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ABSTRACT

Excitation dynamics of polariton quantum fluctuations arising in direct-gap semiconductor as a result of parametric decay of non-equilibrium polariton condensate with non-zero wave vector is studied. The predominant mechanism of polariton scattering is supposed to be exciton-exciton interaction. Steady state which corresponds to the case of dynamic equilibrium between the polariton condensate and quantum fluctuations is obtained. Distribution functions of non-condensate polaritons are localized in the resonant regions, corresponding to two-particle excitation of polaritons from the condensate. The spectrum of elementary excitations in steady state coincides with usual polariton energy with the shift proportional to initial density of polariton condensate.

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1 Introduction

It is known [1] that the coherent electromagnetic radiation resonant to the isolated exciton energy level excites in the crystal the coherent polariton wave with the wave vector $\mathbf{k}_0 \neq 0$ — the non-equilibrium polariton condensate. Different scattering processes accompanying its propagation lead to the loss of initial coherence of polariton wave, complete or partial exhaustion of condensate, excitation of polaritons characterized by some statistics with wave vector $\mathbf{k} \neq 0$ and other phenomena.

In the present paper the effect of exciton-exciton scattering processes on the properties of coherently excited polariton system are discussed. This scattering mechanism is of considerable interest due to the recent experimental investigations [2] and many interesting physical results (see, e.g., [3,4]) obtained in theoretical study of dynamic and kinetic processes in the system of interacting polaritons.

According to [4,5] exciton-exciton scattering is of significant importance in the situation when coherent polaritons are excited in a certain spectral region in which energy and momentum conservation laws allow real processes of two-quantum excitation of polaritons from the condensate. These processes lead to the instability of completely condensed state of the polariton system. The existence of this spectral region situated around the isolated exciton resonance is due to the peculiarities of the polariton dispersion law.

In [5] the energy spectrum of non-condensate polaritons, arising as the result of decay of coherent polariton wave, is studied. According to [5] in some regions of $\mathbf{k}$-space energy spectrum does not exist.

It should be noted that investigations made in [5] are based on the model formally analogous to that used by N.N. Bogoliubov in [6] to study equilibrium system of weakly non-ideal Bose-gas. In the non-equilibrium situation considered in [5], when decay of polariton condensate and excitation of non-condensate polaritons take place, this model is adequate to the real situation only at the initial stage of the condensate decay when the number of condensate polaritons is still much greater than the total number of non-condensate polaritons. This stage is essentially non-stationary while the introduction of energy spectrum implies the steady state of the system (see [7], p.46). Thus the results of [4,5] on the energy spectrum of the system based on the above-mentioned model do not match the reality.

2 Kinetic Equations

Because of the essential non-stationarity of the processes in the system the methods of the non-equilibrium mechanics must be used to describe it adequately. The derivation of equations that describe the kinetics of the polariton condensate decay and the excitation of quantum fluctuations has some specific features owing to the degeneracy in the system. As the energy $E_{\mathbf{k}_1} + E_{\mathbf{k}_2}$ and resulting wave vector $\mathbf{k}_1 + \mathbf{k}_2$ of two non-condensate polaritons can be equal to the energy $2E_{\mathbf{k}_0}$ and wave vector $2\mathbf{k}_0$ of two condensate polaritons, respectively, there is degeneracy of two-particle states. Moreover, the presence of the condensate in the system also leads to degeneracy due to its macroscopic amplitude [8]: adding to the condensate (or taking from it) a finite number of polaritons does not change the energy of the system.

The correct description of the system with degeneracy requires the introduction of abnormal distribution functions [9]

$$\Psi_{\mathbf{k}_0} = \langle \Phi_{\mathbf{k}_0} \rangle$$

and

$$F_{\mathbf{k}} = \langle \Phi_{\mathbf{k}} \Phi_{2\mathbf{k}_0 - \mathbf{k}} \rangle$$

together with the usual (normal) ones

$$N_{\mathbf{k}} = \langle \Phi^{+}_{\mathbf{k}} \Phi_{\mathbf{k}} \rangle.$$

Here $\Phi^{+}_{\mathbf{k}}(\Phi_{\mathbf{k}})$ are Bose-operators of creation (annihilation) of polariton on the lower branch with the wave vector $\mathbf{k}$. The appearance of abnormal averages and the coherent part of the polariton field indicates a breaking of the selection rules connected with the gauge invariance of the system [9,10,11]. It can take place as a result of action of external classic sources, spontaneously, or because of non-invariant initial conditions due to action of external sources in the preceeding time moments.

An earlier attempt to obtain kinetic equations for polaritons excited in semiconductors by an external classic field has been made in [12,13]. However in [12,13] the degeneracy of two-particle states was not taken into account and abnormal distribution functions $F_{\mathbf{k}}$ were not introduced. So one should expect the equations obtained in [12,13] to possess unphysical singularity.

Kinetic equations describing the spatially inhomogeneous time evolution of a system of partially coherent polaritons with taking into account the degeneracy were obtained in [14,15] using the Keldysh method [16], presented by the authors in terms of functionals. They coincide with the equations obtained by the authors in [17] using the method of non-equilibrium statistical operator [18] and do not possess unphysical singularities. The extended versions of [14,15,17] one can find in [19,20,21].

According to [15,17] kinetics of partially coherent polaritons is described in the Born approximation by the closed set of nonlinear integro-differential equations for the coherent part of polariton field $\Psi_{\mathbf{k}_0}$ and the normal

$$n_{\mathbf{k}} = N_{\mathbf{k}} - \delta_{\mathbf{k},\mathbf{k}_0} |\Psi_{\mathbf{k}_0}|^2$$

and abnormal

$$f_{\mathbf{k}} = F_{\mathbf{k}} - \Psi^2_{\mathbf{k}_0}$$

distribution functions. In the absense of quantum fluctuations described by the functions $n_{\mathbf{k}}$ and $f_{\mathbf{k}}$ the equations for them become identities, and the equation for $\Psi_{\mathbf{k}\alpha}$ ($\alpha = 1, 2$ is a number of polariton branch) coincides with that obtained in [22] for the system of coherent excitons and photons interacting with each other. In another particular case, when $\Psi_{\mathbf{k}_0} = 0$ and $f_{\mathbf{k}} = 0$ the equations obtained in [15,17] are reduced to the usual kinetic equation for the distribution function $N_{\mathbf{k}}$ (see, e.g. [23]).

The right-hand sides of equations obtained in [15,17] include terms linear in the constant of exciton-exciton interaction ν and the ones $\sim \nu^2$. Terms $\sim \nu$ correspond to the self-consistent field approximation, which neglects the higher correlation functions. This approximation is sufficient to describe the initial stage of evolution. It is shown [15,21] that exhaustion of the coherent part $\Psi_{\mathbf{k}_0}$ and excitation of quantum fluctuations $n_{\mathbf{k}_0}$ and $f_{\mathbf{k}_0}$ in the condensate mode occurs at this stage. Thus it loses partially its coherence.

There exist two regions of wave vector $\mathbf{k} \neq \mathbf{k}_0$ values. In the first one, called instability region, polariton number increases monotonously, while in the second one it is a periodic function of time with the amplitude decreasing with the increase of the distance from the instability region.

As usual, the terms $\sim \nu^2$ describe the difference between the number of processes of polariton creation in the state with wave vector $\mathbf{k}$ (per unit of time) and that of polariton annihilation. They differ from zero only if non-condensate polaritons exist in the system. They describe the evolution which is slower than that described by terms $\sim \nu$, which correspond to resonant scattering with participation of two condensate polaritons.

It should be mentioned that the self-consistent field approximation takes into account integral backward influence on the condensate by the excited non-condensate polaritons. On the other hand it describes the fastest processes in the system. So one should expect the establishment of steady state in the isolated polariton system after the time interval $\tau \sim \hbar/\nu n$ (n is the condensate initial density). Terms in the kinetic equations $\sim \nu^2$ are responsible for its further evolution.

3 Steady-State Solution

Using considerations similar to that used in [24] we have found exact steady-state solution of evolution equations, obtained in the self-consistent field approximation (see also [25]). Formally, it has the following nature. In the Heisenberg representation the self-consistent field approximation corresponds to the Hamiltonian [15,21] which has operator terms $\Phi^+_{\mathbf{k}}\Phi^+_{2\mathbf{k}_0-\mathbf{k}}$ with time-dependent coefficients

$$\xi = \Psi^2_{\mathbf{k}_0} + \Delta.$$

Here

$$\Delta = \sum_{\mathbf{k}} f_{\mathbf{k}}.$$

At the initial stage of evolution when $|\Psi_{\mathbf{k}_0}|^2 \gg |\Delta|$ these terms are responsible for arising and development of polariton condensate instability [4,5]. In the steady state obtained two terms in ξ compensate each other, so that $\xi = 0$. Thus these terms disappear from the Hamiltonian.

It is worth mentioning that in [4,5] the backward influence of non-condensate polaritons on the condensate was not taken into account and the term Δ in the expression for ξ had not appeared. So in [4,5] the very possibility to obtain steady-state solution, which is the result of mutual compensation of $\Psi^2_{\mathbf{k}_0}$ and Δ, does not exist.

From the physical point of view the results obtained mean that while condensate is exhausting and polaritons are being excited in the instability region, which is modified by the concentration-dependent corrections, the backward processes (when the scattering of two non-condensate polaritons creates two polaritons in the condensate) become more important. It results in slowing down the parametric instability, then in establishment of steady state. This state corresponds to dynamic equilibrium between the condensate and non-condensate polaritons.

The steady state is characterized by the renormalized frequency of polariton condensate

$$\hbar\omega_{\mathbf{k}_0} = \hbar\Omega_{\mathbf{k}_0} + \sqrt{2}\nu n_0$$

and renormalized energies of non-condensate polaritons

$$E_{\mathbf{k}} = \hbar\Omega_{\mathbf{k}} + 2\nu n_0$$

(here $\Omega_{\mathbf{k}_0}$ is the frequency of non-interacting polaritons). This result crucially differs from that obtained in [4,5]. Distribution functions of non-condensate polaritons are localized in the regions of **k**-space determined by the resonant condition

$$E_{\mathbf{k}} + E_{2\mathbf{k}_0-\mathbf{k}} - 2\hbar\omega_{\mathbf{k}_0} = 0.$$

Future investigations of this problem require numerical analysis of the set of integro-differential equations corresponding to self-consistent field approximation. Another important problem in our opinion is the study of polariton kinetics under the action of external stationary source.

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