\sqrt{3}$. The time at which the temperature has fallen to its critical value is $\tau_c = 2\sqrt{2}R_{nuc}$. The fact that these times are proportional to the nuclear radius is a consequence of the assumption of a scaling solution to the hydrodynamic equations in three dimensions. For numerical purposes we shall use $\tau_c = 18$ fm/c, corresponding to the half-density radius of a gold nucleus.

In fig. 1 we show the temperature versus time for longitudinal expansion. One curve shows the idealized Maxwell construction for phase equilibrium. The other three curves are for $\sigma = 20, 50$ and 100 MeV/fm^2. The curve for $\sigma = 50$ MeV/fm^2 is identical to the corresponding curve in figure 2 of ref. [3], at least to within the width of the line. This verifies the essential equivalence of the set of simplified dynamical equations (11) used here with the original integral equation (6). The features of this figure are as discussed in [3]; namely, the system cools below T_c before nucleation sets in, then the nucleation of bubbles and bubble growth heats the system, thus shutting off nucleation at later times. The system reheats to close to T_c but can never quite reach T_c because bubble growth decreases to zero as T_c is approached, eq. (8).

The nucleation rate decreases with decreasing surface free energy, forcing a greater supercooling. However, one should not let σ decrease to zero without also changing the latent heat Δw. The nucleation rate depends on both, as seen in eq. (5). As a strong first order phase transition weakens, Δw decreases, and one naturally expects σ to decrease as well. In the limit that the first order transition goes over to a second order one, both the latent heat and the surface free energy go to zero. So, although our knowledge of the properties of the phase transition from quark-gluon plasma to hadron gas is still poor, we must at least asknowledge that these two parameters cannot be varied completely independently of each other.

Figure 2 shows what happens when a dilution factor is included in the bubble growth volume given in eq. (8). Compared to no dilution factor, fig. 1, nucleation proceeds at about the same rate, but bubble growth is partly compensated by expansion of the system. It is as if the expansion of the bubbles has difficulty matching the expansion of the rest of the system. The matter reheats to a lower temperature, and it takes somewhat longer to complete the transition. It should be recognized, however, that inclusion of the dilution factor is clearly nonsense late in the transition because the system is composed almost entirely of hadronic bubbles, and it is *their* expansion velocity which defines the expansion velocity of the system as a whole. Since the initial time evolution is not sensitive to the inclusion or exclusion of the dilution factor, and since the applicability of the dilution factor is lost at later times, we will not consider this issue any further.

Figure 3 is analogous to fig. 1 except that it assumes spherical expansion rather than longitudinal expansion. The degree of supercooling is about the same in both scenarios. The main difference is that expansion in three dimensions is more rapid than in one dimension. Hence the system spends relatively less time near T_c than it does in fig. 1, and the effect of finite nucleation rate and finite bubble growth is therefore more apparent.

Figure 4 shows the effect of bubble fusion on the longitudinal expansion with the default surface free energy of 50 MeV/fm^2. Clearly, bubble fusion enhances the phase transition rate, causing the system to reheat sooner, and shortening the total time it takes to complete the transition. Apart from the first few fm/c, the system is very close to the idealized Maxwell construction.

Figure 5 shows the effect of bubble fusion for the case of spherical expansion. As in fig. 4, bubble fusion causes the system to reheat earlier, and completion of the transition is correspondingly quickened.

Figures 6 and 7 show the average bubble radius as a function of time, with and without fusion, for the scenarios of longitudinal and spherical expansion, respectively. Of course, at T_c the radius of critical size bubbles is infinite. There is a time lag before bubbles are nucleated, and when they are nucleated, they have a size of several fm. In the absence of fusion the average bubble always remains less than about 5 fm in radius. With fusion, bubbles are larger on the average. At the end of the transition the system really is one big bubble of hadronic gas surrounded by vacuum. The difference in average bubble size between fusion and no fusion is quite remarkable. At 10 fm/c after the transition begins, the average bubble radius is already 10 fm, larger than the radius of the colliding nuclei! After this time the details of bubble fusion must certainly be modified to take into account the finite size of the nuclear system. Anyway, we may conclude that in general bubble fusion hastens the transition and brings it closer to the Maxwell idealization.

7 Conclusions

In this paper we investigated a number of issues relating to the dynamics of a strong first order phase transition in a central collision between massive nuclei at RHIC or LHC.

The nucleation rate increases with increasing surface free energy, although the time evolution of the system is not very sensitive to σ in the range 20 to 100 MeV/fm^2.

Growth of hadronic bubbles may either follow the volume expansion of the system or not (with or without a dilution factor in eq. (8)); to settle this issue probably requires a detailed microscopic calculation. At the beginning of the phase transition the dilution factor has practically no effect. Inclusion of the dilution factor does slow the transition later on, but by that time it is incorrect to include it because volume expansion is essentially defined by the expansion rate of the bubbles themselves. Therefore the issue of dilution is probably irrelevant anyway.

Fusion of bubbles increases the speed of the transition, bringing the temperature versus time curve closer to the Maxwell idealization. The procedure we presented for including bubble fusion is perhaps more relevant for the early universe than for heavy ion collisions because, as figs. 6 and 7 showed, the average bubble radius eventually exceeds the radius of the expanding nuclear matter. A system-size-dependent cut-off may be needed in future studies to take this into account.

A spherically expanding system generally will cool faster than a longitudinally expanding system. We found that the plasma must supercool about 20% below T_c before nucleation and bubble growth causes reheating. The bottoming out of the temperature takes about 4 to 7 fm/c after the matter has cooled to T_c. These numbers are rather insensitive to whether the matter expands spherically or longitudinally and to the precise numerical values of the parameters. They therefore can be taken as indicative of a strong first order phase transition in these collisions. It is also generally true that the matter reheats to within 1 to 5% of T_c before the transition is completed. The main difference between the two scenarios is the length of time spent close to T_c, when nucleation has shut off and completion occurs only because of bubble growth and fusion.

Assuming that QCD undergoes a strong first order phase transition at a temperature of order 150-200 MeV, we may conclude that the matter supercools about 20%, reheats to near T_c, and the transition is completed within 25-45 fm/c. These results seem to be general features not dependent upon details of the expansion nor upon special numerical values of the parameters. How to verify this conclusion experimentally is challenging. One possibility is to measure the spectrum of photons or lepton pairs, since those spectra involve space-time integration over the cooling history of the exploding matter.

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Figure Captions

Figure 1: The temperature as a function of time for matter undergoing longitudinal expansion. The dotted curve is the idealized adiabatic Maxwell construction for complete phase equilibrium. The other three curves take into account finite nucleation rate and (nondiluted) bubble growth. The numbers label the surface free energy σ = 20, 50 and 100 MeV/fm^2.

Figure 2: Same as fig. 1 but diluting bubble growth according to the discussion after eq. (8).

Figure 3: Same as fig. 1 but for matter undergoing spherical expansion.

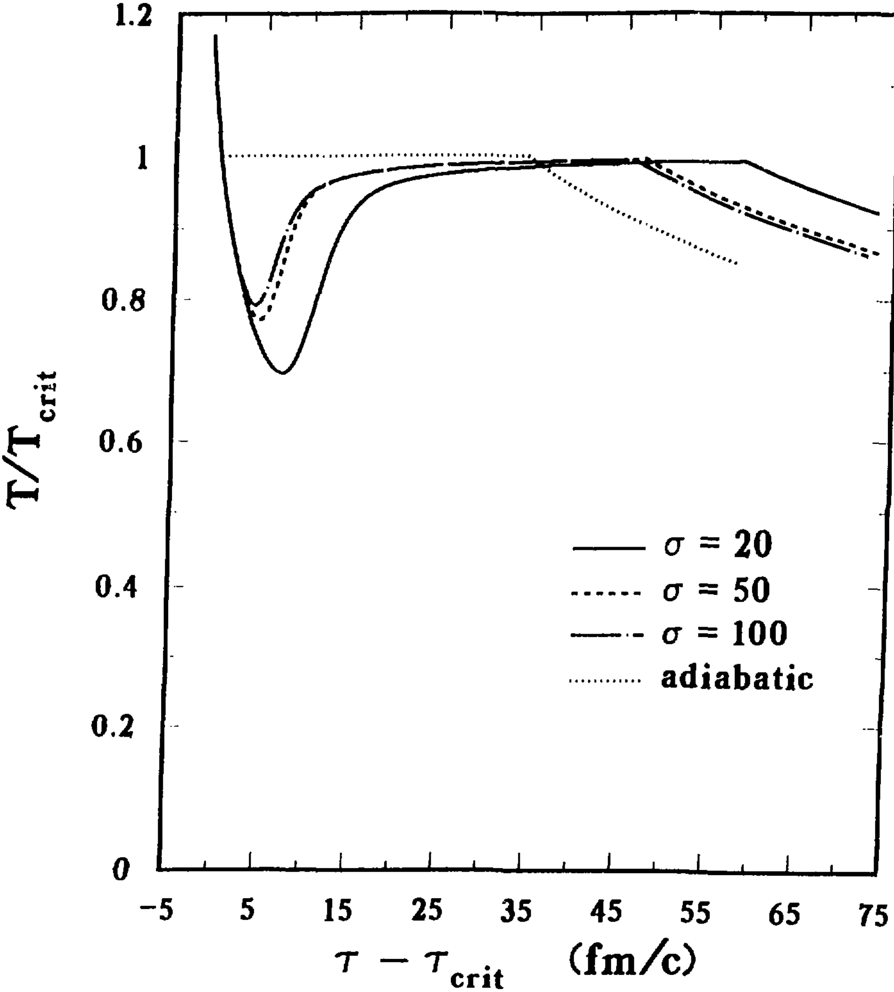
Figure 4: The effect of bubble fusion on the temporal evolution of the temperature, for matter undergoing longitudinal expansion, with σ = 50 MeV/fm^2.

Figure 5: The effect of bubble fusion on the temporal evolution of the temperature, for matter undergoing spherical expansion, with σ = 50 MeV/fm^2.

Figure 6: The average bubble radius for matter undergoing longitudinal expansion and with σ = 50 MeV/fm^2. One curve includes fusion of bubbles, the other doesn't.

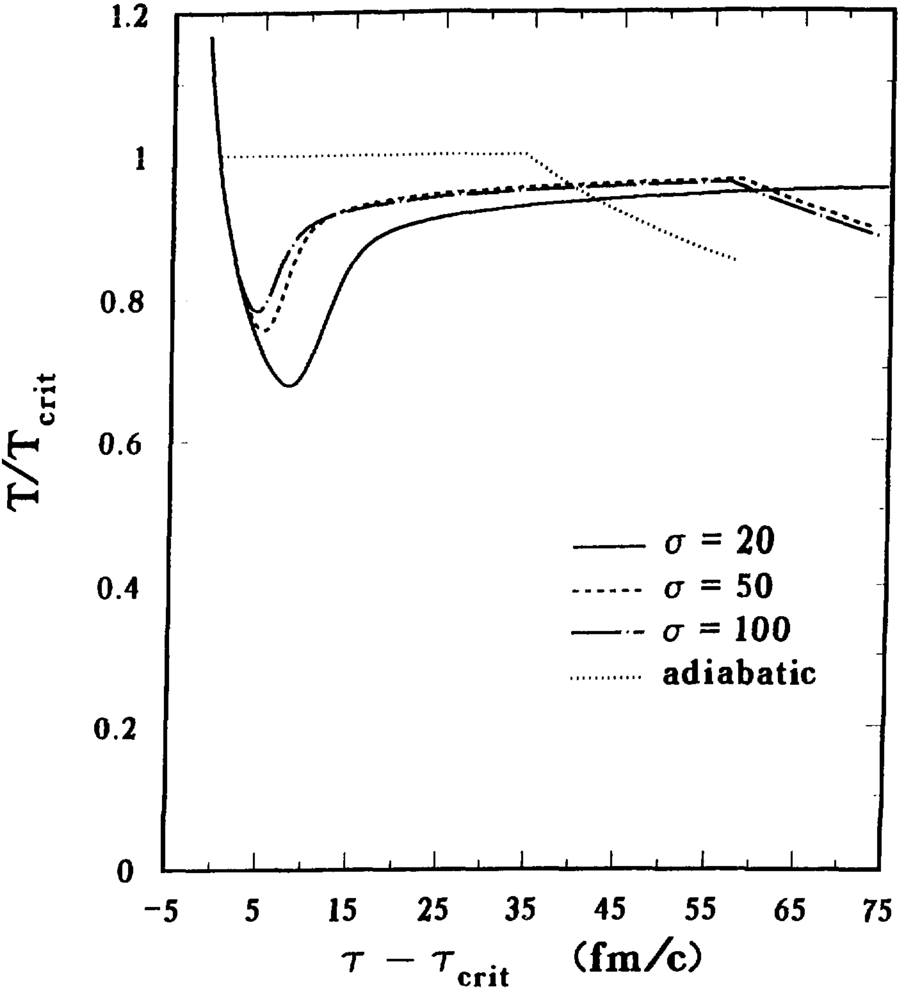
Figure 7: The average bubble radius for matter undergoing spherical expansion and with σ = 50 MeV/fm^2. One curve includes fusion of bubbles, the other doesn't.

Figure 1



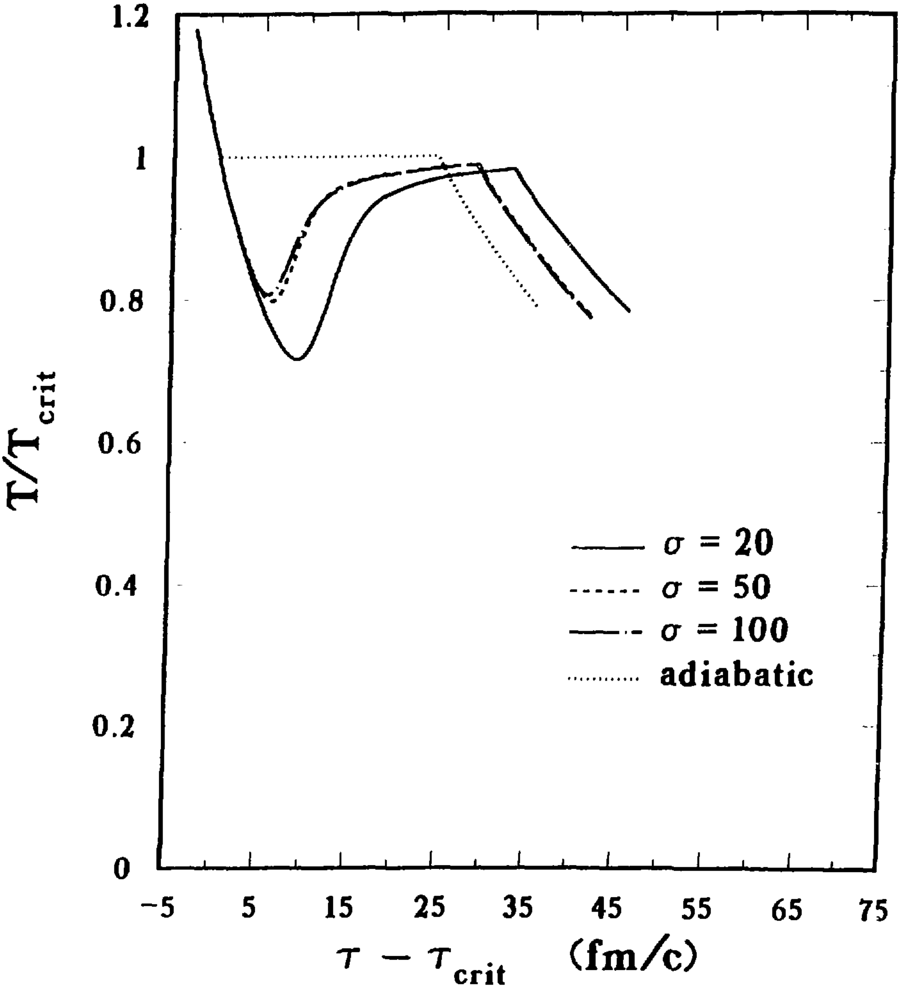
1.2
1
0.8
0.6
0.4
0.2
0
T/T_crit
σ = 20
σ = 50
σ = 100
adiabatic
−5
5
15
25
35
45
55
65
75
τ − τ_crit (fm/c)

Figure 2



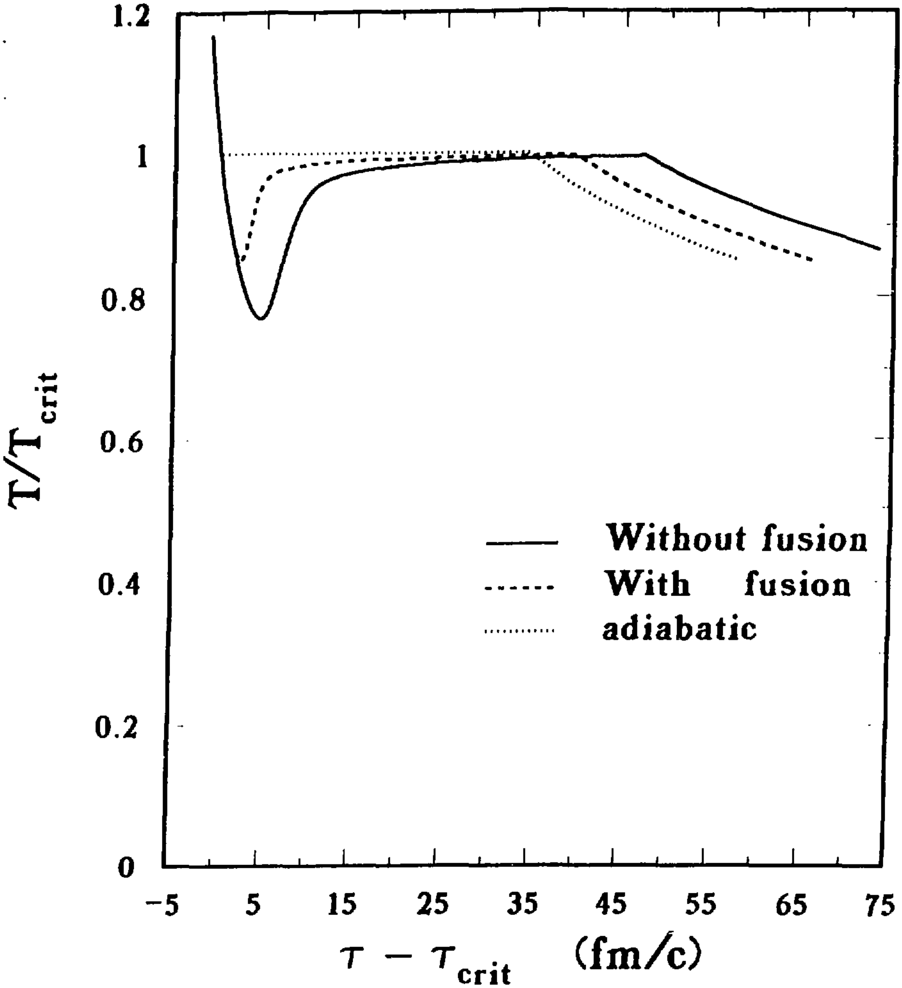
1.2
1
0.8
0.6
0.4
0.2
0
T/T crit
σ = 20
σ = 50
σ = 100
adiabatic
−5
5
15
25
35
45
55
65
75
τ − τ crit (fm/c)

Figure 3



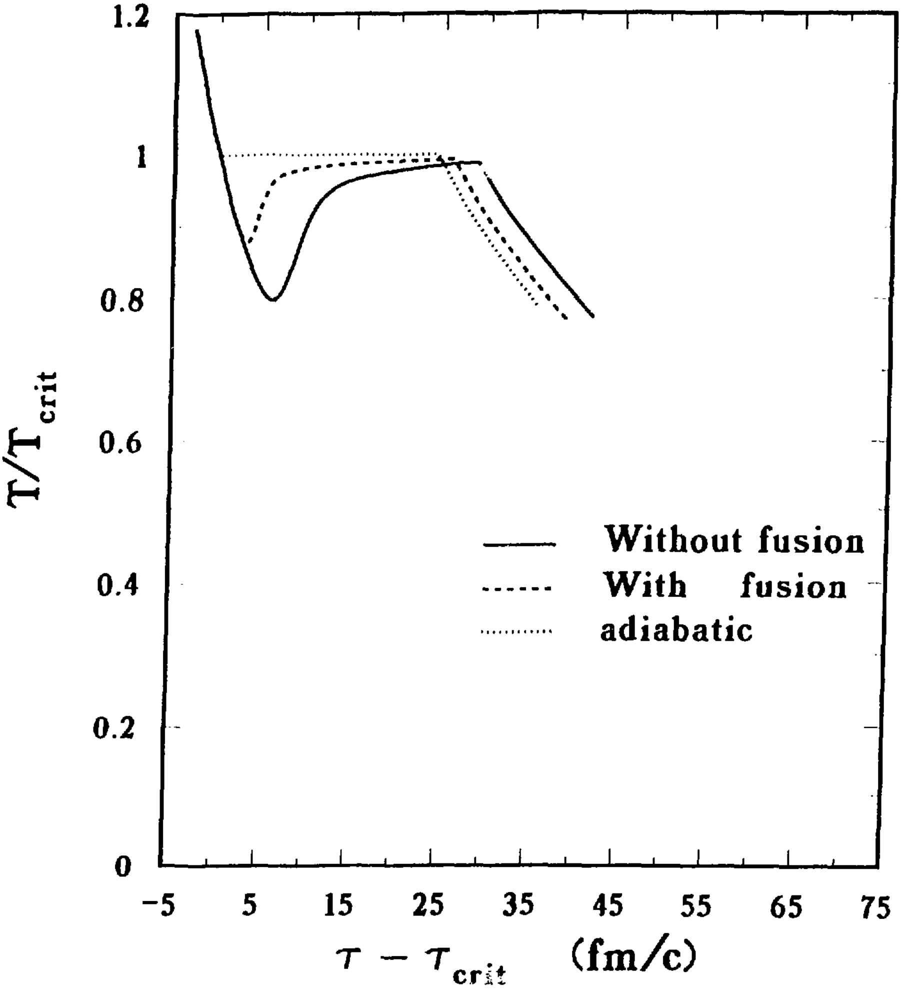
1.2
1
0.8
0.6
0.4
0.2
0
T/T$_{crit}$
σ = 20
σ = 50
σ = 100
adiabatic
−5
5
15
25
35
45
55
65
75
τ − τ$_{crit}$ (fm/c)

Figure 4



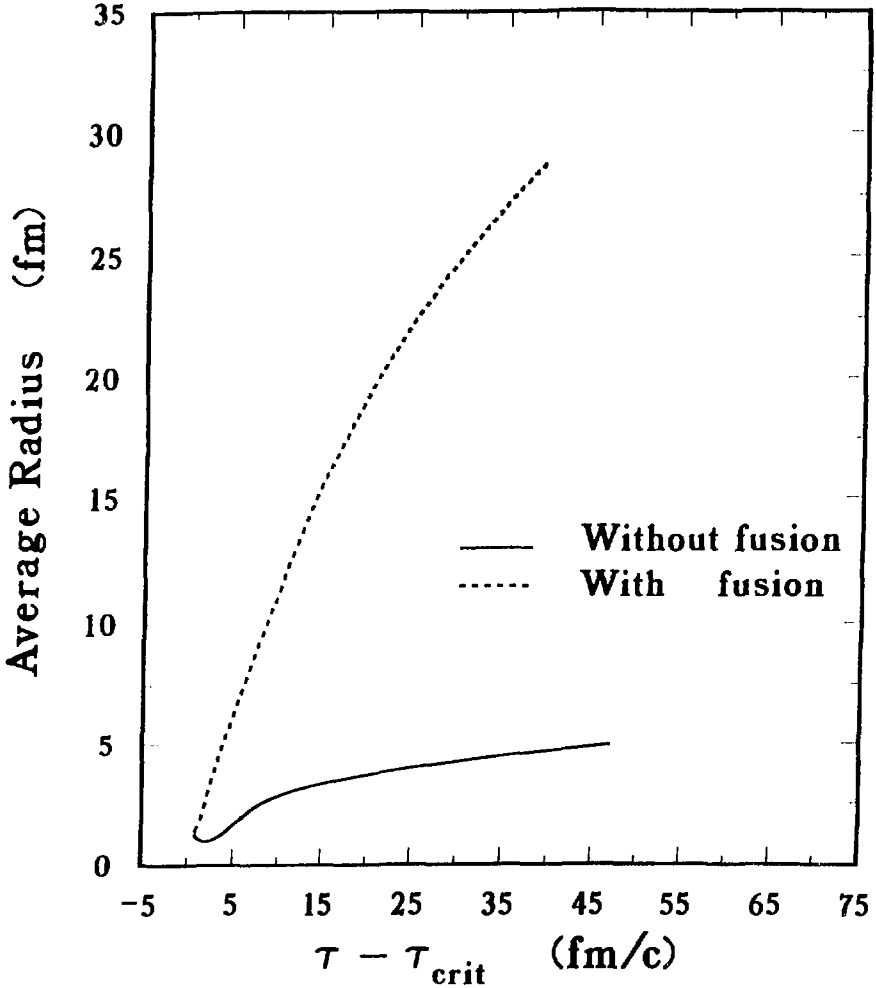
1.2
1
0.8
0.6
0.4
0.2
0
T/T crit
Without fusion
With fusion
adiabatic
-5
5
15
25
35
45
55
65
75
τ − τ crit (fm/c)

Figure 5



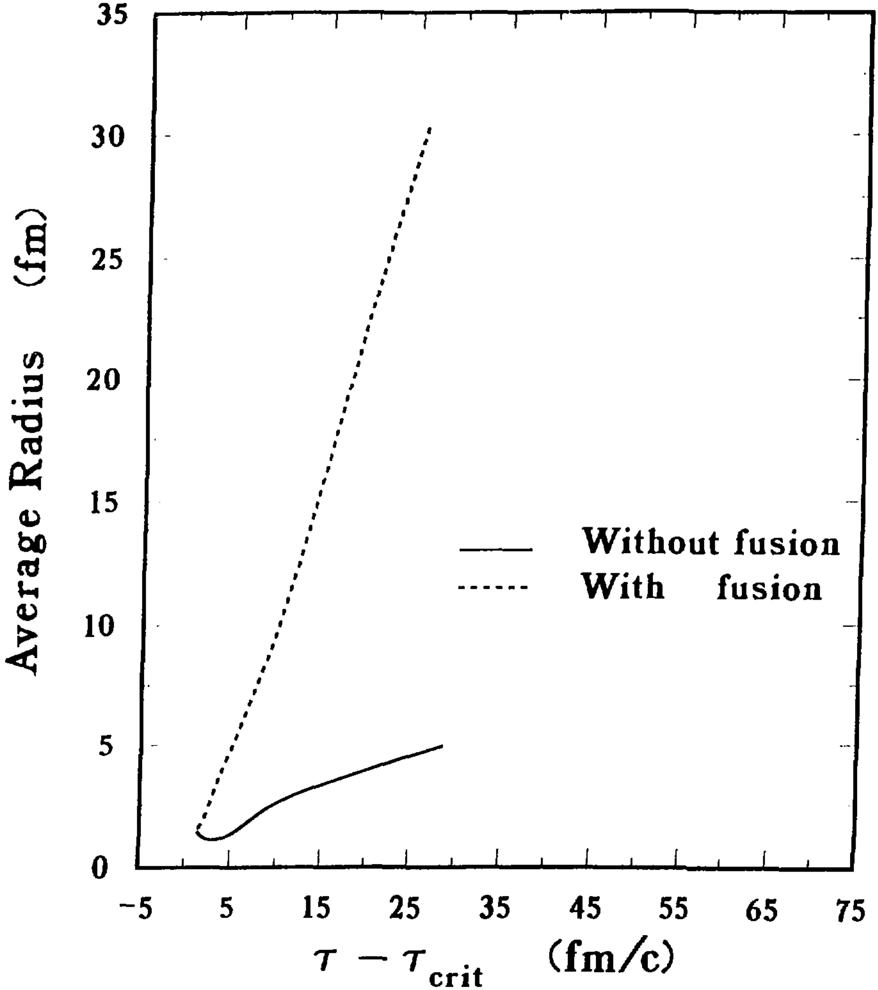
1.2
1
0.8
0.6
0.4
0.2
0
T/T_crit
Without fusion
With fusion
adiabatic
-5
5
15
25
35
45
55
65
75
τ − τ_crit (fm/c)

Figure 6



35
30
25
20
15
10
5
0
Average Radius (fm)
Without fusion
With fusion
−5
5
15
25
35
45
55
65
75
τ − τ crit (fm/c)

Figure 7



35
30
25
20
15
10
5
0
Average Radius (fm)
Without fusion
With fusion
-5
5
15
25
35
45
55
65
75
$\tau - \tau_{crit}$ (fm/c)