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APPARENT UNAMBIGUOUSNESS OF RELATIVISTIC TIME DILATION

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INTRODUCTION

At the present time two transformation formulae of longitudinal sizes of relativistic objects are considered. They correspond to two forms of relativity theory (see, e.g., [1]). The generally adopted Lorentz contraction formula corresponds to the instant form which leans upon the notion of instant distance (and the Einsteinian definition of moving rod length). The other normal form of relativity operates with light or retarded distances (and it is based on the radar method of distance measurement). Relativistic or radar length [2] introduced on the basis of this method increases with increasing velocity according to "the elongation formula". The question like this: "Are the longitudinal sizes of moving material objects in fact contracted or elongated?" - does not appear for time. The more so the experiments on measuring the lifetime (period) of elementary particles testify simply in favour of time dilation. However, as noted at one time [3], it is in fact supposed when deriving the time dilation formula that an object being a clock or an elementary particle has vanishingly small sizes, i.e. it is a point one. Taking into account finiteness of space sizes, the deviation from the conventional formula is possible.

However the most striking example here is undoubtedly the Doppler effect. In this case the element of a light wave (its wave length) can be considered as the simplest lengthy object. The behaviour of the corresponding time interval, wave period, presents various forms of relativistic time transformation depending on observation conditions.

Therefore despite a widespread opinion we have a finite analog with the behaviour of apparent sizes of relativistic objects (see, e.g., [4]) and a full analogy with the dependence of retarded distances on the observation conditions (in particular, on the "retardation factor"). Usual relativistic time dilation would be treated as the average of two observable quantities.

OBSERVABLE WAVE PERIOD FROM A MOVING SOURCE

Using the transformation law of the wave 4-vector k^i, it is easy to consider the Doppler effect. Here we present it as the change of period T of a wave emitted by a source moving relative to an observer (in the S-system) in comparison with proper period T* of the same source in the S*-system where it rests.

Let v be the source velocity, i.e. the velocity of the S*-system relative to S. According to general transformations of 4-vectors we have

$$k^0_* = (k^0 - \vec{\beta}\vec{k})\gamma = k^0(1 - \vec{\beta}\vec{n}_r)\gamma, \tag{1}$$

where $\beta = v/c$, $\gamma = (1-\beta^2)^{-1/2}$. Substituting $k^0 = 2\pi/cT$, $k^1 = k\cos\theta = 2\pi\cos\theta/cT$ here, where the θ is the angle between the direction of emission and the direction of source motion, and expressing T through T*, we obtain

$$T = T^* (1 - \beta\cos\theta)\gamma. \tag{2}$$

It is evident that at different angles of observation* we have different values for the wave period**. In so doing, the known relativistic time dilation formula takes place only for $\theta = \pi/2$, i.e. in case of the transversal or relativistic Doppler effect. At the same time, for example,

$$T = T^* \text{ for } \cos\theta_1 = (1 - \gamma^{-1})\beta^{-1}, \tag{3}$$

i.e. we have the "classical" result, and

$$T = T^* \gamma^{-1} \text{ for } \cos\theta_2 = \beta, \tag{4}$$

i.e. we have relativistic time contraction! On the other hand, for two limiting values $\theta = 0$ and $\theta = \pi$ we obtain

$$T_B = (1 - \beta)T^*\gamma \quad \text{and} \quad T_R = (1 + \beta)T^*\gamma, \tag{5a, b}$$

respectively. From here it follows that the known quantity of relativistic time dilation T_r should be also presented as an average of quantities T_B and T_R

$$T_r = \frac{1}{2}(T_B + T_R). \tag{6}$$

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* Or depending on the "retardation factor" $(1 - \vec{\beta}\vec{n}_r)$, where $\vec{n}_r = \vec{R}_{ret}/R_{ret}$ and R_{ret} is the retarded distance.

** In this connection see also [5].

Here T_B corresponds to the case when the light is emitted in the direction of source notion and T_R in the opposite direction.

On the other hand, introduced formula (6) should be considered as a modification (generalization) of time definition for moving lengthy objects. As is seen, it resembles in a surprising image the formula figuring in the definition of relativistic length [2], in particular if one transits to wave lengths by multiplying both sides of (6) by light velocity c.

RELATIVISTIC DOPPLER EFFECT

As a result of the indicated transition, we have

$$\lambda_r = \frac{1}{2} (\lambda_B + \lambda_R). \quad (7)$$

Using (5), it is easy to obtain the transformation formula for λ_r. It represents the "elongation formula" for wave length

$$\lambda_r = \lambda^* \gamma, \quad (8)$$

i.e. it predicts a red displacement of spectral lines. But that just this quantity is measured in experiments on studying the transversal or relativistic Doppler effect [6-8] is the most important. The first of such experiments [6] having confirmed formula (8), i.e. the displacement of spectral lines by the value

$$\delta\lambda_r = \lambda_r - \lambda^* \simeq \frac{1}{2} \beta^2 \lambda^*, \quad (9)$$

was carried out by Ives and Stilwell in 1938. It should be noted that Einstein attached much importance to experiments of this type [9]. According to Sommerfeld [10]: "Einstein saw in expected red displacement experimentum crucis for relativity theory".

CONCLUSION

Relativity theory has established equality of space and time coordinates having joined them in unified space-time contiuum (Minkowski's space).

The conducted consideration allowed one to remove a kind of "asymmetry" between the notions of length and time for moving objects. On the other hand it was shown that "ambiguity" similar to the existence of two transformation formulae for longitudinal sizes takes place also for time. On the other hand, we came in fact to the generalization of time notion to relativistic lengthy objects.

Since Nature deals with light or retarded, i.e. "nonsimultaneous", distances, a finite time interval relates to them. Just so some space size (e.g., object size) is connected with time duration in the general case. Therefore a deep analogy between the definition of relativistic length and the considered generalization of relativistic time is revealed here. As a result, the mathematical equivalence of space and time coordinates disseminates to corresponding physics notions.

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Стрельцов В.Н. Д2-92-114
Кажущаяся однозначность
релятивистского замедления времени

Указывается на аналогию между поведением запаздывающего расстояния и периода волны, испускаемой движущимся источником, в зависимости от условий наблюдения ("фактора запаздывания"). Отмечается, что определение времени для движущихся протяженных объектов, приводящее к релятивистскому замедлению, соответствует определению релятивистской (локационной) длины, приводящему к "формуле удлинения".

Работа выполнена в Лаборатории высоких энергий ОИЯИ.

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Streltsov V.N. D2-92-114
Apparent Unambiguousness of Relativistic Time Dilation

It is indicated on the definite analogy between the dependence of visible sizes of relativistic objects and period of the wave, emitted by the moving source from the observation conditions ("retardation factor"). It is noted that the definition of time for moving extended objects, led to relativistic dilation, corresponds to the definition of the relativistic (radar) length led to the "elongation formula".

The investigation has been performed at the Laboratory of High Energies, JINR.

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