IC/93/301

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

MAGNETIC PROPERTIES OF A TRANSVERSE SPIN–1 ISING MODEL WITH RANDOM CRYSTAL FIELD INTERACTIONS

A. Benyoussef

and

H. Ez–Zahraouy

INTERNATIONAL ATOMIC ENERGY AGENCY

UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

MIRAMARE-TRIESTE

IC/93/301

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

MAGNETIC PROPERTIES OF A TRANSVERSE SPIN-1 ISING MODEL WITH RANDOM CRYSTAL FIELD INTERACTIONS

A. Benyoussef
Laboratoire de Magnetisme et Physique des Haute Energies,
Department of Physique, Faculte des Sciences,
B.P. 1014, Rabat, Morocco

and

H. Ez-Zahraouy
International Centre for Theoretical Physics, Trieste, Italy
and
Laboratoire de Magnetisme et Physique des Haute Energies,
Department of Physique, Faculte des Sciences,
B.P. 1014, Rabat, Morocco.

ABSTRACT

We study the phase diagram and magnetic properties of a transverse spin-1 Ising model with random longitudinal crystal field interactions within an expansion technique for cluster identities of spin-1 localized spin systems. The partially ordered phases appear for a particular two valued distribution $P(D_i) = 1/2[\delta(D_i-(1+\alpha)D)+\delta(D_i-(1-\alpha)D)]$, and for sufficiently small transverse field Ω. The longitudinal and transverse magnetizations, the quadrupolar moments are calculated. General formulae applicable to structures with arbitrary coordination number, are given.

MIRAMARE - TRIESTE
September 1993

1.Introduction

Spin systems are widespread in very different fields of physics ,e.g., in the theory of magnetism, superconductivity, nuclear physics, etc. Special methods of theoretical physics are needed to describe the systems since commutation relations for spin components differ from the corresponding relations in both Bose and Fermi systems The study of phase transitions in the Ising and the Heisenberg models has been the subject of much interest [1-7]. Phase diagrams of such models show various types of multicritical phenomena [3,4,7]. The Ising model in the presence of a transverse field serves for the study of cooperative phenomena and phase transitions in many physical systems [8-10]. The diluted three-dimensional spin-1 Ising model with crystal field interactions is studied by Saber[11] within Finite cluster approximation. The spin-1 Ising model with a random crystal field is studied by Benyoussef et al[12], Boccara et al[13] within mean field solution. Our aim is to study the influence of the transverse magnetic field on the phase diagram and magnetic properties of spin-1 Ising model with random crystal field interactions. We use the finite cluster approximation [14,15] with an expansion technique for cluster identities of spin-1 localised spin systems established by Ez-Zahraouy et al[16]. Phase diagram for coordination number N=6 is represented in the T-Ω-D space for fixed value of α, where T, Ω and D are respectively temperature, transverse field and longitudinal crystal field. General formula of the magnetizations and quadrupolar moments are determined for an arbitrary coordination number N. The dependence of longitudinal and transverse magnetizations on the crystal field for several values of the transverse field and on temperature for several values of the crystal field are calculated for N=6.

In section 2 we give the method and we calculate the state equations. Section 3 is reserved to results and discussion.

2. Finite cluster approximation.

We consider a spin-1 Ising system in a simple cubic lattice described by a Hamiltonian corresponding to paramangnet of the longitudinal-axis type in a transverse magnetic field.

$$H = -\sum J_{ij} S_i S_j - \Omega \sum S_{ix} + \sum D_i S_{iz}^2 \qquad (1)$$

Where S_{ix} and S_{iz} are respectively the x-component and the z-component of spin-1 operator at site i, Ω represents the transverse field and J_{ij} is the exchange interaction between spins at site i and j, in this paper J_{ij} is constant and equal J. <ij> runs over all nearest -neighbour pairs of spins , and D_i is the random crystal field governed by the probability distribution law given by:

$$P(D_i) = \frac{1}{2}[\delta(D_i - (1+\alpha)D) + \delta(D_i - (1-\alpha)D)]$$

Using a single site cluster approximation in which attention is focused on a cluster comprising just a single selected spin labelled 0, and the neighbouring spins with which it directly interacts, then the Hamiltonian containing 0, namely

$$H_0 = AS_{0z} + BS_{0x} + D_0 S_{0z}^2 \qquad (2)$$

where

$$A = -J\theta \quad ; \quad B = -\Omega \qquad (3)$$

with $$\theta = \sum_{j=1}^{z} S_{jz}$$

This single-site Hamiltonian can readily be diagonalised and its eigenvalues and eigenvectors found. The three eigenvectors corresponding to the eigenvalues

$$\lambda_k = 2(D_0 + \sqrt{\rho}\ \cos(\theta_k))/3 \qquad (4)$$

with $$\theta_k = \frac{1}{3} Arc\cos(-27q/2\rho^3) + 2(k-1)\Pi/3 \qquad (5)$$

and $$\rho = \frac{3\sqrt{3}}{2}\sqrt{27q^2 + |4p^3 + 27q^2|} \qquad (6)$$

$$p = -(A^2 + B^2) - D_0^2/3 \quad ; \quad q = -D_0(2A^2 + 2/9 D_0^2 + B^2)/3 \qquad (7)$$

are

$$|\Psi\rangle_k = \alpha_k|+\rangle + \beta_k|-\rangle + \gamma_k|0\rangle \qquad (8)$$

with

$$\alpha_k = \frac{|B(\lambda_k - D_0 + A)|}{\sqrt{2}\sqrt{B^2((\lambda_k - D_0)^2 + A^2) + ((\lambda_k - D_0)^2 - A^2)^2}} \qquad (9)$$

$$\beta_k = \frac{\lambda_k - D_0 - A}{\lambda_k - D_0 + A}\alpha_k \qquad ; \qquad \gamma_k = \frac{\sqrt{2}}{B}(\lambda_k - D_0 - A)\alpha_k \tag{10}$$

In a representation in which S_{0z} is diagonal. The starting point of single-site cluster approximation is a set of formal identities of the type

$$\left\langle \left\langle S^p_{0\alpha} \right\rangle_c \right\rangle = \left\langle \frac{\mathrm{trace}_0\left[S^p_{0\alpha}\exp(-\beta H_0)\right]}{\mathrm{trace}_0\left[\exp(-\beta H_0)\right]} \right\rangle \tag{11}$$

Where $S^p_{0\alpha}$ is the α-component of the spin operator S_0 raised to the power p, $\left\langle S^p_{0\alpha} \right\rangle_c$ denotes the mean value of $S^p_{0\alpha}$ for a given configuration c of all other spins, i.e when all other spins S_i $(i \neq 0)$ have fixed values. $\langle \ldots \rangle$ denotes the average over all spin configurations. trace_0 means the trace performed over S_0 only. $\beta = \frac{1}{K_B T}$, T the absolute temperature and K_B is the Boltzmann constant. The equations (11) are not exact for an Ising system in a transverse field, they have nevertheless, been accepted as a reasonable starting point in many studies of that system[17]. Let $< S_{0\alpha} >^{\pm}_c$ and $< S^2_{0\alpha} >^{\pm}_c$ denotes respectively the mean value of $S_{0\alpha}$ and $S^2_{0\alpha}$ for a fixed configuration $\pm$ of the random crystal field and for a given configuration c for all other spins.

To calculate, $< S_{0\alpha} >^{\pm}_c$ and $< S^2_{0\alpha} >^{\pm}_c$, one has to effect the inner traces in eqs.(11) over the states of the spin 0 and this is the most easily performed using the eigenstates of eqs.(8) as the basic states. In this way, it follows on setting p=1 and 2 it turn in eqs.(11) that

$$\left\langle S_{0z} \right\rangle^{\pm}_c = \frac{\sum_{k=1}^{3} (\alpha_k^{\pm 2} - \beta_k^{\pm 2}) \exp(-\beta\lambda_k^{\pm})}{\sum_{k=1}^{3} \exp(-\beta\lambda_k^{\pm})} \tag{12}$$

$$\left\langle S_{0x} \right\rangle^{\pm}_c = \sqrt{2}\, \frac{\sum_{k=1}^{3} (\alpha_k^{\pm} + \beta_k^{\pm}) \gamma_k^{\pm} \exp(-\beta\lambda_k^{\pm})}{\sum_{k=1}^{3} \exp(-\beta\lambda_k^{\pm})} \tag{13}$$

$$\left\langle S^2_{0z} \right\rangle^{\pm}_c = \frac{\sum_{k=1}^{3} (\alpha_k^{\pm 2} + \beta_k^{\pm 2}) \exp(-\beta\lambda_k^{\pm})}{\sum_{k=1}^{3} \exp(-\beta\lambda_k^{\pm})} \tag{14}$$

$$\left\langle S^2_{0x} \right\rangle^{\pm}_c = \frac{\sum_{k=1}^{3} ((\alpha_k^{\pm} + \beta_k^{\pm})^2 / 2 + \gamma_k^{\pm 2}) \exp(-\beta\lambda_k^{\pm})}{\sum_{k=1}^{3} \exp(-\beta\lambda_k^{\pm})} \tag{15}$$

where $\begin{cases} \alpha_k^{-} = \alpha_k \\ \beta_k^{\pm} = \beta_k \\ \gamma_k^{\pm} = \gamma_k \\ \lambda_k^{\pm} = \lambda_k \end{cases}$ when $D_0 = (1 \pm \alpha) D$

The magnetizations m_α (α=z,x) and the quadrupolar moments q_α (α=z,x) are given by :

$$m_\alpha = \langle f_\alpha(\theta) \rangle \qquad : \qquad q_\alpha = \langle g_\alpha(\theta) \rangle$$

with

$$f_\alpha(\theta) = \frac{1}{2}(\langle S_{+\alpha} \rangle_c^* + \langle S_{-\alpha} \rangle_c^*) \tag{16}$$

$$g_\alpha(\theta) = \frac{1}{2}(\langle S_{+\alpha} \rangle_c^* + \langle S_{-\alpha} \rangle_c^*) \tag{17}$$

Where $\langle \ldots \rangle$ denotes the average over all configurations of the spins $S_{j\alpha}(j \neq 0)$. To calculate $\langle f_\alpha(\theta) \rangle$ and $\langle g_\alpha(\theta) \rangle$ we have used the expansion technique for spin-1 Ising systems as follows[14]:

Suppose one considers the general product $\prod_{i=1}^{\Lambda}(\sum_{p_i=0}^{2} S_{i\alpha}^{p_i})$ that contains 3^Λ terms. From these terms one may collect together all those terms containing p factors of $S_{i\alpha}^2$ and q factors of $S_{i\alpha}$. Such a group is to be denoted by $\{S_{i\alpha}^2, S_{i\alpha}\}_{\Lambda,p,q}$. For example, if N=4, p=1 and q=2, then

$$\begin{aligned}\{S_{i\alpha}^2, S_{i\alpha}\}_{4,1,2} = {} & S_{1\alpha}^2(S_{2\alpha}S_{3\alpha} + S_{2\alpha}S_{4\alpha} + S_{3\alpha}S_{4\alpha}) + S_{2\alpha}^2(S_{1\alpha}S_{3\alpha} + S_{1\alpha}S_{4\alpha} + S_{3\alpha}S_{4\alpha}) \\ & + S_{3\alpha}^2(S_{1\alpha}S_{2\alpha} + S_{1\alpha}S_{4\alpha} + S_{2\alpha}S_{4\alpha}) + S_{4\alpha}^2(S_{1\alpha}S_{2\alpha} + S_{1\alpha}S_{3\alpha} + S_{2\alpha}S_{3\alpha})\end{aligned} \tag{18}$$

Our aim is to expand the functions of eqs.(16) and (17) in terms of these $\{S_{i\alpha}^2, S_{i\alpha}\}_{\Lambda,p,q}$. Thus, if one writes

$$f_\alpha(\sum_{i=1}^{\Lambda} S_{i\alpha}) = \sum_{q=0}^{\Lambda}\sum_{p=0}^{\Lambda-q} A_{pq}^{\alpha}(N)\{S_{i\alpha}^2, S_{i\alpha}\}_{\Lambda,p,q} \tag{19}$$

$$g_\alpha(\sum_{i=1}^{\Lambda} S_{i\alpha}) = \sum_{q=0}^{N}\sum_{p=0}^{N-q} B_{pq}^{\alpha}(N)\{S_{i\alpha}^2, S_{i\alpha}\}_{N,p,q} \tag{20}$$

the problem is to find the coefficients $A_{pq}^{(\alpha)}(N)$ and $B_{pq}^{(\alpha)}(N)$. To achieve this it is advantageous to transform the spin-1 system to a spin-$\frac{1}{2}$ representation containing the Pauli operators $\sigma_{i\alpha} = \pm 1$. This may be accomplished by setting $S_{i\alpha} = \tau_{i\alpha}\sigma_{i\alpha}$ with $\tau_{i\alpha} = 0, 1$. In this representation, eqs(19) and (20) become

$$f_\alpha(\sum_{i=1}^{N} \tau_{i\alpha}\sigma_{i\alpha}) = \sum_{q=0}^{N}\sum_{p=0}^{N-q} A_{pq}^{\alpha}(N)\{\tau_{i\alpha}^2, \tau_{i\alpha}\sigma_{i\alpha}\}_{N,p,q} \tag{21}$$

$$g_\alpha(\sum_{i=1}^{\Lambda} \tau_{i\alpha}\sigma_{i\alpha}) = \sum_{q=0}^{\Lambda}\sum_{p=0}^{\Lambda-q} B_{pq}^{\alpha}(N)\{\tau_{i\alpha}^2, \tau_{i\alpha}\sigma_{i\alpha}\}_{\Lambda,p,q} \tag{22}$$

and must hold for arbitrary choices of $\tau_{i\alpha}$. Suppose one now chooses the first r out of the N operators $\tau_{i\alpha}$ to be unity, and the remainder zero. Then eqs.(21) and (22) give

$$f_\alpha(\sum_{i=1}^{r} \sigma_{i\alpha}) = \sum_{q=0}^{r}\sum_{p=0}^{r-q} A_{pq}^{\alpha}(N) C_p^{r-q}\{\sigma_{i\alpha}\}_{r,q} \tag{23}$$

$$g_\alpha(\sum_{i=1}^{r} \sigma_{i\alpha}) = \sum_{q=0}^{r}\sum_{p=0}^{r-q} B_{pq}^{\alpha}(N) C_p^{r-q}\{\sigma_{i\alpha}\}_{r,q} \tag{24}$$

where $\{\sigma_z\}_{r,q}$ is the sum of all possible products of q spins operators, σ_{iz}, out of a maximum of r, and the C_n^m are the binomial coefficients $m!/n!(m-n)!$. That is,

$$f_\alpha\left(\sum_{i=1}^{r}\sigma_{iz}\right)=\sum_{q=0}^{r}b_q^{(\alpha)}(r)\{\sigma_z\}_{r,q} \tag{25}$$

$$g_\alpha\left(\sum_{i=1}^{r}\sigma_{iz}\right)=\sum_{q=0}^{r}d_q^{(\alpha)}(r)\{\sigma_z\}_{r,q} \tag{26}$$

with

$$b_q^{(\alpha)}(r)=\sum_{p=0}^{r-q}A_{pq}^{(\alpha)}(N)C_p^{r-q} \tag{27}$$

$$d_q^{(\alpha)}(r)=\sum_{p=0}^{r-q}B_{pq}^{(\alpha)}(N)C_p^{r-q}. \tag{28}$$

The spin-1 problem of eqs.(19) and (20) containing N spins has thus been transformed to a spin-1/2 problem containing r spins. The advantage of doing this is that it now enables one to use directly the results already established in ref.[18] for the spin-1/2 system. It may also be noted that whereas the coefficients $b_q^{(\alpha)}(r)$ and $d_q^{(\alpha)}(r)$ for the spin-1/2 problem depend on the total number of spins present, the coefficients $A_{pq}^{(\alpha)}(N)$ and $B_{pq}^{(\alpha)}(N)$ are in fact independent of N, as is clear from eqs.(27) and (28). Thus the label N is superfluous and may henceforth be dropped. This could, of course, have been inferred directly from eqs.(19) and (20) by setting one of the S_{iz} spins equal to its zero value throughout. Specialising the results of ref.[18] to a single group of r spins, one has for the current problem

$$b_q^{(\alpha)}(r)=\frac{1}{2^r C_q^r}\sum_{i=0}^{r}C_i^r\varepsilon_i(r,q)f_{\alpha i}(r) \tag{29}$$

$$d_q^{(\alpha)}(r)=\frac{1}{2^r C_q^r}\sum_{i=0}^{r}C_i^r\varepsilon_i(r,q)g_{\alpha i}(r) \tag{30}$$

where

$$\varepsilon_i(r,q)=\sum_{\mu=0}^{i}(-1)^\mu C_\mu^i C_{q-\mu}^{r-i} \tag{31}$$

and

$$f_{\alpha i}(r)=f_\alpha(r-2i) \tag{32}$$

$$g_{\alpha i}(r)=g_\alpha(r-2i) \tag{33}$$

Once the coefficients $b_q^{(\alpha)}(r)$ and $d_q^{(\alpha)}(r)$ have been calculated, the coefficients $A_{pq}^{(\alpha)}$ and $B_{pq}^{(\alpha)}$ may be found by the following procedure. First, $A_{0q}^{(\alpha)}$ and $B_{0q}^{(\alpha)}$ are got by setting r=q in eqs.(29) and (30). That is

$$A_{0q}^{(\alpha)}=b_q^{(\alpha)}(q) \quad ; \quad B_{0q}^{(\alpha)}=d_q^{(\alpha)}(q) \tag{34}$$

Then, the other $A_{pq}^{(\alpha)}$ and $B_{pq}^{(\alpha)}$ may be obtained by expressing eqs.(29) and (30) as a recurrence relation, namely as

$$A_{r-q,q}^{(\alpha)}=b_q^{(\alpha)}(r)-\sum_{p=0}^{r-q-1}A_{pq}^{(\alpha)}C_p^{r-q} \tag{35}$$

$$B^{\alpha}_{r,q,q} = d^{\alpha}_{q}(r) - \sum_{p=0}^{r-q-1} B^{(\alpha)}_{pq} C^{r-q}_{p} \tag{36}$$

Then the magnetizations $m_\alpha(\alpha=z,x)$ and the quadrupolar moments $q_\alpha(\alpha=z,x)$ are given for an arbitrary coordination number N, by

$$m_{\alpha} = \sum_{q=0}^{N} \sum_{p=0}^{N-q} A^{\alpha}_{pq} \left\langle \left\{ S_z^2, S_z \right\}_{N,p,q} \right\rangle \tag{37}$$

$$q_{\alpha} = \sum_{q=0}^{N} \sum_{p=0}^{N-q} B^{\alpha}_{pq} \left\langle \left\{ S_z^2, S_z \right\}_{N,p,q} \right\rangle \tag{38}$$

Using the simplest approximation of the Zernike decoupling of the type

$$\left\langle S_{i_1} S_{i_2} \ldots S_{i_r} \ldots \right\rangle \equiv \left\langle S_{i_1} \right\rangle \left\langle S_{i_2} \right\rangle \ldots \left\langle S_{i_r} \right\rangle \ldots \quad for\ i \neq j \neq k \neq \ldots,$$

and seeing that the number of elements of the group $\left\{ S_z^2, S_z \right\}_{N,p,q}$ is equal $C_p^N C_q^{N-p}$, the eqs.(37) and (38) become

$$m_{\alpha} = \sum_{q=0}^{N} \sum_{p=0}^{N-q} A^{\alpha}_{pq} m_z^q q_z^p C_p^N C_q^{N-p} \tag{39}$$

$$q_{\alpha} = \sum_{q=0}^{N} \sum_{p=0}^{N-q} B^{\alpha}_{pq} m_z^q q_z^p C_p^N C_q^{N-p} \tag{40}$$

Let put $m = m_z = \langle S_z \rangle$ and $x = q_z = \langle S_z^2 \rangle$, and if we replace x in (39) by its expression taken from (40), we obtain an equation for m of the form

$$m = am + bm^3 + \ldots \tag{41}$$

where

$$a = N\left(A^{(z)}_{01} + \sum_{p=1}^{N-1} A^{(z)}_{p1} C_p^{N-1} x_0^p \right) \tag{42}$$

with x_0 the solution of the following equation

$$x_0 = B^{(z)}_{00} + \sum_{p=1}^{N} B^{(z)}_{p0} C_p^N x_0^p \tag{43}$$

The critical temperature of the second order transition is determined by a=1. In the vicinity of second order transition the magnetization m_z is determined by:

$$m_z^2 = \frac{1-a}{b} \tag{44}$$

At this temperature the transverse magnetization is given by

$$m_x = \sum_{p=0}^{N} B^{(z)}_{p0} C_p^N x_0^p \tag{45}$$

and the quadrupolar moments q_α $(\alpha=z,x)$ are given by

$$q_{\alpha} = \sum_{p=0}^{N} B^{(\alpha)}_{p0} C_p^N x_0^p \tag{46}$$

The right-hand side of eq.(44) must be positive. If this is not the case the transition is of first order. The point at which a=1 and b=0 is the tricritical point. To obtain the expression for b one has to solve (40) for small m. The solution is of the form

$$x = x_0 + x_1 m^2 \tag{47}$$

Where x_1 is given by :

$$x_1 = \sum_{p=0}^{N-2} B_{p2}^{(N)} C_2^N C_p^{N-2} x_0^p + \sum_{p=1}^{N} p B_{p0}^{(N)} C_p^N x_0^{p-1} x_1 \tag{48}$$

That is

$$x_1 = \frac{\sum_{p=0}^{N-2} B_{p2}^{(N)} C_2^N C_p^{N-2} x_0^p}{1 - \sum_{p=1}^{N} p B_{p0}^{(N)} C_p^N x_0^{p-1}} \tag{49}$$

This yields

$$b = \sum_{p=0}^{N-4} A_{p4}^{(N)} C_4^N C_p^{N-4} x_0^p + N \sum_{p=1}^{N-1} p A_{p1}^{(N)} C_p^{N-1} x^{p-1} x_1 \tag{50}$$

3-Results and discussion.

In this section we present results of the Hamiltonian (1) on a simple cubic lattice (N=6). The study of the phase diagram in the T-Ω-D space yields three different situations depending on the value of α[13], here after we shall consider a case (α=3/4) in which the partialy ordered phases exist. The resulting phase diagram is shown in Fig.1, where partly there exists a

tricritical line TCL separating the surfaces of second and first order transitions and on the other part there exist a first order surface transition separating the ordered phases and the partialy ordered phases; this surface exists at smaller temperature and $\Omega < \Omega_C$. The dependence of the magnetizations m_z, and m_x on the crystal field for a fixed value of the temperature(T/J=0.1) and α=3/4, are shown respectively in Fig2.a and Fig.3.a for several values of the transverse field Ω. First order transition is characterised by the gap of the longitudinal magnetization m_z at the crystal field transition, hence, for $\Omega < \Omega_c$ and for sufficiently low temperature, in Fig.2.a(and for Ω/J=0.01).we have two first order transitions, one is from the ordered phase (mz=1) to the partialy ordered phases(mz=1/2), the second is from the partialy ordered phases to the disordered one. Such transitions are also observed in Fig.3.a in which the transverse magnetization passes through a peak for first order transitions(Fig.3.a for Ω=0.2, 0.3, 0.5) and a shoulder for second order transition. The transverse magnetization m_x increases when increasing the transverse field Ω at low temperature in agreement with Ref.[19]. While the magnetization m_z decreases with Ω. On the other hand, for fixed value of Ω, m_z decreases continuousely in the vicinity of the transition temperature and vanishes at $T=T_C$, for the second order transition and exhibit a discontinuity at first order transition(Fig.2.b, D/J=5.6, 5.5, 2.6). The transverse magnetization m_x increases with the strength of the crystal field at low temperature and passes through a peak for the first order transition and a cusp for the second order transition temperature of m_z and then falls off rapidly (see Fig.3.b) as determined by the relation (45). Finally we remark that the method used here[16] allow us to see the re-

entrant part observed in Fig.2.b(D/J=5.5, 5.6), such phenomena is not observed within mean field theory[13] for $\Omega/J=0$.

Acknowledgments

One of the authors (H.E-Z.) would like to thank Professor Abdus Salam, the International Atomic Energy Agency, and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste.

References.

[1]A.Aharony,Phys.Rev.B18 (1978) 3318

[2]A.Aharony,Phys.Rev.B18 (1978) 3328

[3]S.Galam and A.Aharony, J.Phys.C 14 (1981) 3603

[4]S.Galam and A.Aharony, J.Phys.C 13 (1980) 1065

[5]V.K.Saxena, J.Phys.C 14 (1981)L 745

[6]I.Morgenstera, K.Binder and R.M.Hornreich,Phys.Rev.B23 (1981) 287

[7]V.K.Saxena, Phys.lett 90 A 71 (1982)

[8]R.B Stinchcombe, J.Phys.C 6 (1973) 2459

[9]J.L.Zhong, J.Liangli and C.Z.Yang, Phys.Stat.Sol(b) 160,329 (1990)

[10]V.V.Ulyanov and O.B.Zaslavskii, Phys.Reports 216 179-251 (1992)

[11]M.Saber, Ic/88/261, Internal, Report

[12]A.Benyoussef, T.Biaz, M.Saber and M.Touzani, J.Phys.C: Solid State Phys.20 (1987) 5349

[13]N.Boccara, A.El-Kenz and M.Saber, J.Phys:Condens.Matter1 (1989)5721

[14]N.Boccara, Phys.Lett.A94 (1983)185

[15]A.Benyoussef and N.Boccara, J.Phys.C 16 (1983) 1143

[16]H.Ez-Zahraouy,M.Saber and J.W.Tucker,J.Magn. Magn.Mater 118 (1993)129

[17]F.C.Sà Barreto, I.P.Fittipaldi and B.Zeks, Ferroelectries 39(1981) 1103

[18]P.Tomczak, E.F.Sarmento, A.F.Siqueira and A.R.Ferchmin, Phys. Stat.Sol.(b)142(1987)551

[19]A.Benyoussef and H.Ez-Zahraouy, to appear in Physica A (1993)

Figure Captions

Fig.1. Phase diagram in T-Ω-D space for N=6. TCL is the tricritical line. Dashed lines correspond to first order transition.

Fig.2. a) The crystal field dependence of the longitudinal magnetization when T/J =0.1. The number accompanying each curve denotes the value of Ω/J.

b) The temperature dependence of the longitudinal magnetization when Ω/J=0.2. The number accompanying each curve denotes the value of D/J.

Fig.3.

a) The crystal field dependence of the transverse magnetization when T/J =0.1. The number accompanying each curve denotes the value of Ω /J.

b) The temperature dependence of the transverse magnetization when Ω/J=0.2. The number accompanying each curve denotes the value of D/J.

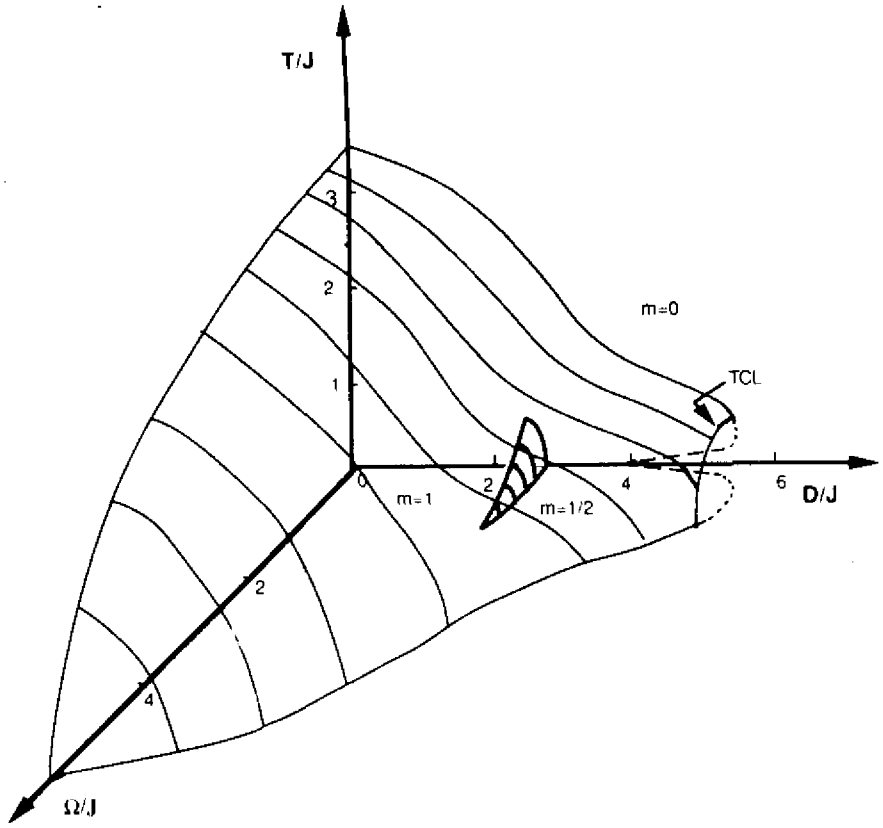

FIG.1

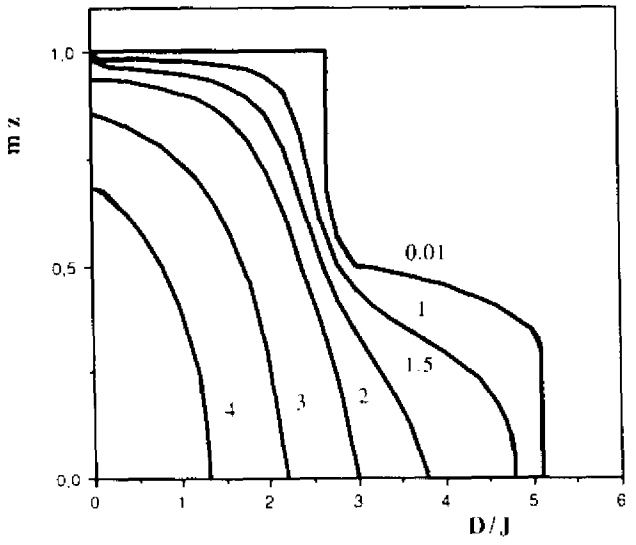

FIG.2.a

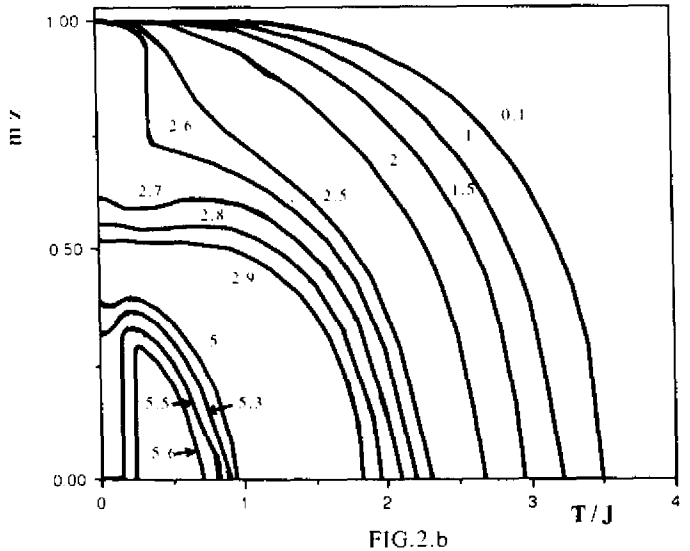

FIG.2.b

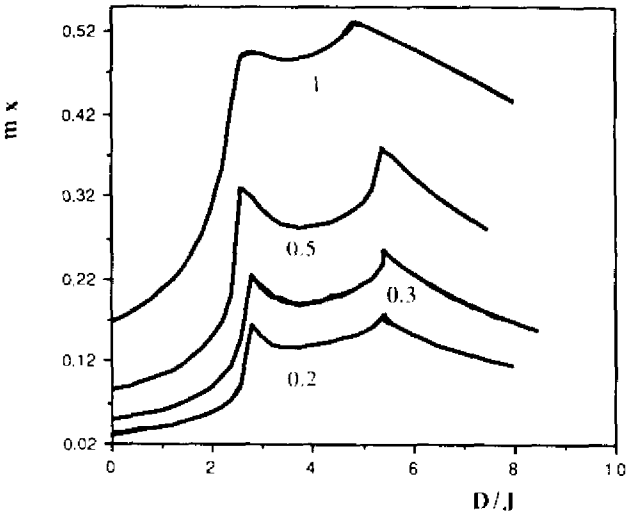

FIG.3.a

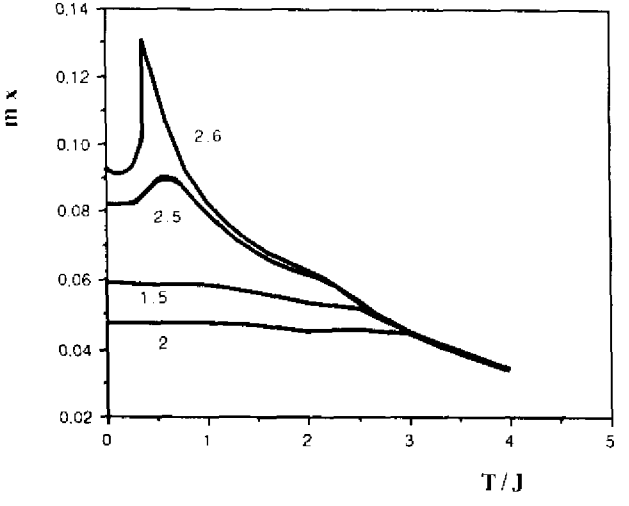

FIG.3.b