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ABSTRACT

Following muon capture by actinide atoms, some of the inner shell muonic transitions proceed by inverse internal conversion, i.e. the excitation energy of the muonic atom is transferred to the nucleus. In particular, the muonic E2:(3d→1s) transition energy is close to the peak of the isoscalar giant quadrupole resonance in actinide nuclei which exhibits a large fission width. Prompt fission in the presence of a bound muon allows us to study the dynamics of large-amplitude collective motion. We solve the time-dependent Dirac equation for the muonic spinor wave function in the Coulomb field of the fissioning nucleus on a 3-dimensional lattice and demonstrate that the muon attachment probability to the light fission fragment is a measure of the nuclear energy dissipation between the outer fission barrier and the scission point.

1. Introduction

Muonic atoms have proven extremely useful in extracting information about electromagnetic properties of nuclei, e.g. electric charge distributions and multipole moments, because the muon has a high position probability density inside the nucleus. Recent experimental studies at LAMPF[1,2], at TRIUMF[3] and at CERN/PSI[4-6] have shown that muons bound to actinide nuclei may induce fission by non-radiative transitions (inverse internal conversion). The nucleus will be surrounded by the muon during the entire fission process, unless the muon is ionized. *Prompt fission in the presence of a* *muon provides a unique tool to study the dynamics of nuclear fission.* In particular, we have demonstrated theoretically[7] that the muon attachment to the light fission fragment depends upon the nuclear friction between the outer fission barrier and the scission point. In this context, nuclear friction is defined as the irreversible flow of energy (and linear or angular momentum) from collective to intrinsic single-particle motion. Through muon-induced fission one expects to gain a deeper understanding of the energy dissipation mechanism in large-amplitude nuclear collective motion. A very important and still unresolved question in nuclear many-body theory is to what extent the dissipation mechanism can be understood in terms of *"one-body friction"* (collisions of the nucleons with the moving walls of the self-consistent mean field) and the role played by *"two-body friction"* (two-body collisions between the nucleons).

From a theoretical point of view, muon-induced fission has several attractive features. Because the nuclear excitation energy exceeds the fission barrier by several MeV, it is permissible to treat the fission dynamics classically (no barrier tunneling). The muon dynamics is determined by the electromagnetic interaction which is precisely known; hence, the process can be calculated, at least in principle, with any desired precision. Our main task is the solution of the Dirac equation for the muon in the presence of a time-dependent external Coulomb field which is generated by the fission fragments in motion. Nonrelativistic 2-D calculations for muon-induced fission have been carried out by Oberacker and Maruhn[8-11] and other theory groups[12,13], but only recently has the Vanderbilt-ORNL collaboration carried out relativistic 3-D lattice calculations[7,14,15].

2. Prompt Muon-Induced Fission

Following the irradiation of a target with a μ^- beam the muons are captured into high-lying atomic states ($n \cong 14$) and form an excited muonic atom. From the outer shells, the muonic atom decays preferentially by emission of Auger electrons. Since ΔE increases rapidly for the inner shells, the transitions between levels with $n \leq 5$ are dominated by mesic X-rays and γ-rays.

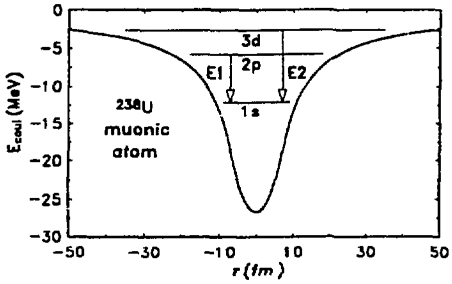

Fig.1 Coulomb interaction potential and energy levels of the muonic atom ^{238}U

Alternatively, the transitions may proceed without emission of radiation via inverse internal conversion, i.e. the muonic excitation energy is transferred to the nucleus, and the muon ends up in the K-shell of the atom. In actinides, the E1(2p→1s) and the E2(3d→1s) muonic transitions (see Fig. 1) are typically of order 6.5 and 10 MeV, respectively; these non-radiative transitions result in excitation of the nuclear giant dipole and giant quadrupole resonance, respectively, which act as doorway states for fission. Since the muon is not annihilated by this process, it can be utilized to study the fission dynamics (see Fig. 2).

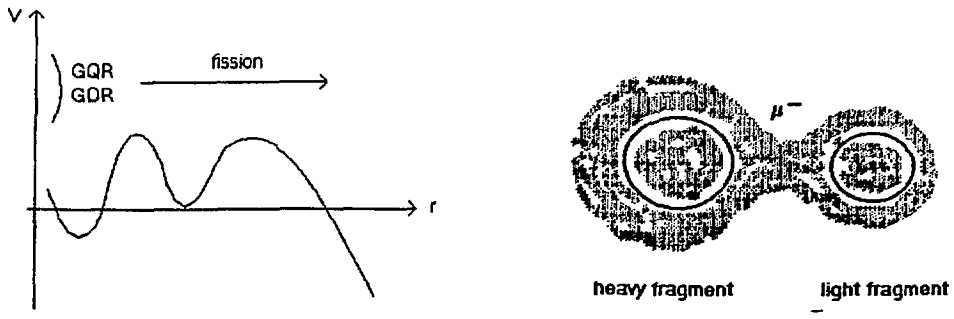

Fig. 2 *Left side:* double-humped fission barrier for an actinide nucleus; the giant dipole resonance (GDR) and giant quadrupole resonance (GQR) are the doorway states to fission.
Right side: prompt fission in the presence of the muon

The prompt muon-induced fission process is most easily understood in terms of a "correlation diagram" in which one plots the binding energies of the transient muonic molecule as a function of the internuclear distance R (Fig. 3). At present, little is known about the nuclear viscosity between the outer fission barrier and the scission point. If the nuclear fission process is slow (large friction), the muon will stay in the lowest molecular energy level 1sσ throughout the fission process and emerge in the 1s bound state of the heavy fission fragment. On the other hand, if the nuclear motion is relatively fast (low friction) there is a nonvanishing probability that the muon may be transferred to higher-lying molecular orbitals, e.g. the 2pσ level from where it may end up attached to the light fission fragment.

Hence, theoretical studies of the *muon-attachment probability to the light fission fragment,* P_L, in combination with experimental data can be utilized to *determine the amount of friction.* An order-of-magnitude estimate for the muon attachment probabilities

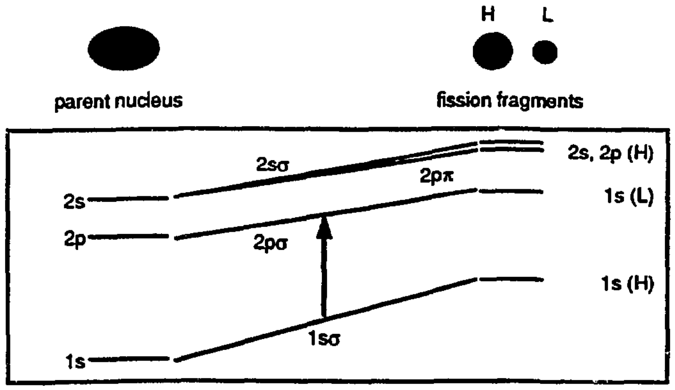

Fig. 3: Schematic "correlation diagram" for the transient muonic molecule ^{238}U

may be obtained in first-order perturbation theory if one restricts the model space to the lowest two molecular levels (1sσ,2pσ); for a fission mass asymmetry $A_H/A_L=1.4$ one obtains muon attachment probabilities P_L of the order of 5 percent[15].

3. Outline of the Theory

In these first relativistic 3D-lattice studies of muon-induced fission, we have chosen a simple parameterization of the nuclear charge distribution (see Fig.4): two spherical segments (radii R_1,R_2) with uniform charge density separated at a distance R. Because of volume conservation during fission, the fragment radii depend upon the elongation, $R_i=R_i(R)$. We describe the fission path by two phenomenological collective coordinates: the elongation parameter R and a mass asymmetry parameter defined as $\xi(R)=(R_1/R_2)^3$.

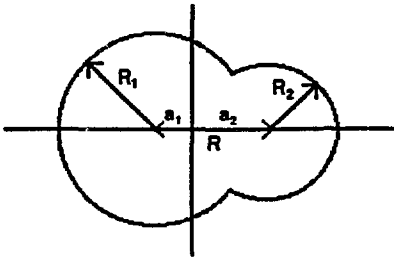

Fig. 4 Parameterization of the nuclear charge distribution

If we denote the coordinate-dependent elongation mass parameter as B(R), the classical collective nuclear energy has the form

$$E_{nuc} = \tfrac{1}{2}B(R)\left(\frac{dR}{dt}\right)^2 + V_{eff}(R). \tag{1}$$

An empirical double-humped fission potential $V_{fis}(R)$ is utilized which is smoothly joined with the Coulomb potential of the fission fragments at large R (see Fig.5).

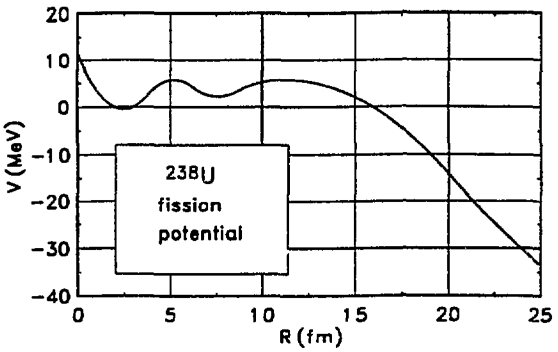

Fig. 5 Phenomenological fission potential for ^{238}U corresponding to the coordinate-dependent mass parameter B(R)

The effective fission potential

$$V_{eff}(R) = V_{fis}(R) + E_{\mu}(R) \tag{2}$$

contains a contribution from the muonic binding energy which depends on the elongation of the fissioning nucleus; this results in an augmentation of the fission barrier. In addition, we introduce a linear friction force which acts between the outer fission barrier and the scission point to account for energy dissipation via neutron and photon emission. In this case, the dissipation function D is a simple quadratic form in the velocity and the time-rate of change of the nuclear collective energy equals twice the dissipation function

$$\frac{dE_{nuc}}{dt} = -2D = -f\left(\frac{dR}{dt}\right)^2 . \tag{3}$$

The adjustable friction parameter f determines the dissipated energy.

Let us now consider the dynamics of the muon. It is convenient to measure the muon position in units of its Compton wavelength and to measure time in units of its Compton time. Hence we introduce the following dimensionless coordinates

$$\begin{aligned} \bar{x} &= \bar{r} / \lambda_c \qquad & \lambda_c &= \hbar / (m_{\mu} c) = 1.87\ fm \\ \tau &= t / \tau_c \qquad & \tau_c &= \lambda_c / c = 6.23 \times 10^{-24}\, s\, , \end{aligned} \tag{4}$$

and the dimensionless Coulomb interaction between the muon and the fissioning nucleus

$$V^0 = -eA^0 / (m_{\mu} c^2). \tag{5}$$

The time-dependent Dirac equation for the muon spinor wave function has the structure

$$\left[-i\bar{\alpha} \cdot \bar{\nabla} + \beta + V^0(\bar{x}, \tau)\right] \psi(\bar{x}, \tau) = i \frac{\partial}{\partial \tau} \psi(\bar{x}, \tau). \tag{6}$$

Note that the nuclear and muonic motions are coupled via the instantaneous muon energy in eq.(2)

$$E_{\mu}(R(t)) = \langle \psi(\bar{x}, t) | -i\bar{\alpha} \cdot \bar{\nabla} + \beta + V^0(\bar{x}, t) | \psi(\bar{x}, t) \rangle . \tag{7}$$

Therefore, the nuclear equation of motion (3) and the Dirac equation (6) must be solved simultaneously.

4. Numerical Implementation

We solve the time-dependent Dirac equation on a three-dimensional Cartesian lattice using the basis-spline collocation method[16]. The calculation proceeds in two steps: we first compute the static wavefunction by a damped relaxation method and then solve the time-dependent problem by a Taylor-series expansion of the infinitesimal time-evolution operator[17,18]. In this way, we reduce the Dirac equation to a series of (matrix)x(vector) operations which can be executed with high efficiency on vector or parallel supercomputers without explicitly storing the matrix in memory[19]. Typical runs on a CRAY-2 supercomputer for cubic or rectangular lattices with up to 29 collocation points in x, y, and z direction and a lattice spacing $\Delta x = 2\lambda_c$ take 4 Megawords of memory and 6.5 CPU hours. As in all lattice calculations, we need to demonstrate convergence in terms of the lattice size and lattice spacing; we estimate that convergent calculations will require a rectangular lattice with approximately 39×39×65 lattice points and a lattice spacing $\Delta x = \lambda_c$. We also plan to examine the possibility of an additional improvement of our numerical method by using variable lattice spacing; the basis-spline collocation method is well suited to this task.

5. Results and Discussion

In the following we present results for prompt fission of ^{238}U induced by the E2:(3d→1s, 9.6 MeV) non-radiative muonic transition. Fig. 6 shows the relative velocity of the fission fragments as a function of time; all times are indicated in units of the muon Compton time, eq. (4). Our model assumes that there is no friction until the outer fission barrier E_B is reached; hence, the velocity profile is essentially the mirror image of the fission potential displayed in Fig.5, with small deviations arising from muonic binding energy contributions. The numerical calculations have been carried out for a variety of friction parameters; for f=500 we observe a time delay in the nuclear relative motion of $\Delta t_{nuc} \cong 600\ \tau_c = 3.7 \times 10^{-21}$ s.

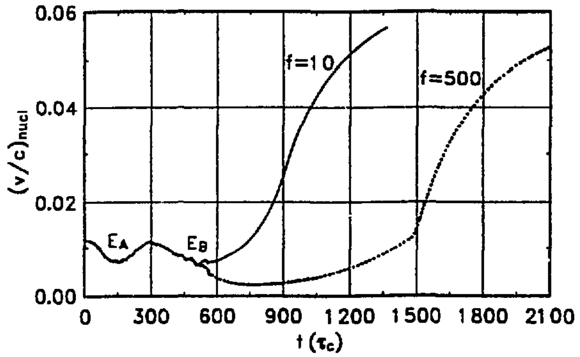

Fig. 6 Fission fragment velocity vs. time for two values of the friction parameter, f=10 and f=500

Fig. 7 shows the nuclear energy dissipation (in form of neutron- and γ-emission) as a function of time; in our model, friction is confined to the region between the outer fission barrier and the scission point; for friction parameters f=10 and f=500, we obtain total dissipated energies $E_{diss} = 0.7$ MeV and 15.8 MeV, respectively.

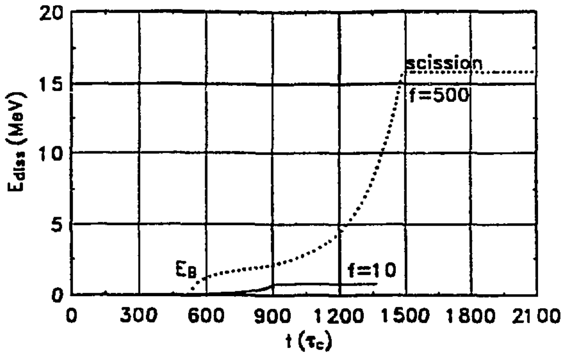

Fig. 7 Energy dissipated during fission

In Fig.8 we depict the time-variation of the instantaneous muon energy defined in eq.(7): at t=0, it corresponds to the energy of a muon bound by the Coulomb field of a quadrupole-deformed ^{238}U nucleus; at t→∞, the binding energy approaches a value somewhere in between the binding energies of the heavy and the light fission fragment, but much closer to that of the heavy fragment.

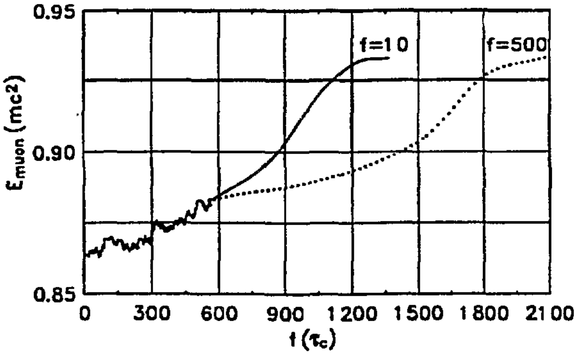

Fig. 8 Total muon energy during fission

Fig. 9 shows the Coulomb interaction energy between the muon and the fission fragments at large separation. The two Coulomb wells are clearly visible; the deeper well on the left is generated by the heavy fission fragment. Also shown (on the right side of Fig.9) is the associated muon position probability density[7,15]. For a fragment mass asymmetry A_H/A_L=1.40 we observe that the muon sticks predominatly to the heavy fragment: the muon attachment probability to the light fragment is represented by the small bump on the right.

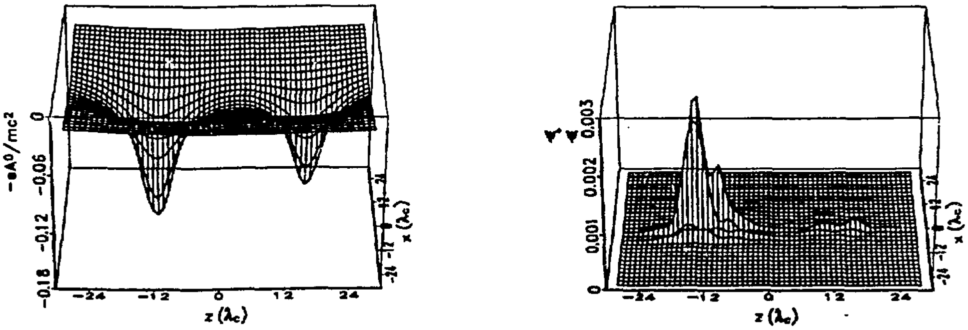

Fig. 9 Coulomb potential (left) and muonic position probability density (right) at time $t = 1301\tau_c$ and internuclear distance $R = 52.9$ fm

Fig. 10 shows the muon attachment probability to the light fission fragment P_L as a function of the dissipated nuclear energy. We observe that P_L decreases with dissipated energy. For a light fission fragment of mass A_L=99 we calculate muon attachment probabilities of order 10%. This is in good agreement with recent experimental data by Polikanov[6]: (0.090±0.027) for light fragments in the mass range 107-117. Since the sticking probability depends strongly on the fragment mass, a more detailed comparison with Polikanov's data is necessary.

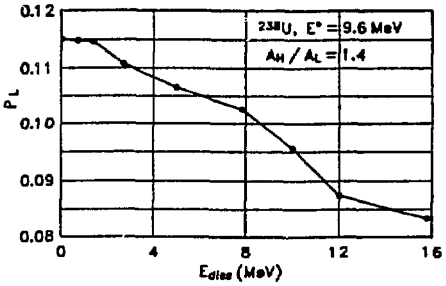

Fig. 10 Muon attachment probability to the light fission fragment as function of the nuclear energy dissipated during fission

6. Summary and Outlook

Prompt muon-induced fission is a tool for studying the dynamics of nuclear fission. In particular, our goal is to determine the nuclear energy dissipation between the outer fission barrier and the scission point; this is possible by a comparison of the theoretical values of P_L for a variety of friction coefficients with the experimental μ^- attachment probabilities. It will be interesting to compare these results to experimental information from other methods such as neutron emission[20] and to theoretical models of friction (macroscopic and microscopic)[21-24]. In this context we will study μ^- attachment probabilities to the fission fragments as a function of dissipated energy and fission fragment mass asymmetry, and we will attempt to analyze all available experimental data. We also plan to make theoretical predictions for actinide nuclei which have not yet been investigated experimentally.

Furthermore, we want to compare the dependence of the final state on different nuclear shape parameterizations and fission models (e.g. hydrodynamic model *vs.* Strutinsky shell-correction method). Ultimately, we want to link our current studies of the time-evolution of the muonic spinor wave function to a microscopic treatment of fission dynamics. This could be achieved via a time-dependent Hartree-Fock description[25] of the nuclear motion between the outer fission barrier and the scission point, providing the first experimentally accessible way to check the concept of mean-field dynamics and the associated one-body dissipation. Discrepancies between theory and experiment would indicate two-body dissipation.

7. Acknowledgment

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