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ABSTRACT

Interband optical absorption in the Wannier-Stark ladder in the presence of the electron-LO-phonon resonance is investigated theoretically. The electron-LO-phonon resonance occurs when the energy spacing between adjacent Stark-ladder levels coincides with the LO-phonon energy. We propose a model describing the polaron effect in a superlattice. Calculations show that the absorption line shape is strongly modified due to the polaron effect under the electron-LO-phonon resonance condition. We consider optical phenomena in a normal magnetic field that leads to enhancement of polaron effects.

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Optical properties of semiconductor superlattices under electric fields applied along the growth axis have attracted considerable interest in recent years [1,2]. One important topic is related to the effect of the electron-phonon interaction on electron states of the Wannier-Stark ladder (WSL) in the case when the energy splitting $eFd$ ($F$ and $d$ are an electric field and the period of a superlattice) between levels of adjacent WSL quantum states becomes equal to the phonon energy $\omega_{LO}$. This problem was considered in the context of electronic transport phenomena [3]. Besides, the electron-LO-phonon resonance condition $\Delta\varepsilon = \omega_{LO}$ ($\Delta\varepsilon$ is the spacing between energies of different quantum states of a superlattice) can lead to an enhancement of LO-phonon Raman scattering observed experimentally in strongly coupled superlattices GaAs-AlAs in a normal electric field [4,5] and in multiple quantum wells [6].

In the present paper we consider interband optical absorption in the WSL theoretically. Under the condition $eFd = \omega_{LO}$ states of the conduction band are polarons because the electron-LO-phonon resonance causes the mixing between electronic WSL states and phonons. In this case polarons play the role of finish states for optical absorption which contains information about the polaron spectrum. We study this problem in the presence of a magnetic field normal to the layers that results in an enhancement of polaron effects due to singularities of the density of electron states in a magnetic field [7].

We shall consider the electron-phonon coupling in a...
tunneling-transparent GaAs-AlAs superlattice under normal electric and magnetic fields \( \mathbf{F} \) and \( \mathbf{B} \) (\( \mathbf{F} \parallel \mathbf{B} \parallel -z \), where \( z \) is the normal to the system) in the vicinity of the internal resonance:

\[ \Omega_{\text{lo}} - eFd \ll \Omega_{\text{lo}} \] at the zero temperature. For instance, we study the resonance between electrons and LO-phonons of AlAs-barriers. In this case the interaction with LO-phonons of GaAs is not essential because of large difference between LO-phonon energies of AlAs and GaAs. The interaction with GaAs-phonons can induce only small polaron shift of the electron energy. Let us note that the Fröhlich electron-phonon interaction constant of AlAs is more than the one of GaAs (\( a_{\text{AlAs}}/a_{\text{GaAs}} \gg 1 \)). Besides, we shall suppose that AlAs-phonons are completely localized in corresponding layers. The latter assumption is valid when the in-plane momentum of confined phonons is small \( \frac{p_H}{\alpha} \ll \frac{\pi}{a} \), where \( a \) is the width of the AlAs-layer. As to electronic wave functions, we shall exploit the nearest-neighbor overlap approximation. Therefore, we shall discuss the following processes (Fig.1): i) resonant tunneling of an electron from \( n \)-quantum well (QW) to \( n+1 \)-QW with the phonon emission in the \( n \)-AlAs-layer, ii) the reverse process in which an electron tunnels from \( n+1 \)-QW to \( n \)-QW with absorption of the phonon which was emitted in the i-process. At the zero temperature the ii-process can follow only after the i-process.


Electron wave-functions of WSL can be represented by

\[ \psi_n(z) = \mid n, i, k_{z} \rangle, \] where \( n, i, \) and \( k_{z} \) are the number of the WSL state centered in the \( n \)-GaAs layer (\( n=0,1,2, \ldots N-1; N \) is the total number of wells in a superlattice), the number of the Landau level, and the in-plane momentum, respectively. We shall take into consideration only the lowest subband and the zero Landau level (\( l=0 \)). The electron energy in the WSL is \( e_n = e^0_n, e^0 = eFd > 0 \). The confined phonon states of AlAs-layers are denoted by \( \mid m, f, p \rangle \), where \( m, f, \) and \( p \) are the number of the AlAs-layer (\( m=0,1, \ldots , N-2 \)), the number of the confined phonon mode, and the in-plane momentum (Fig.1). The Hamiltonian of the Fröhlich electron-phonon interaction is \([9] (n=1)\)
assuming that electrons and phonons are strongly localized, we shall take into account only amplitudes $I_{n+1,n;0}^{J}$ corresponding to the tunneling between nearest QWs with the emission of a phonon in the AlAs-layer between above-mentioned QWs. The integrals $I_{n+1,n;0}^{J}$ can be neglected because of a strong localization of electrons and phonons. As to the integrals $I_{n,n;0}^{J}$, they describe no tunneling and lead to a small energy shift of order $\Delta \omega_{lo}$. It should be pointed out that amplitudes of the resonant tunneling $I_{n}^{J}$ lead to polaron shifts of order $\gamma \Delta \omega_{lo}$.

The Green function of an electron in the state $|\psi\rangle$ at the zero temperature is

$$G_{\psi}(\epsilon) = -\int_{0}^{\infty} \epsilon_{\psi}(t) c_{\psi}^\dagger(0) |O\rangle e^{i\epsilon t - \Gamma t} dt = \frac{1}{\epsilon - \epsilon_{\psi} - \Sigma_{\psi}(\epsilon) + i\Gamma},$$

where $|0\rangle$ is the vacuum state without any particles, $\epsilon_{\psi}$ is the creation operator of an electron, $\epsilon$ and $\Gamma$ are the energy and the lifetime broadening, $\Sigma_{\psi}(\epsilon)$ is the self-energy. Fig. 2a shows the main resonant diagrams for $\Sigma_{\psi}(\epsilon)$, assuming that the constant $\alpha < 1$. Within our model we ignore the diagrams with crossings of phonon lines (Fig. 2b) because they include integrals $I_{n+1,n;0}^{J}$ with $m \neq n$. For the function $\Sigma_{\psi}(\epsilon)$ we can write ($|\psi\rangle = |n, 0, k_{x}\rangle$)

$$\Sigma_{\psi}(\epsilon) = \frac{\gamma^{2}}{\epsilon - \epsilon_{n+1} - \omega_{lo} - \sum_{n+1}^{n} (\epsilon - \omega_{lo}) + i\Gamma},$$

where $\Sigma_{\psi}(\epsilon)$ is independent of $k_{x}$. Matrix elements in $\Sigma_{\psi}(\epsilon)$ involve phonons with in-plaee momentums $p = (c/eB)^{-1/2}$. Thus, long-wave limit $p \ll \hbar / a$ mentioned in the introduction can be achieved if $\alpha \ll 1$. Eq. 3 contains the sum over all "enclosed" diagrams. The polaron spectrum is given by $\epsilon - \epsilon_{n} - \Sigma_{\psi}(\epsilon) = 0$. Using this equation and Eq. 3 we obtain the expression with the continued fraction

$$\lambda = \frac{\lambda - \lambda - \lambda - \lambda}{\lambda - \lambda - \lambda - \lambda},$$

where $\Delta = (\omega_{lo} - \epsilon_{0})/\gamma$. By mathematical methods of continued fractions [10], one can prove that Eq. 4 is equivalent to $\det[H - \lambda \hat{1}] = 0$, where $\hat{1}$ is the Hamiltonian of WSL without phonons under the effective electric field $e\Phi = e\Phi - \omega_{lo}$. The operator $\hat{1}$ describes a linear chain of $N = N - n$ states in a tight-binding approximation [2,11].

Consider the electron-LO-phonon resonance. When $\Delta = 0$ Eq. 4 has simple solutions ($F^{2} = 0$)

$$\epsilon_{n} = \frac{\epsilon_{n} + 2\omega_{lo} \cos[(\pi / (N + 1))]}{N - n} - if, \quad f = 1, 2, \ldots, N - n.$$

It is seen that the polaron spectrum includes bands of the width $\Delta \omega$ near the energies $\epsilon = \epsilon_{n}$ and the quantity of states in these bands is $N - n$ for the certain momentum $k_{x}$. Let us note that

$$\gamma^{2} = \frac{2\omega_{lo}^{2}}{\pi} \frac{\omega_{C}}{(\omega_{lo} F)} \sum_{n} \frac{1}{f_{n}^{2}} |f_{n}|^{2},$$

where $\omega_{C} = \frac{2 \omega_{lo}^{2}}{\pi} \frac{\omega_{C}}{(\omega_{lo} F)}$. The function $\Sigma_{\psi}(\epsilon)$ is independent of $k_{x}$. Matrix elements in $\Sigma_{\psi}(\epsilon)$ involve phonons with in-plane momentums $p = (c/eB)^{-1/2}$. Thus, long-wave limit $p \ll \hbar / a$ mentioned in the introduction can be achieved if $\alpha \ll 1$. Eq. 3 contains the sum over all "enclosed" diagrams. The polaron spectrum is given by $\epsilon - \epsilon_{n} - \Sigma_{\psi}(\epsilon) = 0$. Using this equation and Eq. 3 we obtain the expression with the continued fraction

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$$\epsilon_{n} = \frac{\epsilon_{n} + 2\omega_{lo} \cos[(\pi / (N + 1))]}{N - n} - if, \quad f = 1, 2, \ldots, N - n.$$
The polaron spectrum is shown in Fig. 1 qualitatively. In the limit $N \to \infty$ ($\Gamma \to V/(N-n), V \to \infty$) the density of states in the polaron bands under the resonant condition $\omega_{LO} = \epsilon^0$ is
\[
\nu_n(\epsilon) = \frac{N-n}{N} \frac{1}{(4V^2 - (\epsilon - \epsilon_n^2)^2)^{1/2}}.
\]

The total density of states is given by $N_0 \nu(\epsilon)$, where $N_0 = A eB/\hbar c$ is the quantity of states for the Landau level. The function $\Sigma_0(\epsilon)$ for the case $N \to \infty$ and $\omega_{LO} = \epsilon^0$ can be found by the equation:
\[
\Sigma_n(\tilde{\epsilon}) = V^2/(\tilde{\epsilon} - \Sigma_n(\tilde{\epsilon})), \quad \tilde{\epsilon} = \epsilon - \epsilon_n^0 + i\Gamma.
\]
So we obtain
\[
\Sigma_n(\tilde{\epsilon}) = \frac{\tilde{\epsilon}^2}{4} - \left( \frac{\tilde{\epsilon}^2}{4} - V^2 \right)^{1/2}, \quad G_n(\tilde{\epsilon}) = \frac{1}{\frac{\tilde{\epsilon}^2}{4} + \left( \frac{\tilde{\epsilon}^2}{4} - V^2 \right)^{1/2}},
\]
where $\text{Re} |G_n(\tilde{\epsilon})| > 0$.

When $\Delta > 0$ solutions of Eq. 4 are discrete and can be obtained by numerical calculations but the energy spacing between the roots is about $|\omega_{LO} - \epsilon^0|$. In the limit $\Delta \to 0$ the electron-phonon interaction is no longer essential and solutions of Eq. 4 are
\[
\epsilon_{n+1} = \epsilon_n + i(\omega_{LO} - \epsilon^0) - i\Gamma = \epsilon_n + i(\omega_{LO} - \epsilon^0), \quad n = 1, 2, \ldots N-n.
\]

Consider the physical sense of obtained equations. In the strict resonance ($\Delta = 0$) the electron is delocalized due to a resonant emission of phonons. Moving along the direction $eF|z$, the electron emits LO-phonons in barriers. Alternatively, the electron motion in the opposite direction is accompanied by absorption of phonons which were omitted earlier. Therefore, we are dealing here with the new quasi-particle named by the polaron in this paper. In our model the polaron wave function with the number $n$ can be found as a linear combination of $N - n$ states [12].

These states are degenerate if $\Delta = 0$. The latter results in a strong mixing of above-mentioned states. If $\omega_{LO} = \epsilon^0$ the electron is localized because the difference between $\epsilon^0$ and $\omega_{LO}$ leads to violating the condition for the resonant tunneling. In this case the electron has to have the discrete spectrum. The length of a localization is $W/|\omega_{LO} - \epsilon^0|$ [12].

2. Interband optical absorption in the Stark ladder.

Let us now calculate the spectrum of absorption associated with transitions between valence and conduction bands; the tunneling of holes is ignored, assuming that the hole effective mass is much more than the electron one. Neglecting the exciton effects, the absorption intensity $K(\omega_1)$ can be expressed by the Green function ([13, 14]) ($N \to \infty$)
\[
K(\omega_1) = \sum_{R=0,i} K_R(\omega_1),
\]
\[
K_R(\omega_1) = N \frac{\hbar}{c} \sum \frac{P^2}{m_0^2 Z_g} \frac{S^2}{R} N_c \text{Re} I_G(\hbar \omega_R + i\Gamma), \quad G_0(\epsilon) = \frac{1}{\epsilon - \Sigma_0(\epsilon)}.
\]
\[ \Sigma_{ij}(\epsilon) = \frac{v_i^2}{\epsilon - \sigma_{ij}^2 - \frac{v^2}{\epsilon - 2\sigma_{ij}^2}} \]

\[ \Delta w_k = \omega_L - \sigma g k ; \sigma g k = E g + E_e n + 1/2 (\omega_c + \omega_{ch}) + \epsilon_0 \beta_k, \beta = 0,1 \]

\[ S_{ij} = \langle \psi_i(z) | \psi_j(z) \rangle, S_{ij} = \langle \psi_{n+1}(z) | \psi_j(z) \rangle, S_{ij} = \langle \psi_{n+1}(z) | \psi_j(z) \rangle, \]

where \( \omega_L \) is the frequency of the incident light, \( \chi_n(z) \) is the wave function of a hole localized in the n-GaAs layer, \( E_e \) and \( E_h \) are intrawell energies of electron (hole), \( E_g \) and \( P \) are the bandgap of GaAs and the interband matrix element, respectively. \( \omega_c \) is the free electron mass, \( \omega_{ch} \) is the hole cyclotron frequency. The optical processes with \( \beta = 0,1 \) relate to transitions between zero Landau levels of holes and electrons (see inset of Fig.3). The spectrum of interband absorption contains the main profile near the frequency \( \omega_L = \sigma g 0 \) and satellites near \( \omega_L = \sigma g 1 \), \( \sigma g 2 \) (Fig.3). For simplicity, we consider overlap only between nearest wavefunctions of electrons and holes. The spectrum near resonant frequencies \( \sigma g k (k = 0,1) \) is given by the function \( P(\omega) = \text{Re} \sigma_0 (\omega + 1) \) containing information about the polaron spectrum.

In the case \( \omega_L = \epsilon_0 \) and \( \Gamma \rightarrow 0 \) it is seen from Eq.6

\[ \text{Re} \sigma_0 (\epsilon) = \frac{(\epsilon^2 - 4\epsilon^2/4)^{1/2}}{v_i^2}, |\epsilon| < 2\epsilon. \]  \hspace{1cm} (8)

Let us note that \( K(\omega_L) \) vanishes in the vicinity of the edges of the polaron band \( \omega_L = \sigma g k \pm 2\epsilon \) (see Eq. 7, 8). At the same time, the density of states for \( n = 0 \) Eq.5 has singularities near \( \epsilon = \pm 2\epsilon \). To understand this fact we can write \( \psi_k = \psi_0(n)(\epsilon)P(\epsilon) \), where \( \epsilon = \omega_L - \sigma g k \), \( P(\epsilon) \) is the probability of the electron being in the state without phonons. From wave functions of polarons [12] it is seen that \( P(\epsilon) \sim (4\epsilon^2 - \epsilon^2) \) and the origin of Eq.8 becomes clear.

Fig.4 represents the function \( P(\epsilon) \) calculated by Eqs.6, 7. In the case \( |\omega_L - \epsilon_0| < \Gamma \) the function \( P(\epsilon) \) contains the peak with the broadening \( 2\Gamma \) correlated with the polaron band. When \( |\omega_L - \epsilon_0| > \Gamma \) the function \( P(\epsilon) \) exhibits the discrete peaks having broadenings \( \Gamma \) corresponding to discrete spectrum of localized electrons. The spacing between peaks is about \( |\omega_L - \epsilon_0| \). In the limit \( \Gamma \rightarrow \infty \) frequencies of peaks are given by \( \omega_L = \epsilon_0 + (\omega_L - \epsilon_0) \) and relate to interband processes with a creation of the electron-hole pair in the states \( \psi_{n+1}(z), \psi_{n}(z) \) and phonons in the states \( n-k \). Intensities of these processes are proportional to \( I_{nk} \sim \sigma g k \Delta^2 \Gamma \). For instance, the main profile with \( n = 0 \) includes interband optical processes \( \psi_{n+1}(z) \rightarrow \psi_{n}(z), |\psi_{n+1}(z)\rangle \rightarrow |\psi_{n}(z)\rangle, |\psi_{n+2}(z)\rangle \rightarrow |\psi_{n+1}(z)\rangle \rightarrow |\psi_{n}(z)\rangle \), etc., shown in Fig.5. It should be pointed out that the polaron effect is essential if \( \Gamma > \Gamma \) or \( |\omega_L - \epsilon_0| > \Gamma \).

Thus, the absorption line shape shows features of the polaron spectrum. When \( \omega_L = \epsilon_0 \) profiles of the interband absorption are broadened due to the electron-phonon interaction. On the other hand, away from the resonance, profiles exhibit a fine structure of peaks.
In the case of two coupled quantum wells under the electric field, the electron-LO-phonon resonance was studied theoretically in Ref. 15, where it was shown that the polaron effect plays the important role in double-resonant Raman scattering by LO-phonons.

As to bulk systems, optical manifestations of polaron effects in a magnetic field were studied experimentally in Ref. 16, 17 and theoretically in Ref. 7, 14 for the case of the resonance between the cyclotron excitation and optical phonons.

Conclusions

By the diagram technique we have calculated optical spectra of the WSL in the presence of the electron-LO-phonon resonance. It is shown that the polaron effect can lead to considerable change of the interband absorption spectrum in the WSL. Under the condition $\omega_0 = \epsilon^0$, the profiles of absorption have broadenings $2\gamma$ (see Eq.8) and are quite different from the usual defect-induced line shapes $\Gamma/(\omega^2 + \Gamma^2)$. Away from the resonance $\omega_0 = \epsilon^0$, the profiles consist of a set of lines corresponding to interband LO-phonon-assisted transitions. Let us point out that predicted effects can be observed experimentally in superlattices of high quality when $V > \Gamma$.

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References

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**FIGURE CAPTIONS**

Fig. 1. The system of coupled quantum wells in the normal electric field, circles denote the confined phonon states of AlAs. The density of polaron states for the case $\Delta = 0$, qualitatively.

Fig. 2. Feynman diagrams for the calculation of the self-energy $\Sigma(t)$, a) the main diagram, b) the ignored diagram with a crossing of phonon lines.

Fig. 3. Qualitative picture of the absorption spectrum in the WS1; the inset shows the diagram of interband transitions for $l=0$.

Fig. 4. The function $F(\omega)$ for a description of interband absorption ($\Gamma V = 0.3$).

Fig. 5. Interband LO-phonon-assisted transitions corresponding to the main profile with $k=0$.
Fig. 3 \( (\omega_z - \varepsilon_{g0})/\varepsilon^0 \)

Fig. 4 \( F(\omega) \)

1 - \( \Delta = 0 \)
2 - \( \Delta = 1 \)