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G. Ganbold

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G.V. Efimov

INTERNATIONAL ATOMIC ENERGY AGENCY

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G. Ganbold [1]
International Centre for Theoretical Physics, Trieste, Italy
and
Laboratory of Theoretical Physics, Joint Institute for Nuclear Research,
141980 Dubna, Russian Federation

and

G.V. Efimov
Laboratory of Theoretical Physics, Joint Institute for Nuclear Research,
141980 Dubna, Russian Federation.

ABSTRACT

We have suggested a regular scheme for calculating systematically the leading term and next corrections to it up to the fourth order for the effective potential in the scalar φ_2^4 theory. The obtained results give evidence in favour of a second-order phase transition at $(g/2\pi m^2)_{crit} \simeq 0.9$ in the theory under consideration.

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[1] On leave of absence from: Institute of Physics and Technology, Enh Taivan Ave. 54B, P.O. Box 51/188, Ulaanbaatar, Mongolia.

1 Introduction

The phenomenon of spontaneous symmetry breaking, or in other words, the vacuum structure arrangement is an important part of many quantum field constructions. The simplest example, where the vacuum exhibits a nontrivial structure, is the φ_2^4 theory. Many papers [1]-[11] are devoted to the investigation of vacuum phase structure in this model. We shortly treat some nonperturbative methods that seemed to be basic among investigations on this subject. Approaches based on perturbative expansion [1,2], allowing for the "one-loop" and "two-loop" contributions to the effective potential (EP) are regular, but they have certain problems with reliability of the obtained estimation in the critical region and also with renormalization procedures. Among the nonperturbative methods, the canonical transformation of the given Hamiltonian [3,4] is used regularly. An original approach [3] using a Hartree-type approximation exhibits satisfactory results in both weak and strong coupling regimes but gives a wrong prediction about the existence of a first order phase transition (PT) in this theory. A similar result was obtained [5] within the Gaussian effective potential (GEP) approach. The dimensionless value of the critical coupling constant, for which the first order PT takes place, is $G = 1.62$ in both papers. These conclusions contradict mathematical theorems [6,7] proving that a second order PT occurs in the φ_2^4 model. Usually, problems of that type are studied by means of variational methods which are popular due to their clear physical meaning and relative simplicity of calculations. There are papers [8]-[12] where different variational methods have been used for solving this problem and a second order PT has been observed in the region $G \sim 1$. Although the results obtained by developing a post-Gaussian approximation [10] or a non-Gaussian "quartic" exponential [11] are interesting, these methods either do not converge in the wide region and one able only to fix roughly an interval where the critical point is located [10], or use a complicated approximate scheme breaking the $-\varphi \leftrightarrow \varphi$ invariance and require complicated numerical calculations [11]. Meanwhile, variational approaches do not give a systematic prescription for choosing a trial wave function (or functional) and also do not allow one to control the obtained approximation accuracy.

We will investigate this problem within the method of EP utilizing the techniques introduced in [13]-[15]. This method is regular, it gives a common prescription for calculating different path integrals defined on Gaussian measures. For Hermitian functionals, its zeroth order approximation reproduces the variational estimation on general Gaussian measure. Our approach also allows one to take systematically into account high order corrections to the zeroth approximation. Besides, this method is valid for both real and complex functionals. We have shown a possible occurrence of a second order PT in this theory and have given an estimation for the corresponding critical coupling constant.

2 Effective potential

In this paper, we consider the scalar field theory with the Lagrangian density

$$\mathcal{L} = -\frac{1}{2}\varphi(\mathrm{x})\cdot(-\Box + m^2)\cdot\varphi(\mathrm{x}) - \frac{g}{4}\cdot N_o\{\varphi^4(\mathrm{x})\}\ , \tag{1}$$

where the normal product N_o of the fields $\varphi(\mathrm{x})$

$$N_o\{\varphi^4(\mathrm{x})\} = \varphi^4(\mathrm{x}) - 6\cdot\varphi^2(\mathrm{x})\cdot D_o(0) + 3\cdot D_o^2(0)\,, \tag{2}$$

$$D_o(\mathbf{x}) = \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{\exp\{i\mathbf{kx}\}}{m^2 + \mathbf{k}^2}.$$

is introduced for removing completely all the "tadpole"-type divergences arising in this superrenormalizable theory. Here $\mathbf{x} = (x_1, x_2) \in \Omega$ and Ω is a finite volume in $\mathbf{R}^2$. Both the mass parameter m and coupling constant g are positive. At small g, the Lagrangian (1) describes a system invariant with respect to the transformation $\varphi \leftrightarrow -\varphi$. The question is whether this symmetry remains for increasing g.

In this section, we will investigate this question by the method of the EP defined [16] as

$$V(\varphi_o) = -\lim_{\Omega\to\infty} \frac{1}{\Omega} \ln I_\Omega(\varphi_o), \tag{3}$$

$$I_\Omega(\varphi_o) = C_o \int \delta\varphi \cdot \delta\{\varphi_o - \frac{1}{\Omega}\int_\Omega d^2\mathbf{x}\varphi(\mathbf{x})\} \exp \int_\Omega d^2\mathbf{x} \cdot \mathcal{L}[\varphi(\mathbf{x})], \tag{4}$$

$$C_o = \sqrt{\det\{-\Box + m^2\}},$$

where the normalization satisfies the usual condition

$$I_\Omega(\varphi_o, g = 0) = 1. \tag{5}$$

We work in the Euclidean space. The functional integral in (4) is well defined by perturbation expansions at small g. The absolute minimum of the EP $V(\varphi_o)$ at the point $\varphi_o = \varphi_*$ determines the true ground state (vacuum) energy in the theory. For $g \ll 1$ the minimum is disposed at the origin: $\varphi_* = 0$. As g increases, there may occur new nontrivial minima $\varphi_* = \pm a$, which means the appearance of a PT in this system.

If a PT takes place at certain coupling $g = g_c$, then for $g < g_c$ the system is still in the original symmetry unbroken phase with $\varphi_c = 0$. At reaching $g = g_c$ the origin $\varphi_o = 0$ is not more the absolute minimum of $V(\varphi_o)$ and the system goes to the new lowest energetic state with $\varphi_c \neq 0$. The first-order PT means that the point $\varphi_o = 0$ remains to be a local but not the absolute minimum of $V(\varphi_o)$. In other words, the first derivative of $V(\varphi_o)$ is zero and the second one is positive at the origin $\varphi_o = 0$. But in the case of the second-order transition, the point $\varphi_o = 0$ becomes a local maximum of EP at $g = g_c$. The second derivative of $V(\varphi_o)$ at $\varphi_o = 0$ is negative in this situation.

Thus, the coefficient $\alpha(g)$ in the representation of $V(\varphi_o)$ for small φ_o

$$V(\varphi_o) = E(g) + \alpha(g)\cdot\varphi_o{}^2 + O(\varphi_o^4) \tag{6}$$

plays an important role in determining the character of a PT. If $\alpha(g)$ becomes zero at certain $g = g_c$ and is negative for $g > g_c$ up to $g \to \infty$, one says that a second-order PT appears here. On the contrary, the positiveness of $\alpha(g)$ for any g excludes the second-order transition.

Let us do some transformations of the functional integral $I_\Omega(\varphi_o)$ in (4).

First, we introduce a transformation of the field variable

$$\varphi(\mathbf{x}) = \phi_o + \phi(\mathbf{x}), \tag{7}$$

where ϕ_o is a constant field and the new field variable $\phi(\mathbf{x})$ corresponding to the new mass μ in

$$D^{-1}(\mathbf{x},\mathbf{y}) = (-\Box + \mu^2)\cdot\delta(\mathbf{x}-\mathbf{y}), \tag{8}$$

satisfies the condition:

$$\int_\Omega d^2\mathbf{x}\cdot\phi(\mathbf{x}) = 0. \tag{9}$$

We substitute (7) into (4) and perform integration over $d\phi_o$ taking into account the functional differential $\delta\varphi = d\phi_o\delta\phi$. Then, we obtain

$$I_\Omega(\varphi_o) = C_o\int \delta\phi\cdot\exp\Big\{\int_\Omega d^2\mathbf{x}\cdot\mathcal{L}[\varphi_o + \phi(\mathbf{x})]\Big\}, \tag{10}$$

where the functional integration is now performed over the fields satisfying the condition (9).

Second, we introduce a new quadratic measure

$$d\sigma = C\,\delta\phi\cdot\exp\Big\{-\frac{1}{2}\int_\Omega d^2\mathbf{x}\int_\Omega d^2\mathbf{y}\,\phi(\mathbf{x})D^{-1}(\mathbf{x},\mathbf{y})\phi(\mathbf{y})\Big\}, \tag{11}$$

$$\int d\sigma\cdot 1 = 1,$$

which is supposed to contain in itself all the quadratic new field configurations ($\sim\phi^2$) of the new Lagrangian. Then, we have to go over to the normal ordering in the new fields $\phi(\mathbf{x})$ with the mass parameter μ using the formula [17]

$$N_o\Big\{\exp\{\beta\varphi(\mathbf{x})\}\Big\} = N\Big\{\exp\{\beta(\varphi_o + \phi(\mathbf{x})) + \frac{\beta^2}{2}\Delta(D_o, D)\}\Big\}, \tag{12}$$

$$\Delta(D_o, D) \equiv D_o(0) - D(0) = \frac{1}{4\pi}\ln\frac{\mu^2}{m^2} + \frac{1}{\mu^2\Omega}, \tag{13}$$

$$D(\mathbf{x}) = \int\frac{d^2\mathbf{k}}{(2\pi)^2}\frac{\exp\{i\mathbf{kx}\}}{\mu^2 + \mathbf{k}^2} - \frac{1}{\mu^2\Omega}. \tag{14}$$

Then, substituting (11) and (13) into (10) we get

$$I_\Omega(\varphi_o) = \exp\Big\{\ln\frac{C_o}{C}\Big\}\cdot\int d\sigma\cdot\exp\Big\{W_{int}(\phi,\varphi_o,D)\Big\}, \tag{15}$$

where the interaction functional is defined:

$$W_{int}(\phi,\varphi_o,D) \equiv -L_o - \frac{g}{4}\int_\Omega d^2\mathbf{x}N\Big[\phi^4(\mathbf{x}) + 4\varphi_o\phi^3(\mathbf{x})\Big] - L_1 - L_2, \tag{16}$$

$$L_o = \Omega\Big\{\frac{m^2\varphi_o^2}{2} + \frac{g}{4}[\varphi_o^4 - 6\Delta\varphi_o^2 + 3\Delta^2] + \frac{m^2-\mu^2}{2}D(0)\Big\}$$

$$L_1 = \varphi_o[m^2 + g(\varphi_o^2 - 3\Delta)]\int_\Omega d^2\mathbf{x}N[\phi(\mathbf{x})], \tag{17}$$

$$L_2 = \frac{1}{2}[m^2 - \mu^2 + 3g(\varphi_o^2 - \Delta)]\int_\Omega d^2\mathbf{x}N[\phi^2(\mathbf{x})].$$

Our basic idea was [14] that the linear and quadratic terms over the new field configuration $\phi(\mathbf{x})$ should be removed from the interaction functional $W_{int}(\phi,\varphi_o,D)$ in (17). In other words, we demand the conditions $L_1 = 0, L_2 = 0$. One can easily see that $L_1 = 0$ automatically owing to the condition (9). Thus, we obtain the following equation:

$$m^2 - \mu^2 + 3g\cdot(\varphi_o^2 - \Delta) = 0 \tag{18}$$

which provides the removing of the term L_2 in (17).

Taking into account Eqs. (13) and (14) one finds that the new mass parameter μ is defined by the equation

$$\mu^2 = m^2 + 3g\cdot\left[\varphi_o^2 - \frac{1}{4\pi}\ln\frac{\mu^2}{m^2}\right] \tag{19}$$

in the limit $\Omega \to \infty$.

Thus, we obtain for the EP

$$V(\varphi_o) = V_o(\varphi_o) + V_{sc}(\varphi_o), \tag{20}$$

where

$$V_{sc}(\varphi_o) = -\lim_{\Omega\to\infty}\frac{1}{\Omega}\ln J_\Omega(\varphi_o). \tag{21}$$

The leading-order term of the EP is presented by

$$V_o(\varphi_o) = -\frac{1}{2}\int\frac{d^2\mathbf{k}}{(2\pi)^2}\left[\ln\left(1+\frac{m^2-\mu^2}{\mu^2+\mathbf{k}^2}\right) - \frac{m^2-\mu^2}{\mu^2+\mathbf{k}^2}\right] + \frac{m^2}{2}\varphi_o^2 + \frac{g}{4}(\varphi_o^4 - 6\Delta\varphi_o^2 + 3\Delta^2) \tag{22}$$

and corrections to it may be calculated by developing the functional integral

$$J_\Omega(\varphi_o) = \int d\sigma\cdot\exp\left\{-\frac{g}{4}\int_\Omega d^2\mathbf{x}N\left[\phi^4(\mathbf{x}) + 4\varphi_o\phi^3(\mathbf{x})\right]\right\}. \tag{23}$$

Eqs. (19)-(23) define the EP at arbitrary coupling g.

We note that our leading term $V_o(\varphi_o)$ and constraint equation (19) reproduce the Hartree-type potential [8] and the corresponding condition of its minimum on the parameter μ. This coincidence may be explained by our particular choices of linear field transformation (7) and Gaussian type of measure $d\sigma$ in (11). It is well known [3,5] that the potential (22), in which μ is defined by (19), corresponds to the sum of "cactus-type" diagrams and leads to the first-order PT appearing at

$$\left(\frac{g}{m^2}\right)_* = 10.211 \quad \text{or} \quad G_o = \left(\frac{g}{2\pi m^2}\right)_* = 1.6251. \tag{24}$$

Further, it will be convenient to work in units of m dealing with dimensionless values for numerical calculations. We define

$$G = \frac{g}{2\pi m^2}, \quad \xi = (\mu/m)^2 \quad \text{and} \quad \Phi_o{}^2 = 4\pi\varphi_o^2. \tag{25}$$

Then, in these units (22) becomes

$$V_o(\Phi_o) = \frac{m^2}{8\pi}\left\{\xi - 1 - \ln\xi + \Phi_o{}^2 + \frac{G}{4}\left[\Phi_o{}^4 + 3\ln^2\xi - 6\Phi_o{}^2\ln\xi\right]\right\}. \tag{26}$$

Eq. (19) can be rewritten as

$$2\xi - 2 + 3G\cdot(\ln\xi - \Phi_o{}^2) = 0. \tag{27}$$

Our idea was to investigate the coefficient $\alpha(G)$ in the expansion (6) of the EP around the point $\Phi_o = 0$. Then the Gaussian EP (26) has the following expansion:

$$V_o(\Phi_o) = \frac{m^2}{8\pi}\left\{\Phi_o{}^2 + O(\Phi_o{}^4)\right\} \tag{28}$$

as $\Phi_o \to 0$ at any G. We obtain

$$\alpha_o(G) = 1. \tag{29}$$

In other words, GEP excludes any occurence of a second-order PT.

3 Post-Gaussian corrections

In the previous section, we have obtained the expression for the EP consisting of two parts. Considering only the "leading" term $V_o(\varphi_o)$ it is impossible to describe reliably the nature of a PT in this theory. To answer this question one should take into account also the remaining part $V_{sc}(\varphi_o)$ of the EP, defined by Eqs. (21) and (23). The problem is to estimate corrections to $\alpha_o(G)$ which come from the path integral $J_\Omega(\varphi_o)$ in (23). This aim can be reached in two approaches: either using a variational estimation of (23) or, by developing a perturbation expansion for $J_\Omega(\varphi_o)$ with an appropriate scheme to sum the obtained series. Below we consider both approaches.

3.1 Variational estimation of $\alpha(G)$

We rewrite (23) in the form correct for small φ_o:

$$J_\Omega(\varphi_o) = \int d\sigma\cdot\exp\left\{-\frac{g}{4}\int_\Omega d^2\mathbf{x}N\phi^4(\mathbf{x}) + \frac{g^2\varphi_o^2}{2}\left[\int_\Omega d^2\mathbf{x}N\phi^3(\mathbf{x})\right]^2\right\}. \tag{30}$$

This representation can easily be obtained due to the validity of the following transformation in (23):

$$\exp(-\varphi_o W) = \cosh(\varphi_o W) \simeq \exp\left\{\frac{1}{2}\varphi_o^2 W^2 + O(\varphi_o^4)\right\} \tag{31}$$

for infinitesimal φ_o and finite functional W.

Applying to (30) Jensen's inequality we get an upper bound

$$V_{sc}(\varphi_o) \le V_{sc}^+(\varphi_o) = -\frac{g^2\varphi_o^2}{2\Omega}\int_\Omega d^2\mathbf{x}\int_\Omega d^2\mathbf{y}\int d\sigma\left\{N\phi^3(\mathbf{x})N\phi^3(\mathbf{y})\right\}. \tag{32}$$

It is easily shown that

$$\int d\sigma N\phi^3(\mathbf{x})N\phi^3(\mathbf{y}) = 6D^3(\mathbf{x}-\mathbf{y}). \tag{33}$$

Then, we rewrite (32) in the form

$$V_{sc}^{+}(\Phi_o) = -\frac{m^2}{8\pi}\cdot\frac{3G^2{\Phi_o}^2}{2\xi}\cdot Q\,, \tag{34}$$

$$Q = \iiint\limits_0^1 d\alpha\, d\beta\, d\gamma\, \frac{\delta(1-\alpha-\beta-\gamma)}{\alpha\beta+\alpha\gamma+\beta\gamma} = 2.3439\,. \tag{35}$$

Substituting the parameter ξ derived from (27) into (34) one gets the behavior of $V_{sc}^{+}(\Phi_o)$ for small values $\Phi_o \sim 0$. Omitting details of calculations we write the result

$$V_{sc}^{+}(\Phi_o) = -\frac{m^2}{8\pi}\cdot\left\{-\frac{3Q}{2}G^2{\Phi_o}^2 + O({\Phi_o}^4)\right\} \tag{36}$$

Finally, taking into account Eqs. (21) and (28) we obtain the following behavior of an upper bound of the EP in the region of small $\Phi_o \sim 0$:

$$V^{+}[\Phi_o] = V_o[\Phi_o] + V_{sc}^{+}[\Phi_o] = \frac{m^2}{8\pi}\cdot\left[\alpha(G)\cdot{\Phi_o}^2 + O({\Phi_o}^4)\right], \tag{37}$$

where

$$\alpha(G) = 1 - 3QG^2/2 \tag{38}$$

One can easily check that the coefficient $\alpha(G)$ in (38) becomes negative as $G > G^{+} = 0.5333$. Obtained value G^{+} is, of course, a rough estimation of the critical coupling constant because it neglect, in fact, the high order corrections generated by the "quartic" exponential $\phi^4(x)$ in $J_\Omega(\varphi_o)$. As the next neglected corrections have opposite signs to the accounted term, we would expect that contributions coming from these could increase the estimated value $G^{+} = 0.53$. We predict that the true G_{crit} would be larger than G^{+}: $G_{crit} > G^{+}$. Nevertheless, in the authors' opinion, it can indicate a possible occurrence of a second order PT in the model under consideration.

3.2 Padé approximation for $\alpha(G)$

As it has been mentioned above in Introduction, there are indications [8]-[12] that the critical region is localized near the value $G \sim 1$. In the region ${\Phi_o}^2 \ll 1$, the parameter ξ is defined by Eq. (27) and equals to

$$\xi = 1 + \frac{3G}{2+3G}\cdot{\Phi_o}^2\,. \tag{39}$$

The effective coupling constant

$$G_{eff}^{o} = \frac{G}{\xi} \to G\,. \tag{40}$$

is not small in the critical region. We have not got any "weak coupling" regime for $G \sim 1$ and the perturbation series in G_{eff}^{o} for the path integral (23)

$$J_\Omega(\Phi_o) = 1 + \Omega\cdot\sum_{n=2}^{\infty}\frac{(-1)^n}{n!}(G_{eff}^{o})^n U_n(\Phi_o) \tag{41}$$

is not convergent. The best we can do in this situation is to try summing this alternate series for the EP employing the method of Padé approximants [18].

Let us expand the path integral (23) in power series in g (or in $G_{eff}^{o} = g/2\pi m^2\xi$). Developing this series up to the fourth order and taking into account the definition (41) we obtain

$$\begin{aligned} U_2(\Phi_o) &= \frac{m^2}{8\pi}\,2!\,\xi(Q_4 + Q_3\cdot{\Phi_o}^2)\,,\\ U_3(\Phi_o) &= \frac{m^2}{8\pi}\,3!\,\xi(Q_6 + Q_5\cdot{\Phi_o}^2)\,,\\ U_4(\Phi_o) &= \frac{m^2}{8\pi}\,4!\,\xi(Q_8 + Q_7\cdot{\Phi_o}^2 + Q_9\cdot\phi_o^4)\,, \end{aligned} \tag{42}$$

where Q_i ($n = 3,\ldots 9$) are numbers expressed by some multidimensional integrals. Employing numerical calculations for them we get

$$\begin{aligned} &Q_3 = 3.52,\quad Q_5 = 9.91,\quad Q_7 = 75.1,\quad Q_9 = 6.58,\\ &Q_4 = 3.15,\quad Q_6 = 3.95,\quad Q_8 = 4.07\,. \end{aligned} \tag{43}$$

We define functions

$$V_N(\Phi_o) = V_o(\Phi_o) - \sum_{n=2}^{N}\frac{(-1)^n}{n!}(G_{eff}^{o})^n U_n(\Phi_o)\,. \tag{44}$$

which are the "N-th" order perturbation approximations for the EP.

Expanding these functions around the point $\Phi_o = 0$ we obtain

$$V_N(\Phi_o) = \frac{m^2}{8\pi}\left[E_N(G) + \alpha_N(G)\cdot{\Phi_o}^2 + O({\Phi_o}^4)\right]. \tag{45}$$

In this paper we investigate the coefficients $\alpha_N(G)$ up to $N = 4$. We write

$$\begin{aligned} \alpha_2(G) &= 1 - \lambda_2(G)\cdot G^2\,,\\ \alpha_3(G) &= 1 - \lambda_2(G)\cdot G^2 + \lambda_3(G)\cdot G^3\,,\\ \alpha_4(G) &= 1 - \lambda_2(G)\cdot G^2 + \lambda_3(G)\cdot G^3 - \lambda_4(G)\cdot G^4\,, \end{aligned} \tag{46}$$

$$\lambda_n(G) = Q_{2n-1} - (n-1)\frac{3G}{2+3G}Q_{2n}\,,\quad (n = 2,3,4)\,. \tag{47}$$

The behaviours of the functions $\alpha_N(G)$ for $N = 2,3,4$ at different values of the dimensionless coupling constant G are shown in Fig.1.

One can see that the functions $\alpha_2(G)$ and $\alpha_4(G)$ are negative at $G \sim 1$ while $\alpha_3(G)$ indicates the absence of the second order PT in this theory. Our final result is the alternate series in (46) for the function $\alpha_4(G)$ which unfortunately does not converge at $G \sim 1$. We try to summarize it using a Padé approximation.

The series $\alpha_4(G)$ in (46) consists of four terms. Then, for its summation we should employ a [3/1] scheme of Padé approximants (see [18])

$$\alpha_4(G) \sim \alpha_{Padé}(G) = \int_0^\infty dt\,\exp(-t)\frac{\mathbf{P}_3(tG)}{\mathbf{P}_1(tG)} \tag{48}$$

where $\mathbf{P}_i(tG)$ is a polynomial of "i-th" order.

We write the final expression

$$\alpha_{Padé}(G) = 1 - \lambda_2(G)G^2 + \frac{2}{3}\lambda_3^2(G)G^3 \int\limits_0^\infty \frac{dt\, t^3 \exp(-t)}{4\,\lambda_3(G) + \lambda_4(G)\,G\,t}, \tag{49}$$

The behaviour of the function $\alpha_{Padé}(G)$ at different G is shown in Fig.1. One can easily see that the function $\alpha_{Padé}(G)$ becomes negative since $G > C_{crit} = 0.9$.

4 Conclusion

We have suggested a regular scheme of calculations for the path integral defining the EP in the scalar $g\varphi_2^4$ theory. We easily get the leading term $V_o(\varphi_o)$ to the EP which coincides with the GEP. Next corrections to $V_o(\varphi_o)$ can be obtained by estimating the transformed functional integral $J_\Omega(\varphi_o)$ in (23). To study the character of PT in the theory under consideration, we have investigated the behaviour of the function $\alpha(G)$ (which is in fact the mass of the scalar particle in the new representation) in the expansion of the EP around $\phi_o = 0$. We have developed a variational approach and a perturbative series up to the fourth order of the effective coupling constant to estimate the path integral $J_\Omega(\varphi_o)$ in the region $G \sim 1$. Since the perturbation series is alternate and not convergent in the expected critical region $G \sim 1$, we have employed a Padé approximation for summarizing this series. The obtained numerical values of the dimensionless critical coupling constant $G^+ = 0.53$ (the variational lower bound) and $G_{crit} = 0.9$ (the Padé approximation's result), at which the system goes to the new vacuum under the second order PT, are in agreement with the results quoted in [8]-[12].

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Figure 1.
The behaviours of the functior $\alpha_N(G)$ for $N = 2, 3, 4$ and $\alpha_{Padé}(G)$ at different values of dimensionless coupling constant G. Functions $\alpha_2(G)$ and $\alpha_4(G)$ are (the curves numb red 2 and 3) decreasing as G increases while $\alpha_3(G)$ (the curve 3) keeps its positiveness at any G. The Padé approximant $\alpha_{Padé}(G)$ (corresponds to the curve 1) of $\alpha_4(G)$ becomes negative for $G > C_{crit} = 0.9$.