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Top and Higgs mass predictions in supersymmetric $SU(5)$ model with big top quark Yukawa coupling constant

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Abstract

From the requirement of the absence of the Landau pole singularity for the effective top quark Yukawa coupling constant up to Planck scale in $SU(5)$ supersymmetric model we find an upper bound $m_t \leq 187$ *Gev* for the top quark mass. For the $SU(5)$ fixed point renormalization group solution for top quark Yukawa coupling constant which can be interpreted as the case of composite superhiggs we find that $m_t \geq 140$ *Gev*. Similar bound takes place in all models with big $\bar{h}_t(m_t)$. For $m_t \leq 160$ *Gev* we find also that the Higgs boson is lighter than m_Z and hence it can be discovered at LEP2.

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At present one of the most urgent problems in experimental high energy physics is the search for the top quark and the Higgs boson. The existing experimental lower bound on the top quark mass $m_t \geq 91$ *Gev* [1] means that the top quark Yukawa coupling constant $\bar{h}_t(m_t)$ is not small and probably the top quark plays an essential role in electroweak symmetry breaking. The precision tests of the standard model based on the account of the radiative corrections allow to extract the value of the top quark mass. Recent analysis [2] gives $m_t = (120 - 170)$ *Gev*. In [3, 4] from the requirement of the absence of the Landau pole singularity up to some scale of about $O(10^{16})$ *Gev* for the top quark effective Yukawa coupling constant an upper bound on the top quark mass has been derived in supersymmetric extension of the Weinberg-Salam model.

In this paper we investigate the spectrum of top quark and Higgs boson masses in models with big top quark Yukawa coupling constant. From the requirement of the absence of the Landau pole singularity for the effective top quark Yukawa coupling constant up to the Planck scale in $SU(5)$ model we find an upper bound $m_t \leq 187$ *Gev* for top quark mass. For the $SU(5)$ fixed point renormalization group solution for top quark Yukawa coupling constant which can be interpreted as the case of composite superhiggs we find that $m_t \geq 140$ *Gev*. Similar bound takes place in all models with big $\bar{h}_t(m_t)$. For $m_t \leq 160$ *Gev* the Higgs boson is lighter than m_Z and hence it can be discovered at LEP2. Some results of this paper has been published in our recent preprint [5].

Consider the supersymmetric $SU(5)$ model [6] with standard kinetic terms and with the superpotential

$$\begin{aligned} W \;=\; & 2^{0.5}\lambda_1 T_{\alpha\beta}(\bar{F}^{\alpha}\bar{H}^{\beta} - \bar{F}^{\beta}\bar{H}^{\alpha}) \\ & +0.25\lambda_2\epsilon^{\alpha\beta\gamma\delta\epsilon}T_{\alpha\beta}T_{\gamma\delta}H_{\epsilon} \\ & +\lambda_3 Tr(\Phi^3) + \lambda_4\bar{H}\Phi H + M_1 Tr(\Phi^2) + M_2\bar{H}H \end{aligned} \tag{1}$$

Here we consider only the third (top quark) supermatter generation and neglect all Yukawa coupling constants of the first and second generations. The model contains the matter supermultiplets $\bar{F}^{\alpha}(\bar{5})$ and $T_{\alpha\beta}(10)$ which correspond to the third generation and the superhiggses $\Phi(24), \bar{H}^{\alpha}(\bar{5}), H_{\alpha}(5)$. The nonzero vacuum solution $< \Phi >= Diag(2,2,2,-3,-3)$ breaks the $SU(5)$ gauge group to the electroweak $SU(3)\otimes SU(2)\otimes U(1)$ gauge group. For energies higher than the grand unified scale M_{GUT} we have effective restoration of the SU(5) gauge symmetry. We shall neglect all Yukawa coupling constants in the effective potential except that one for the top quark Yukawa coupling constant λ_2. The corresponding renormalization group equations in one loop approximation have the form (for the case of three generations) [7]

$$\frac{d\bar{h}_t^2}{dt} = A(\bar{h}_t^2)^2 - B\bar{h}_t^2\bar{g}_5^2 \quad (2)$$

$$\frac{d\bar{g}_5^2}{dt} = -b(\bar{g}_5^2)^2 \quad (3)$$

Here g_5 is the SU(5) gauge coupling, $(16\pi^2)t = \ln(M/\mu)$, b =6, A = 18, B = 192/5, and $h_t \equiv \lambda_2$. The solution of the eqn.(2,3) has the form

$$\bar{g}_5^2 = \frac{g_5^2}{(1 + g_5^2 bt)}, \quad (4)$$

$$\frac{1}{\bar{h}_t^2} = A(g_5^2(B-b))^{-1}(1 + g_5^2 bt) + (1/h^2 - A((B-b)g_5^2)^{-1})(1 + g_5^2 bt)^{B/b} \quad (5)$$

We shall use the numerical value $\alpha_{GUT} = g_5^2/4\pi = 1/24$ for the $SU(5)$ gauge coupling constant at the unification scale $M_{GUT} = 10^{16}$ *Gev*. From the requirement of the absence of the Landau pole singularity for the effective top quark coupling constant $\bar{h}_t$ up to the Planck scale $M_{PL} = 1.2 \times 10^{19}$ *Gev* we find that the top quark Yukawa coupling constant $\bar{h}_t(M_{GUT}) \le 1.3$. For the renormalization group equations (2,3) the infrared fixed point solution [8] is

$$\bar{h}_t^2 = k\bar{g}_5^2, \; k = (B-b)/A = 1.8 \quad (6)$$

For the infrared fixed point solution (6) we find that $\bar{h}_t^2(M_{GUT}) = 0.94$. It should be noted that in our analysis we neglected all Yukawa coupling constants in the superpotential (1) except that one for the top quark constant. However the neglected Yukawa coupling constants give positive contribution to the β-functions that can only make the bound $\bar{h}_t(M_{GUT}) \le 1.3$ more stringent.

For the special fixed point solution (6) the ultraviolet asymptotics for the propagator of the superfield $H_\alpha(5)$ is (for the scalar component of the superfield)

$$D_H(p^2) \sim -(ip^2)^{-1}[\ln(-p^2/\mu^2)]^{0.2} \quad (7)$$

From the equal time commutation relation

$$[\partial_0 H_\alpha, H_\beta^*]\,|_{x^0=y^0} = \delta_{\alpha\beta}(1/iZ_H)\delta^3(x-y) \quad (8)$$

for the scalar component of the superfield $H_\alpha(5)$ and the Kallen-Lehmann representation for the propagator $D_H(p^2)$ we find that the ultraviolet asymptotics for the $D_H(p^2)$ propagator is $i/p^2 Z_H$ (here Z_H is the wave function renormalization). Thus we find from the relation (7) that the wave function renormalization Z_H is equal to zero in the limit of the regularization removing. Therefore we can treat the superfield $H_\alpha(5)$ as composite, namely: the probability $Z_H = |< \varphi \mid \phi >|^2$ (where φ is the "bare" state and ϕ is the "renormalized" state) of the physical state being at the "bare" state is equal to zero and besides the kinetic term for the renormalized superfield vanishes in the limit of the regularization removing [9]. So the "compositeness" condition allows to determine the top quark Yukawa coupling constant and hence to predict the top quark mass.

The renormalization group equations for the $SU(3) \otimes SU(2) \otimes U(1)$ electroweak supersymmetric gauge theory allow to connect the top quark Yukawa coupling constant at grand unified scale with the Yukawa coupling constant at the scale of the supersymmetry breaking M_{SUSY} ; to relate the Yukawa coupling constant at the M_{SUSY} scale with the observable Yukawa coupling constant at the electroweak scale M_W we have to use the the renormalization group equations for the standard Weinberg-Salam model. The renormalization group equations have the form [10, 11]

$$\frac{d\bar{h}_t^2}{dt} = A_1(\bar{h}_t^2)^2 - \bar{h}_t^2(B_1\bar{g}_1^2 + B_2\bar{g}_2^2 + B_3\bar{g}_3^2) \quad (9)$$

$$\frac{d\bar{g}_i^2}{dt} = -b_i(\bar{g}_i^2))^2 \quad (10)$$

with $b_1 = -22,\ b_2 = -2,\ b_3 = 6,\ A_1 = 12,\ B_1 = 26/9,\ B_2 = 6,\ B_3 = 32/3$ for the supersymmetric case and with $b_1 = -41/3,\ b_2 = 19/3,\ b_3 = 14,\ A_1 = 9,\ B_1 = 17/6,\ B_2 = 9/2,\ B_3 = 16$ for the standard case. The solution of the equation (9) can be represented in the form

$$1/\bar{h}^2(M) = \prod_{i=1}^{3}(1 + g_i^2 b_i t)^{B_i/b_i}(1/\bar{h}^2(\mu) - A_1 K(t)) \quad (11)$$

$$K(t) = \int_0^t (\prod_{i=1}^{3}(1 + g_i^2 b_i x)^{-B_i/b_i})dx$$

Numerically for $M = 0.1\ Tev$ we find

$$(\bar{h}_t^2(0.1\ Tev))^{-1} = \frac{0.088}{\bar{h}_t^2(M_{GUT})} + 0.815 \quad (12)$$

Using our previously derived bound $\bar{h}_t(M_{GUT}) \leq 1.3$ we find that $\bar{h}_t(0.1\,Tev) \leq 1.08$. The top quark mass is equal to $m_t = \bar{h}_t(m_t) < H >$, where $v^2 = (174\ Gev)^2 = < H^2 > + < \bar{H}^2 >$ and $< H >= v\cos(\beta)$. So our bound on the top quark mass reads

$$m_t = \bar{h}_t(m_t)v\cos(\beta) \leq 187\ Gev \quad (13)$$

Note that from the requirement of the absence of the Landau pole singularity for the effective top quark Yukawa coupling constant up to GUT scale we find that $m_t \leq 193\ Gev$. For the fixed point solution the Yukawa coupling constant $\bar{h}_t(0.1\ Tev) = 1.05$ and $m_t = 183\ Gev$ for $< H >= 174\ Gev$. The top quark Yukawa coupling constant at the electroweak scale depends rather weakly on the Yukawa coupling constant at GUT scale. For instance, for $0.5 \leq \bar{h}_t^2(M_{GUT}) \leq 1.7$ we find that $1 \leq \bar{h}_t(0.1\ Tev) \leq 1.08$. In the rest of our paper we shall consider the models with $\bar{h}_t(0.1\ Tev) \geq 1$ (according to our definition such models are the models with big Yukawa coupling constant). It should be noted that our formula (12) has been derived for $\bar{\alpha}_3(0.1\ Tev) = 0.116$. The main uncertainty in the determination of the $\bar{h}_t(0.1\ Tev)$ comes from the uncertainty related with the determination of the strong coupling constant $\bar{\alpha}_3(0.1\ Tev)$ and roughly speaking $\bar{h}_t(0.1\ Tev) \sim \sqrt{\bar{\alpha}_3(0.1\ Tev)}$, so for $\bar{\alpha}_3(0.1\ Tev) = 0.115 \pm 0.01$ we have 4% uncertainty in the determination of the $\bar{h}_t(0.1\ Tev)$. Besides there are uncertainties related with threshold effects. We shall assume that the accuracy of the $\bar{h}_t(0.1\ Tev)$ calculation is 5% . For the SUSY breaking scale $M_{SUSY} = 0.1\ Tev$ the radiative corrections to the Higgs boson mass are numerically small and the tree level formulae for the Higgs boson mass are valid. In the supersymmetricc Weinberg-Salam model with soft supersymmetry breaking the following inequality [12] for the Higgs boson mass takes place:

$$m_H \leq m_Z |\cos(2\beta)| \ , \ \tan(\beta) \ = < \bar{H} > / < H > \tag{14}$$

Using the inequality (14) and the experimental bound [14] $m_H \geq 42\ Gev$ for the supersymmetric Higgs boson mass we find that

$$|\cos(\beta)| \geq 0.85 \tag{15}$$

$$|\cos(\beta)| \leq 0.30 \tag{16}$$

For the solution (16) $m_t = \bar{h}_t(m_t)\cos(\beta)v \leq 60\ Gev$ and hence it is excluded (remember that the experimental bound on the top quark mass is $m_t \geq 91\ Gev$ [1]). From the bound (15) we find that for the SU(5) fixed point solution

$$m_t = \bar{h}_t(m_t) v \cos(\beta) \ \geq \ (155 \pm 8)\ Gev \tag{17}$$

The error in (17) is nothing but the assumed 5% uncertainty in the determination of the $\bar{h}_t(0.1\ Tev)$. So we find that in the models with big top quark Yukawa coupling constant ($\bar{h}_t(0.1\ Tev) \geq 1$) the top quark has to be heavier than 145 Gev. Assuming that $m_t = (145 \pm 25)\ Gev$ [2] we find that $|\cos(\beta)| \leq 0.93(1 \pm 0.05)$ and hence according to the inequality (14) the Higgs boson mass is lighter than

$$m_H \leq 67^{+20}_{-11}\ Gev \tag{18}$$

For the case of relatively big supersymmetry breaking scale M_{SUSY} the radiative corrections are very important [14] for big top quark masses and we have to take into account them. We shall assume that the supersymmetry breaking scale M_{SUSY} (we identify the M_{SUSY} with the mass of the top squark) is less than 1 *Tev* (for M_{SUSY} bigger than 1 *Tev* it is very difficult to explain why m_Z is small compared to M_{SUSY}) and at the scale between m_Z and M_{SUSY} we have effectively the Weinberg-Salam model with the single Higgs isodoublet , so the superpartners of quarks and leptons have the masses $\sim M_{SUSY}$. To estimate the Higgs boson mass we have to solve [14] the renormalization group equation for the selfinteraction selfcoupling constant $\bar{\lambda}$ for the light Higgs doublet which at one loop level reads

$$\frac{d\bar{\lambda}}{dt} = 12[\bar{\lambda}^2 + (\bar{h}_t^2 - \bar{g}_1^2/4 - 3\bar{g}_2^2/4)\bar{\lambda} - \bar{h}_t^4 + \bar{g}_1^4/16 + \bar{g}_1^2\bar{g}_2^2/8 + 3\bar{g}_2^4/16] \quad (19)$$

Here λ is the Higgs doublet selfinteraction coupling constant. Besides we have to solve the eqn. (9,10) for for the energies between m_t and M_{SUSY} . We have found that for $M_{SUSY} = 1$ *Tev* the value of the $\bar{h}_t(m_t)$ (for 120 *Gev* $\leq m_t \leq$ 200 *Gev*) is increased approximately by 3% compared to the case $M_{SUSY} = 0.1$ *Tev*. We have solved the renormalization group equation (15) together with the boundary condition

$$\bar{\lambda}\mid_{p^2=M^2_{SUSY}} = \frac{\bar{g}_1^2 + \bar{g}_2^2}{4} \ (\cos(2\beta))^2 \quad (20)$$

Remember that $\cos(\beta) = \frac{m_t}{\bar{h}_t(m_t)v}$ and the knowledge of the $\bar{h}_t(m_t)$ allows to relate the top quark mass with $\bar{\lambda}(m^2_H)$ and hence to determine the Higgs boson mass

$$m_H^2 = 2\bar{\lambda}(m_H^2)v^2 \quad (21)$$

The results of our calculations for the top quark masses in the interval 120 *Gev* $\leq m_t \leq$ 170 *Gev* are presented in the table 1. For the case when $M_{SUSY} \sim 1$ *Tev*, at smaller energies the standard Weinberg-Salam model is valid and we can use the experimental bound [13] $m_H \geq 60$ *Gev* for the non-supersymmetric Higgs boson. From the table 1 we see that for $m_t \geq 140$ *Gev* the Higgs boson is lighter than 60 *Gev* and hence in our model (and in all models with $\bar{h}_t(m_t) \geq 1$) the top quark has to be heavier than 140 *Gev*. Moreover another important feature of the model is that for $m_t \leq 160$ *Gev* the Higgs boson mass is lighter than the Z-boson mass and hence it will be discovered at LEP2.

In conclusion we would like to stress that the considered model and all models with big $\bar{h}_t(m_t)$ predict that the top quark mass is heavier than $m_t \geq 140$ *Gev*. Moreover, for $m_t \leq 160$ *Gev* the Higgs boson (for $M_{SUSY} \leq$ 1 *Tev*) is lighter than the Z-boson and hence it will be discovered at LEP2. It should be noted that the inequality $m_t \geq 140$ *Gev* has been obtained in [15] where the supersymmetric model with "top quark condensate" has been considered. The model with the "top quark condensate" is nothing but the standard model plus the assumption that the top quark effective Yukawa coupling constant has Landau pole at some scale Λ.

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Table 1. The dependence of the Higgs mass on the top quark mass in the assumption that the supersymmetry breaking scale is $M_{SUSY} = 1\ Tev$. All masses are in Gev. The uncertainty in the calculation of m_t reflects the assumed 5% uncertainty in the determination of $\bar{h}_t(m_t)$.

m_t	m_H
120	31^{+5}_{-3}
125	35^{+4}_{-1}
130	41^{+2}_{-1}
135	$47^{+.5}_{-.2}$
140	53^{-1}_{+3}
145	59^{-2}_{+5}
150	66^{-3}_{+6}
155	73^{-4}_{+8}
160	81^{-6}_{+9}
165	89^{-7}_{+10}
170	97^{-8}_{+11}

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