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Sedov variational principle which is the generalization of the least action principle for the dissipative and irreversible processes and the classical dissipative mechanics in the phase space is considered. Quantum dynamics for the dissipative and irreversible processes is constructed. As an example of the dissipative quantum theory we consider the nonlinear two-dimensional sigma-model The conformal anomaly of the energy momentum tensor trace for closed bosonic string on the affine-metric manifold is investigated. The two-loop metric beta-function for nonlinear dissipative sigma-model was calculated. The results are compared with the ultraviolet two-loop conterterms for affine-metric sigma model.

Introduction

Vectorial Newtonian mechanics describes the motion of the mechanical systems subjected to forces. The Forces usualy are divided into potential and dissipative forces. The Newtonian approach doesn't restrict the nature of the force [1]. Variational Lagrangian and Hamiltonian mechanics describes the systems subjected to the potential forces only [1,2]. The dissipative forces are beyond the sphere of the variational principles [3-6]. For this reason the statistical mechanics does not describe the irreversible and nonequilibrium processes. It is caused by the absence of the Liapunov function [7] in the phase space in the Hamiltonian mechanics (Poincare-Misra theorem) [8-10]. To describe the dissipative and irreversible processes we must introduce the additional postulate in statistical mechanics (for example, the Bogolubov principle of weakening (relaxation) correlation [11] and the hypothesis of the relaxation time hierarchy [12]) [10,13,14]. Therefore this processes is considered in the sphere of the physical kinetics [13-15]. It is known that the initial point of the quantum mechanics formalism is Hamiltonian mechanics [16]. Therefore the quantum theory describes the physical objects in the potential force fields only. The irreversible and dissipative quantum dynamics is beyond the sphere of quantum mechanics. Sedov L.I. suggests the variational principle [3-6] which is the generalization of the least actional principle for the dissipative and irreversible processes. The holonomic and nonholonomic functionals are used to include the dissipative processes in the field of the variational principle. In this paper we consider the Sedov variational principle and the classical ($\hbar \to 0$) dissipative mechanics in the phase space. Taking into account this Hamiltonian dissipative mechanics we generalize the quantum dynamics for the dissipative and irreversible processes. In the dissipative quantum mechanics we get that the operator of the nonholonomic quantity is nonassociative and consider the properties of this operator. We obtain the dissipative analogues of the Schroedinger equations and the Feynman represantation of the Green's function. The dissipative quantum scheme suggested in this paper allows to formulate the approach to the quantum dissipative field theory. As an example of the dissipative quantum theory we consider the sigma-model approach [18] to the quantum string theory [17]. The conformal anomaly of the energy momentum tensor trace [18] for closed bosonic string on the curved affine-metric manifold (or in dissipative and nondissipative background fields) is discussed. The two-loop metric ultraviolet renormalization group beta-function [19-20] for two-dimensional non-linear dissipative bosonic sigma-model is obtained. The results are compared with the ultraviolet two-loop metric counterterms for affine-metric sigma-model suggested in the papers [21,22].

1 Sedov Variational Principle.

The equations of motions of the mechanical systems in n-dinensional configurational space are

$$D_i T(q,u,t) + Q_i = 0 \qquad (1)$$

where T is the kinetic energy, which can be written in the form

$$T(q,u,t) = \frac{1}{2}a_{ij}(q,t)u^i u^j + a_i(q,t)u^i + a_0(q,t) \qquad (2)$$

$$D_i \equiv \frac{d}{dt}\frac{\partial}{\partial u^i} - \frac{\partial}{\partial q^i}$$

$u^i \equiv dq^i/dt$ and $Q_i = Q_i(q,u,t)$ is the sum of external forces. In general case, Q_i is the sum of the potential Q_i^p and the dissipative Q_i^d forces. The potential force is the force for which a function $V = V(q,u,t)$:

$$D_i V = -Q_i^p \qquad (3)$$

exist. The dissipative force Q_i^d is the force which can not be written in the form (3). Then the Euler-Lagrange equations take the form

$$D_i L + Q_i^d = 0 \qquad (4)$$

where $L = L(q,u,t) \equiv T - V$ is Lagrangian. In the dissipative case ($Q^d \not\equiv 0$), the equation (4) can not be followed from the least actional principle [1,2]:

$$\delta S(q) \equiv \delta \int dt\ L(q,u,t) = 0 \qquad (5)$$

The basic variational principle for dissipative processes is the principle suggested by L.I.Sedov [3-6]. It is a generalization of the least action principle. The Sedov variational principle has the form:

$$\delta S(q) + \delta\tilde{W}(q) = 0 \qquad (6)$$

where S(q) is the holonomic functional called action and $\tilde{W}(q)$ is the nonholonomic functional. The nonholonomic functional is defined by the nonholonomic equation. Let a variation of the nonholonomic functional be linear in the variations δq^i and δu^i that is

$$\delta \tilde{W} = \delta \int dt\ w(q,u) = \int dt\ (w_i^1(q,v)\delta q^i + w_i^2(q,u)\delta u^i) \qquad (7)$$

where w_i^1 and w_i^2 are the vector functions in the configurational space. The classical ($\hbar \to 0$) field theories (in the continium mechanics) for the systems with $\tilde{W} \not\equiv 0$ are considered in [3-6]. Let us consider the Hamiltonian approach to the variational classical mechanics with dissipative forces.

2 Hamiltonian Dissipative Mechanics.

One direct corollary of the Sedov variational principle are the following dissipative equations of motions

$$\frac{d}{dt}(\frac{\partial L(q,u)}{\partial u^i} + w_i^2(q,u)) = \frac{\partial L(q,u)}{\partial q^i} + w_i^1(q,u) \qquad \frac{dq^i}{dt} = u^i \qquad (8)$$

Let us define a canonically conjugate momentum by the equations

$$p_i \equiv \frac{\partial L(q,u)}{\partial u^i} + w_i^2(q,u) \qquad (9)$$

and represent this relation in the form $u^i = v^i(q,p)$. The Hamiltonian is given by

$$h(q,p) = p_i v^i(q,p) - L(q,v(q,p)) \qquad (10)$$

If we consider the variation of the Hamiltonian, we obtain the dissipative Hamiltonian equations of motion

$$\frac{dq^i}{dt} = \frac{\delta(h-w)}{\delta p_i} \qquad \frac{dp_i}{dt} = -\frac{\delta(h-w)}{\delta q^i} \qquad (11)$$

where

$$\delta w(q,p) = \delta w(q,v(q,p)) = w_i^q \delta q^i + w_p^i \delta p_i \qquad (12)$$

Let the coordinates p_i, q^i, w, t of the (2n+2)-dimensional extended phase space be connected by the equations

$$\delta w - a_i(q,p,t)\delta q^i - b^i(q,p,t)\delta p_i = 0 \qquad (13)$$

where a_i , b^i are the vector functions in phase space. Let us call the dependence w on the coordinate q and momentum p the holonomic-nonholonomic function (HNF) and denote $w = w(q,p) \in \Phi$. If the vector functions satisfy the relation

$$\frac{\partial a_i(q,p)}{\partial p_j} = \frac{\partial b^j(q,p)}{\partial q^i} \qquad (14)$$

the coordinate w is the holonomic function ($w \in F$). If these vector functions don't satisfy the relation (14) the object w(q,p) we call the nonholonomic function or the Sedovian ($w \in \hat{F}$). Let us define the variational Poisson brackets (VPB) for $\forall a, b \in \Phi$ in the form:

$$[a,b] \equiv \frac{\delta a}{\delta q^i}\frac{\delta b}{\delta p_i} - \frac{\delta a}{\delta p_i}\frac{\delta b}{\delta q^i} \qquad (15)$$

The basic properties of the VPB:

$$1) \quad \forall a, b \in \Phi \qquad [a,b] = -[b,a] \in F;$$

$$2) \quad \forall a, b, c, \in \Phi: \ a \vee b \vee c \in \hat{F} \qquad AS[a,b,c,] \not\equiv 0;$$

$$3) \quad \forall a \wedge b \wedge c \in F \qquad AS[a,b,c] = 0;$$

where $AS[a,b,c] \equiv [a,[b,c]] + [b,[c,a]] + [c,[a,b]]$. It is easy to verify that this properties for the holonomic functions coincide with the properties of the usual Poisson brackets [1,2]. Let us consider now the characteristic properties of the physical quantities:

$$1) \quad [p_i, p_j] = [q^i, q^j] = 0 \qquad [q^i, p_j] = \delta^i_j$$

$$2) \quad [w, p_i] = w^q_i \qquad [w, q^i] = -w^i_p$$

$$3) \quad [[w,p_i],p_j] \not\equiv [[w,p_j],p_i] \qquad [[w,q^i],q^j] \not\equiv [[w,q^j],q^i]$$

$$4) \quad AS[q^i, w, p_j] = \Omega^i_j \not\equiv 0$$

where

$$\Omega^i_j \equiv \frac{\partial w^q_j}{\partial p_i} - \frac{\partial w^i_p}{\partial q^j} = \frac{\delta^2 w}{\delta p_i \delta q^j} - \frac{\delta^2 w}{\delta q^j \partial p_i} \qquad (16)$$

This object Ω^i_j characterizes the deviation from the condition of integrability (14) for the equation (13) and by the Stokes theorem

$$\oint_{\partial M} \delta w = \int_M \Omega^i_j \, dq^j \wedge dp_i \not\equiv 0$$

If we take into account VPB the dissipative Hamiltonian equation of motion (11) takes the form

$$\frac{dq^i}{dt} = [q^i, h - w] \qquad \frac{dp_i}{dt} = [p_i, h - w] \qquad (17)$$

The total derivative of the physical quantity $A = A(q,p) \in F$ with respect to the time t is written in the form

$$\frac{dA(q,p,t)}{dt} = \frac{\partial A(q,p,t)}{\partial t} + [A, h - w] \qquad (18)$$

Note that the equation of motion (17) can be derived from the equation (18) as a particular case. Let us consider the solution of the equation (17) in the form

$$q^i = q^i(q_0, p_0, t) \qquad p_i = p_i(q_0, p_0, t) \qquad (19)$$

Let us assume that the points of the volume $J_0 = \int \delta q_0 \delta p_0$ in the phase space are initial points at the moment $t = t_0$ [23]. Then the equations (19) transform the volume J_0 to the volume $J = \int \delta q \delta p = \int I \delta q_0 \delta p_0$, where $I = \frac{\partial(q,p)}{\partial(q_0,p_0)} = \frac{\partial q^i}{\partial q^k}\frac{\partial p_i}{\partial p_k} - \frac{\partial p_i}{\partial q^k}\frac{\partial q^i}{\partial p_k} = [q^i, p_i]_0$. The following equation is easily verified

$$\frac{dJ}{dt} = \int \delta q \delta p \, \Omega \qquad (20)$$

where $\Omega = \sum_{i=1}^n \Omega^i_i = \sum_{i=1}^n AS[q^i, w, p_i]$. The fundamental hypothesis of the statistical mechanics [10,13,23,24] is that the state at the moment t is defined by the distribution function $\rho(q,p,t)$, called density, which satisfies the normalization condition

$$\int dq dp \; \rho(q,p,t) = 1 \qquad (21)$$

The average of the physical quantity A(q,p,t) is defined [23,24] by

$$< A >_\rho = \int dq dp \; \rho(q,p,t) \; A(q,p,t) \qquad (22)$$

Using for equation (21) formula (20), we obtain the dissipative analogue of the Liouville equation [10,13]:

$$\frac{d\rho}{dt} = -\Omega\rho \qquad i\frac{\partial\rho}{\partial t} = \hat{L}\rho \qquad (23)$$

where

$$\hat{L} = i\left(\frac{\delta(h-w)}{\delta q^k}\frac{\partial}{\partial p_k} - \frac{\delta(h-w)}{\delta p_k}\frac{\partial}{\partial q^k} - \Omega \right)$$

called Liouville operator [10,13,14]. In addition to the Poincare-Misra theorem [8-10] (" The Liapunov function [7] of the point of the phase space does not exist in the Hamiltonian dynamics.") we obtain the statement: " There exists the Liapunov function of the coordinate and momentum in the dissipative Hamiltonian mechanics". Let us define the function $\eta(q,p,t) \equiv -ln\rho(q,p,t)$ and assume $\Omega > 0$. The equation (23) shows that $d\eta/dt = \Omega$. and the function η satisfies the relations $d\eta/dt > 0$. It is convenient to introduce the entropy of the distribution [10,13,25,26] defined as follows

$$s \equiv <\eta> = -\int \delta q \delta p \ \rho(q,p,t) \ ln\rho(q,p,t) \qquad (24)$$

The relation $ds/dt > 0$ is easily verified. In the general case, any function $f(q,p,t)$ which is the composite function $f(q,p,t) = g(\rho(q,p,t))$ and satisfies the relation $\Omega \ (\partial g(\rho))/(\partial\rho) < 0 \ (\forall t)$ is the Liapunov function [7], that is $(df)/(dt) > 0$.

3 Quantum Dissipative Mecanics.

Let us use the usual rule of definition of the quantum physical quantities, which have the classical analogues [16,27,28]: If we consider the operators A,B,C of the physical quantities a,b,c which satisfy the classical Poisson brackets $[a,b] = c$, then the operators must satisfy the relation: $[A,B] \equiv (AB) - (BA) = i\hbar C$. If we take into account the characteristic properties the physical quantities operators are defined by the following relations:

$$1) \quad [Q^i,Q^j] = [P_i,P_j] = 0 \qquad [Q^i,P_j] = i\hbar\delta^i_j$$

$$2) \quad [W,P_i] = i\hbar W^q_i \qquad [W,Q^i] = -i\hbar W^i_p$$

$$3) \quad [[W,P_i],P_j] \neq [[W,P_j],P_i] \qquad [[W,Q^i],Q^j] \neq [[W,Q^j],Q^i] \quad i \neq j$$

$$4) \quad [Q^i,[W,P_j]] \neq [P_j,[W,Q^i]] \quad or \quad AS[Q^i,W,P_j] = \Omega^i_j \neq 0 \qquad (25)$$

where $AS[A,B,C] = -1/(\hbar^2)$ ($[A[BC]] + [B[CA]] + [C[AB]]$) and $Q^\dagger = Q$; $P^\dagger = P$; $W^\dagger = W$; $\Omega^\dagger = \Omega$. Let us require that the usual quantum commutational rules be a part of this rules (25). To satisfy the commutation rule (25) the operators of the nonholonomic quantities must be nonassociative. It is sufficient to require that the operator W *satisfy the following conditions:*

1) left and right associativity:

$$((WA^i)B^j) = (W(A^iB^j)) \quad and \quad (A^i(B^jW)) = ((A^iB^j)W)$$

2) left-right nonassociativity:

$$((A^iW)A^j) \not\equiv (A^i(WA^j)) \quad if\ i \not\equiv j$$

$$and \quad ((A^iW)B^j) \not\equiv (A^i(WB^j)) \quad if \quad A^i \not\equiv B^i$$

where A and B are P or Q operators. The state in the quantum dissipative mechanics can be represented by the density operator $\rho(t)$ as usual [19]. This operator is defined by the usual conditions: $\rho^\dagger(t) = \rho(t)$ *and* $Sp(\rho(t)) = 1$. The average of the physical quantity A is defined by $< A >= Sp(A(t)\rho(t))$. The time variations of the operator of physical quantity $A(t) \equiv A(Q,P,t)$ and of the operator of state $\rho(t)$ are written in the form

$$\frac{dA}{dt} = \frac{\partial A}{\partial t} + \frac{i}{\hbar}[H - W, A] \qquad (26)$$

$$\frac{d\rho}{dt} = -\frac{1}{2}[\ \rho\ ,\ \Omega\ ]_+ \qquad (27)$$

where anticommutator $[\ ,\ ]_+$ is the consequence of the hermiticity for the density operator ρ and for the operator Ω. The solution of the first equation may by written in the form

$$A(t) = S(t,t_0)A(t_0)S^\dagger(t,t_0)$$

where

$$S(t,t_0) = Texp\frac{i}{\hbar}\int_{t_0}^{t} d\tau\ (H - W)(\tau) \qquad (28)$$

T-exponent is defined as usual [29-31] and we add the following flow chart $exp\ A = 1 + A + \frac{1}{2}(AA) + \frac{1}{6}((AA)A) + \frac{1}{24}(((AA)A)A) + ...$ The solution of the equation (27) has the form

$$\rho(t) = U(t,t_0)\rho(t_0)U^{\dagger}(t,t_0) \quad where \quad U(t,t_0) = Texp\frac{1}{2}\int_{t_0}^{t} d\tau\ \Omega(\tau) \qquad (29)$$

In this way the time evolution of the physical quantity operator is unitary and the evolution of the state operator is nonunitary. It is easy to verify that the pure state [28] at the moment $t = t_0$ ($\rho^2(t_0) = \rho(t_0)$) is not a pure state at the next time moment $t \neq t_0$. We can define the entropy of the state [32,33,10,25] $s = - < ln\rho >= - Sp(\rho\ ln\rho)$. If $Sp(\Omega\rho) > 0$ the entropy satisfies the equation $(ds)/(dt) > 0$. In the general case the time evolution equation for the entropy has the form

$$\frac{d}{dt}s = < \Omega > \qquad (30)$$

Let us define the canonical (unitary) transformation [16,27] of an operator $A_H(t) = A(t)$ in the form

$$A_S(t,t_0) = S^{\dagger}(t,t_0)A_H(t)S(t,t_0)$$

The operator $A_S(t,t_0)$ satisfies the condition $A_S(t_0,t_0) = A_H(t_0)$. In this case the equations (26), (27) take the form

$$\frac{d}{dt}A_S(t,t_0) = S^{\dagger}(t,t_0)\frac{\partial A_H(t)}{\partial t}S(t,t_0) \qquad (31)$$

$$\frac{d}{dt}\rho_S(t,t_0) = \frac{i}{\hbar}[\rho_S,\ (H-W)_S] - \frac{1}{2}[\Omega_S,\ \rho_S]_+ \qquad (32)$$

This is the dissipative analogue of the Schroedinger equations, the operators $A_H(t)$ and $A_S(t)$ called Heisenberg and Schroedinger representations accordingly [16,27]. The solution of the equation (32) has the form

$$\rho_S(t,t_0) = U_S^{\dagger}(t,t')\rho_S(t',t_O)U_S(t,t')$$

where

$$U_S^{\dagger}(t,t') = Texp\frac{-i}{\hbar}\int_{t'}^{t} d\tau\ (H - W - \frac{i\hbar}{2}\Omega)_S(\tau,t_0) \qquad (33)$$

Let us consider some important features of the basis vectors [16]. Account is to be taken of the time dependence of the state operator $\rho_H(t) = \sum_\alpha \rho_\alpha |\psi_\alpha, t>_H < \psi_\alpha, t|_H$ and of the wave vectors in the Heisenberg representation $|\psi, t>_H$. That is $|q,t_1>_H \not\equiv |q,t_2>_H$ contrary to usual quantum mechanics. Let us define the basis vectors $\{|q,t>\}$ [16] at the fixed time point $t = t_f$:

$$1)\quad Q_H(t)|q,t>_H = |q,t>_H q_f \qquad 2)\quad <q,t|_H|q',t>_H = \delta(q-q')$$

$$3)\quad \int dq\ |q,t>_H <q,t|_H = 1 \qquad 4)\quad Q_H(t) = \int dq\ |q,t>_H q_f <q,t|_H$$

$$5)\quad |\psi,t>_H = \int dq\ |q,t_f>_H\ \Psi_H(q,t,t_f)$$

where $\Psi_H(q,t,t_f) = <q,t_f|_H|\psi,t>_H$. It is easy to prove the following statements: " 1. The basis vector unitary transformed is a basis vector ; 2. There exists a unitary transformation for any two basis vectors defined at the non equal time points." Thus, Schroedinger representation of the basis vector $|q\ t,t_0>_S \equiv S^\dagger(t-t_0)|q,t>_H$ might be considered as the unitary transformation of the basis vector $|q,t_0>_H = S^\dagger(t-t_0)|q,t>_H = |q,t,t_0>$. Let us consider now Green's functions and its Feynman representation [34-36]. If we take into account the equation (27) we can write the dissipative Schroedinger equation for wave vector in the the form

$$i\hbar\frac{d}{dt}|\psi,t,t_0>_S = (H - W - \frac{i\hbar}{2}\Omega)_S(t,t_0)|\psi,t,t_0>_S$$

The simple example of this equation for the harmonic oscillator with friction is considered in Appendix 2. Account is to be taken of the time dependence of state in Heisenberg representation. Therefore we make the distinctions between following Green's functions

$$\Psi_S(q,t) = \int dq'\ G(q,q',t-t')\ \Psi_S(q',t')$$

$$\Psi_H(q,t) = \int dq'\ G_H(q,q',t-t')\ \Psi_H(q',t')$$

where

$$\Psi_S(q,t) \equiv <q,t|_S|\psi,t>_S \equiv <q,t|_H|q,t>_H$$

$$\Psi_H(q,t) \equiv <q,t|_H|\psi,t>_S \equiv <q,t|_S|\psi,t>_H$$

$$G_S(q,q',t-t') \equiv <q,t|_S\ U_S^\dagger(t,t')\ |q',t'>_S\ \theta(t-t') \equiv$$

$$\equiv <q,t|_H\ U_H^\dagger(t,t')\ |q',t'>_H\ \theta(t-t') \qquad (34)$$

$$G_H(q.q',t-t') \equiv <q,t|_S\ U_H^\dagger(t-t')\ |q',t'>_S\ \theta(t-t') \qquad (35)$$

and $|q,t>_H \equiv |q,t=t_{fixed}>_H$

The Green's function satisfies the time-dependent equation

$$i\frac{d}{dt}G_S(q,q',t) = (H - W - \frac{i\hbar}{2}\Omega)_S G_S(q,q',t) \; and \; G_S(q,q',0) = \delta(q-q')$$

Let us use the Fadeev's method [36] and the conditions

$$< p,t_f]_H(H - W - \frac{i\hbar}{2}\Omega)_H[q,t_f >= (h - w - \frac{i\hbar}{2}\Omega_{cl})(q_f,p_f) < p,t_f]_H[q,t_f >_H$$

$$< q^{n+1},t_n]_S U_S^{\dagger}(t_{n+1}-t_n)[q^n,t_n >_S \simeq$$

$$\simeq < q^{n+1},t_n]_H exp\frac{-i(t_{n+1}-t_n)}{\hbar}(H - W - \frac{i\hbar}{2}\Omega)_H(t_n)[q^n,t_n >_H$$

The Feynman representation of the Green's functions has the form

$$G_S(q,q',t-t') \approx$$

$$= \int Dq \; Dp \; exp\frac{i}{\hbar}\int_{t'}^{t} d\tau \; (p\frac{dq}{d\tau} - h(q,p,\tau) + w(q,p,\tau) + \frac{i\hbar}{2}\Omega_{cl}) \quad (36)$$

In the same way we can formulate the path integration and generating functional in the quantum field theory [29,31,36-38].

4 Two-Dimensional Non-Linear Dissipative Sigma Model.

Let us consider now the closed bosonic string theory [17] in curved space [39-41] or more exactly the nonlinear two-dimensional sigma-model [19,20,42,43] and the sigma-model approach to the string theory [40,46]. The world sheet swept out by the string is described by map X(x) from two-dimensional parametr space N into space-time manifold M, i.e., $X(x) : N \to M$. The two-dimensional parametr are $x = (\tau,\sigma)$ and the map X(x) is given by space-time coordinates $X^k(x)$. Let us choose the holonomic and nonholonomic functionals in the form

$$S(X) = S(G,\Phi,g) = \frac{1}{2}\int d^2x \; G_{kl}(X)\partial_\mu X^k \sqrt{g}g^{\mu\nu}\partial_\nu X^l +$$

$$+ \frac{\alpha'}{2}\int d^2x \; \sqrt{g}R^{(2)}(g)\Phi(X) \qquad (37)$$

$$\delta\hat{W} = -\int d^2x\ D_{ikl}(X)\partial_\mu X^k\sqrt{g}g^{\mu\nu}\partial_\nu X^l\delta X^i \qquad (38)$$

where $g^{\mu\nu}(x)$ is the two-dimensional metric tensor; α' is the inverse of the string tension; $\Phi(X)$ is the dilaton field. The Lagrangian and Sedovian define a closed bosonic string propagating in the presence of dissipative and nondissipative background fields or in curved affine-metric space-time. The classical equation of motion for this model is an equation of the geodesic two-dimensional flow on the affine-metric manifold (the two-dimensional analogue of the geodesic line), where $D_{ikl}(X)$ is a connection defect tensor [47,48]. Let us choose a parametrization for two-dimensional metric tensor $g^{\mu\nu}$ in the form [49]

$$g_{\mu\nu}(x)dx^\mu dx^\nu = c(x)(n^2(x)(d\tau)^2 - (d\sigma + m(x)d\tau)^2) \qquad (39)$$

In this case the densities of the Hamiltonian without the dilaton field term, Sedovian and Omega are rewritten in the form

$$h = -\frac{n}{2}G^{kl}(X)\Pi_k\Pi_l + m\Pi_k X'^k - \frac{n}{2}G_{kl}(X)X'^k X'^l \qquad (40)$$

$$w = \frac{n}{2}(\Delta_1^{kl}\Pi_k\Pi_l + \Delta_{kl}^2 X'^k X'^l)\ ; \quad \Omega = 2nD^k(X)\Pi_k \qquad (41)$$

where $D^k(X) \equiv D^k_{ij}(X)G^{ij}(X)$; $X'^k \equiv (dX^k)/(d\sigma)$; Π_k is the canonical momentum ; Δ are tensorial integral operators, which can be written in the conditional form of indefinite multiple integral δX^k :

$$\Delta_1^{kl} = 2\int \delta X^i D_i^{kl}(X) \quad \Delta_{kl}^2 = -2\int \delta X^i D_{ikl}(X) \qquad (42)$$

Unfortunately we have no correct mathematical definition of this operators. This difficulty can be removed by expressing the nonholonomic functional as a power series in a covariant field $\xi^k(x)$ which is the tangent vector to the geodesic line containing X_0^k and $X^k = X_0^k + f^k(X_0, \xi)$. The background field expansions of the Δ - operators are written in the form

$$\Delta_1^{kl} = 2D_i^{kl}(X_0)\xi^i + O(\xi^2) \quad \Delta_{kl}^2 = -2D_{ikl}(X_0)\xi^i + O(\xi^2) \qquad (43)$$

The covariant background field method [48,50,51] in the phase space is defined by the usual expansion of the coordinates $X^k(x)$ only. Note that the model defined by (37) (and (38)) in the conformal gauge $n = 1, m = 0$ called two-dimensional nonlinear (dissipative) sigma-model. Let us define the generating functional for connected Green functions [29,31] in the form

$$W(J,g) = -i\hbar \; ln \int DX D\Pi \; exp\frac{i}{\hbar} \int d^2x \; (Z_1(X,\Pi,g) + Z_2(X,J)) \qquad (44)$$

where

$$Z_1(X,\Pi,g) \equiv \Pi_k \frac{d}{d\tau} X^k - h + w + \frac{i\hbar}{2}\Omega + \frac{\alpha'}{2}\sqrt{g}R^{(2)}\Phi(X) \qquad (45)$$

and $Z_2(X,J)$ is the source term discussed in [53-60]. We derive the covariant background field expansion of Z_1, Z_2 and define a new generating functional $W(X_0,g,J)$ by

$$exp\frac{i}{\hbar}(\; W(X_0,g,J) + \tilde{W}(X_0) \;) =$$

$$= \int D\xi D\Pi \; exp\frac{i}{\hbar} \int d^2x(\; Z_1(X(X_0,\xi),\Pi,g) + J_k(x)\xi^k(x) \;).$$

The functional integral over momentum Π is Gaussian integral. It is easy to derive the path integral for the generating functional:

$$W(X_0,g,J) = -i\hbar \; ln \int D\xi \; exp\frac{i}{\hbar}(\; A(X(X_0,\xi)) \; +$$

$$+ \int d^2x \frac{i\hbar}{2}\delta(0) ln det(G^{-1} + \Delta_1)^{-1} \;) \qquad (46)$$

where

$$A(X) = -\frac{1}{2n}[G^{-1} + \Delta_1]^{-1}{}_{kl}(\dot{X}^k - mX'^k + inD^k(X))(\dot{X}^l - mX'^l + inD^l(X)) +$$

$$+ \frac{n}{2} \; [\; G + \Delta^2]_{kl} X'^k X'^l \qquad (47)$$

and $\dot{X}^k = (dX^k)/(d\tau)$ and $D^l(X) = G^{lk}G^{ij}D_{kij}(X)$. The effective action $A(X)$ is rewritten in the form $A(X) = S(G,\Phi,g) + S(D,g)$, where

$$S(D,g) = S_1(D,g) + S_2(D,g) + S_3(D,g)$$

$$S_1 = -\int d^2x \; \frac{1}{2}\Delta^2_{kl}\partial_\mu X^k \sqrt{g} g^{\mu\nu}\partial_\nu X^l \; = -\tilde{W}(X) \qquad (48)$$

$$S_2 = \int d^2x \; \frac{1}{2} f_{kl}(X)\partial_\mu X^k \sqrt{g}\kappa^{\mu\nu}\partial_\nu X^l \quad (49)$$

$$S_3 = \int d^2x\sqrt{g}(V_{k\mu}g^{\mu\nu}\partial_\nu X^k + B(X))$$

$$f_{kl} = [G^{-1} + \Delta_1]^{-1}{}_{kl} - [G + \Delta^2]_{kl} = 4D_i{}^n{}_k D_{jnl}\xi^i\xi^j + O(\xi^3) \qquad (50)$$

$$V_{k\mu} \equiv \frac{1}{2}g_{\mu\nu}k^\nu[G^{-1} + \Delta^1]^{-1}{}_{kl}D^l(X)$$

$$B(X) \equiv \frac{1}{2}c^{-1}(x)[G^{-1} + \Delta^1]^{-1}_{kl}D^k(X)D^l(X) \qquad (51)$$

$$k^\mu = (k^\tau, k^\sigma) = (-2ic^{-1}, 2imc^{-1})$$

$$\kappa^{\mu\nu} = (\kappa^{\tau\tau}, \kappa^{\tau\sigma}, \kappa^{\sigma\sigma}) = (-n^{-2}c^{-1}, mn^{-2}c^{-1}, -m^2n^{-2}c^{-1}) \quad (52)$$

Note that the effective action is conformally invariant, because has no $c(x)$ dependence . Account is to be taken of the parametrization of the two-dimensional tensors $\kappa^{\mu\nu}$ and k^μ are connected with the parametrization of two-dimensional metric tensor $g^{\mu\nu}$, i.e. $\kappa = \kappa(g)$ and $k = k(g)$. The energy-momentum tensor is defined as usually [18,61] by

$$T^{\mu\nu}(x) = -\frac{2}{\sqrt{g}}\frac{\delta}{\delta g_{\mu\nu}}S(G, \Phi, g) \qquad (53)$$

In the general case , the objects in $S(D, g)$ are the nonholonomic objects. We use the covariant background field method [21,22]. Therefore this object is the power series in quantum fields $\xi^k(x)$. We note that the background field expansion of the nonholonomic functionals and the two-dimensional metric variation of it are not commutative operations, i.e. $(\delta\tilde{W}(X))/(\delta g^{\mu\nu}) = 0$ and $(\delta\tilde{W}(X_0, \xi))/(\delta g^{\mu\nu}) \not\equiv 0$. Therefore the vacuum expactation value of the energy-momentum tensor [61]

$$< T^{\mu\nu} > \equiv N exp(\frac{-i}{\hbar}W(J)) \int D\xi \; T^{\mu\nu}(x) \; exp\frac{i}{\hbar}A(X) \qquad (54)$$

can not be written in the form $-\frac{2}{\sqrt{g}}(\delta W(J))/(\delta g_{\mu\nu})$ because we consider the nonholonomic functionals as the background field power series. (In the opposite case, we must prove the Gaussian momentum integration formulas for the tensorial indefinite integral operator Δ in functional integral.) Let us define the two-metrical generating functional in the form

$$W(g^{\mu\nu}, a^{\mu\nu}, X_0, J) \equiv$$

$$\equiv -i\hbar \; ln \int D\xi exp\frac{i}{\hbar}(S(G, \Phi, g^{\mu\nu}) + S(D, a^{\mu\nu}) + \int d^2x J_k(x)\xi^k(x))$$

The usual functional $W(g, X_0)$ is derived by $W(g, X_0) = W(g, a, X_0)_{j=a}$. The vacuum expectation value (54) can be written in the form

$$< T_{\mu\nu} > = [-\frac{2}{\sqrt{g}}\frac{\delta}{\delta g^{\mu\nu}} W(g, a, X_0)]_{g=a} \quad (55)$$

It is easy to derive the conformal anomaly of the trace of the energy - momentum tensor [18] in the form

$$< T^{\mu}_{\mu} > = \frac{1}{2}\hat{\beta}^{G}_{kl}\, \partial^{\mu} X^{k}_{0}\partial_{\mu} X^{l}_{0} + \frac{\alpha'}{2} R^{(2)}(g)\hat{\beta}^{\Phi} \qquad (56)$$

where

$$\hat{\beta}^{G}_{kl} = \bar{\beta}^{G}_{kl} + ... ; \quad \hat{\beta}^{\Phi} = \bar{\beta}^{\Phi} - \frac{1}{4}\bar{\beta}^{G}_{ij}(G^{ij} + ...)$$

$$\bar{\beta}^{G}_{kl} = \beta^{G}_{kl} + 2\alpha'\hat{\nabla}_{k}\hat{\nabla}_{l}\Phi ; \quad \bar{\beta}^{\Phi} = \beta^{\Phi} + \alpha'\hat{\nabla}_{k}\Phi\hat{\nabla}^{k}\Phi$$

$$\beta^{G}_{kl} = \mu\frac{d}{d\mu}G^{Renorm}_{kl} ; \ \hat{\nabla}_{k}V_{l} \equiv \partial_{k}V_{l} + (\{^{i}_{kl}\} + D^{i}_{(kl)})V_{i} \qquad (57)$$

The law of the change for the energy momentum tensor has the form

$$\nabla^{\mu}T_{\mu\nu} = -\frac{1}{\sqrt{g}}\frac{\delta S(G,\Phi,X)}{\delta X^{k}}\partial_{\nu}X^{k} \quad (58)$$

If we take into account the background field expansion [21,22,50] of $\partial_{\mu}X^{k} = C^{k}_{l}(X_0,\xi)\partial_{\mu}X^{l}_{0}$ the vacuum expectation value of this law can be written in the form

$$< \nabla^{\mu}T_{\mu\nu} > = < \frac{\delta S(X_0,\xi)}{\delta X^{k}_{0}} > \partial_{\nu}X^{k}_{0} \qquad (59)$$

Let us choose the following solution of the classical equation of motion $X^{k}_{0}(x) = const$. The equation (59) is rewritten in the usual form [40] $< \nabla^{\mu}T_{\mu\nu} >= 0$. It is *easy to derive that the central charge of the Virasoro algebra [62] is proportional* to the dilaton β -function as usualy [46]. The sufficient condition for the validity of this relation [40] is $\beta^{\Phi} = const$ and $\beta^{G}_{kl} = 0$. Where β^{G} is the metric beta -function of the two-dimensional nonlinear dissipative sigma-model. In the two-loop metric beta-function calculation we use affine-metric background field method [42,44], introduce an auxilliary mass term [44,66], the dimensional regularization $2 \to n = 2 - 2\epsilon$ [63-65] and the minimal subtraction with the general prescription for contraction for the two-dimensional $\kappa^{\mu\nu}$ tensor $\kappa^{\mu\nu}\eta_{\mu\nu} = z(n)$ where $z(n) =$

$1+z_1\epsilon+O(\epsilon^2)$ and η_{μ_ν} is two-dimensional Minkowski metric. Different prescriptions may correspond to different renormalization shemes and thus their results should be related through redefenition of the couplings G_{kl} and f_{kl} by analogy to Riemannian two-dimensional non-linear sigma-model with the Wess-Zumino term [71]. It is known that propagator of the quantum fields $\xi^k(x)$ is not standard. Therefore we introduce an m-bein $e^a_k(X)$, where m is dimension of the manifold M and define $\xi^a(x) = e^a_k\xi^k(x)$, where $\hat{\nabla}_k e^a_l = 0$. After this modification the kinetic terms become $\hat{\nabla}_\mu\xi^a\hat{\nabla}_\nu\xi^a$, where

$$\hat{\nabla}_\mu\xi^a = \partial_\mu\xi^a + \hat{\Lambda}^a_{bc}e^b_k\,\partial_\mu X^k_0\,\xi^c$$

This covariant derivative for the affine-metric manifold involves the Schouten-Vranceanu connection [67-69] $\Lambda_{abc} = \hat{\Lambda}_{abc} + 2Q_{(ij)l}\,e^i_a e^j_c e^l_b$, (where Q^k_{ij} is the torsion tensor of the affine-metric manifold) which is Ricci rotation coefficient on the Riemannian manifold [70]. Note in addition to [42,44] we take into account the diagrams whose external background field lines involve the Schouten-Vranceanu conection. This diagrams must not cancel [45] in contrary to the usual non-linear sigma-model [50]. It caused by the relation $\Lambda_{(a/b/c)} = K_{ijl}\,e^i_a e^j_c e^l_b$, where K_{ijl} is nonmetricity tensor of affine-metric manifold. The two-loop metric beta-function of the dissipative sigma-model is $\beta^G = \beta^G_{AM} + \beta^G_1 + \beta^G_2$, where β^G_{AM} is the metric beta-function [19,20,50] of the affine-metric sigma-model defined in [21,22] , i.e. the part of the metric beta-function from the action $S(G,\Phi,g)$ only, where two-dimensional metric is the Minkowski metric; β^G_K $K = 1,2$ is the part of metric beta-function from $S_K(D,g)$ in equations (48), (49). The full expression of the two-loop ultraviolet counterterms is very complicated , but it is easy to see the following ultraviolet finiteness conditions. The one and two loop counterterms for two-dimensional non-linear dissipative sigma-model vanish if the correlation between the affine connection and the metric structures on the manifold N is given by *the ultraviolet finiteness* condition [45]:

$$\hat{\nabla}_k G_{ij} = N_{ijk} = N_{(ijk)}\ ; \quad \hat{\nabla}_{(l}N_{k)ij} = N^p_{i(k}N_{l)jp}\ ;$$

$$\hat{R}_{(k/(ij)/l)} = \frac{3}{4}N^p_{(k/(i}N_{j)/l)p} \qquad (60)$$

It is easy to see that the ultraviolet finiteness conditions have not the z_1 dependence. Note that the affine-metric beta-function is zero in all loops if the affine-metric manifold with the nonmetrisity tensor K_{ijl} and torsion tensor Q^i_{kl} is defined [45] by

$$\hat{R}_{kijl} = R_{kijl} - 2\hat{\nabla}_{[j/}Q_{ki/l]} - 2Q^n_{i[l/}Q_{kn/j]} = 0 \qquad (61)$$

$$\hat{\nabla}_k G_{ij} = K_{ijk} - 2Q_{(ij)k} = 0 \qquad (62)$$

It is easy to see that this affine-metric manifold is not flat space.

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Appendix 1

The equation of the geodesic line in the affine-metric manifold [48,21,22] has the form

$$\frac{d}{dt}u^i + (\left[{}^{i}_{kl}\right] + D^i_{kl})u^k u^l = 0$$

where $\left[{}^{i}_{kl}\right]$ is Christoffel symbol for the metric $a_{ij}(q)$. It is well known that this equation can not be derived from the least action principle when the connection defect is other then null. Note the Riemannian geodesic line can be derived from this variational principle if the Lagrangian defined by $L(q,u,t) = T(q,u) = \frac{1}{2}a_{ij}(q)u^i u^j$. Let us choose Sedovian in the form $\delta w(q,u) = -a_{in}(q)\ D^n_{kl}(q)\ u^k u^l\ \delta q^i$. Then the geodesic line equation can be derived from the Sedov's variational principle [45].

Appendix 2

Let us choose Hamiltonian and Sedovian in the form

$$h = \frac{p^2}{2m} + \frac{m\omega^2 q^2}{2} \ ; \quad \delta w = \gamma m p\ \delta q$$

This objects define the one-dimensional harmonic oscillator with friction. Let us use the background field method and expand the objects in Taylor series of $Q = q - q_0$, where q_0 is the solution of the classical equation of motion in the coordinate space. Let us choose $q_0 = 0$. In this case the dissipative Schroedinger equation takes the form

$$i\hbar\frac{d}{dt}\Psi(t) = [-\frac{\hbar}{2m}\frac{\partial^2}{\partial Q^2} +$$

$$+ i\hbar\gamma\ Q\frac{\partial}{\partial Q} \ + \ \frac{m\omega^2}{2}Q^2 - \frac{i}{2}\gamma\ ]\ \Psi(t)$$

The stationary state $\Psi(\xi,t) = u(\xi)exp - \frac{i}{\hbar}Et$ is defined by the equation

$$u''(\xi) - a\xi u'(\xi) + (\epsilon - \xi^2)u(\xi) = 0$$

where

$$a = \frac{2i\gamma}{\omega}\ ;\ \xi = \sqrt{\frac{m\omega}{\hbar}}Q\ ;\ \varepsilon = \frac{2}{\hbar\omega}(E - \frac{i}{2}\gamma)$$

Let us consider the function $u(\xi)$ in the form

$$u(\xi) = (\sum_{k=0}^{n} A_k\xi^k)exp - \frac{1}{2}s\xi^2$$

where s is the solution of the equation $s^2 + as - 1 = 0$ and $n < \infty$. As a result we obtain the following eigenvalues $E_n = \hbar\sqrt{\omega^2 - \gamma^2}(n + \frac{1}{2}) - i\gamma$ when $0 < \gamma^2/\omega^2 < \frac{1}{2}$ and the continuous spectrum when $\gamma^2/\omega^2 > \frac{1}{2}$. Note that the life time for the state is $T = \frac{\hbar}{2\gamma} < \infty$. We can rewrite the result in the form

$$\Delta E_n(\omega) = (\hbar\sqrt{\omega^2 - \gamma^2}\ when\ \omega^2 > 2\gamma^2) \bigwedge (0\ when\ \omega^2 < 2\gamma^2)$$

Note that the jump in the point $\omega = \sqrt{2}\gamma$ is the purely quantum dissipative effect.

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ДИССИПАТИВНАЯ КВАНТОВАЯ ДИНАМИКА И НЕЛИНЕЙНАЯ СИГМА-МОДЕЛЬ

Рассматриваются вариационный принцип Седова обобщающий принцип наименьшего действия на диссипативные и необратимые процессы и классическая диссипативная механика в фазовом постанстве. Строится квантовая динамика для диссипативных и необатимых процессов. В качестве примера диссипативной квантовой теории рассматривается двумерная нелинейная сигма-модель. Исследуется конформная аномалия тензора энергии импульса замкнутых бозонных струн на аффинно-метрическом многообразии. Вычислена двухпетлевая метрическая бета-функция нелинейной диссипативной сигма-модели. Результаты сравниваются с ультрафиолетовыми двухпетлевыми контрчленами аффинно-метрической сигма-модели.

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