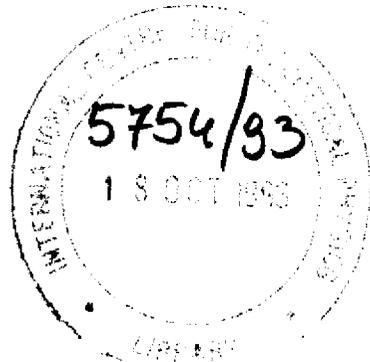


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**INTERNATIONAL CENTRE FOR
THEORETICAL PHYSICS**



**THE BOND DILUTED
SPIN-1 BLUME-EMERY-GRIFFITHS MODEL
IN A TRANSVERSE FIELD**

H. Ez-Zahraouy



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IN A TRANSVERSE FIELD**

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ABSTRACT

The effect of Bond-dilution on the magnetic properties of a quantum transverse spin-1 Blume-Emery-Griffiths model is investigated within an expansion technique for cluster identities of a spin-1 localized spin system. The longitudinal and transverse magnetizations and quadrupolar moments are studied for several values of the bond concentration. A general formula, applicable to structures with arbitrary coordination number N , are given.

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1. Introduction

The phase transition and tricritical behavior of the Ising model have received a great amount of interest in recent years because spin systems are widespread in very different fields of physics, e.g., in the theory of magnetism, superconductivity, nuclear physics, etc. In particular, because of the fundamental interest in the multi-critical phenomena of physical systems, spin-1 model with single-ion anisotropy[1,2] have been already investigated in some detail using a number of methods, namely the mean field approximation[1,2], effective field approximation [3,4], high-temperature series expansion[5], constant coupling approximation[6], Monte-Carlo technique[7], finite cluster approximation with an expansion technique[8] and so on. All these methods suggest the existence of a tricritical point at which the system changes from second order transition to the first one.

The spin-1 Ising model with nearest-neighbor interactions, both bilinear and biquadratic, and with a crystal-field interaction was introduced by Blume, Emery and Griffiths (BEG)[9] to describe phase separation and superfluid ordering in He^3 - He^4 mixtures. With vanishing biquadratic interactions the model is known as the Blume-Capel model [10-11]. The BEG model was subsequently reinterpreted to describe phase transitions in simple[12] and multi-component fluids[13,14]. This model has been extensively studied by different techniques. It has been analyzed using mean field theory[9,12-16], an effective field theory based on the differential operator technique[17,18], real space renormalization group methods[19,21], high- and low-temperature series expansion[22], and Monte-Carlo simulations [16,23-26]. The Ising model in the presence of a transverse field serves for the study of cooperative phenomena and phase transitions in many physical systems[27-29]. The transverse-field

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or crystal field effects of spin-1 Ising model has been studied by a number of authors[1-31].

This model including both transverse and crystal-field has been studied by Yang et al[32] within mean field approximation, and by Benyoussef et al[8,33] within an expansion technique for cluster identities of spin-1 localized spin systems[8,33].The site diluted spin-1 and spin-1/2 Ising ferromagnet in a transverse field are investigated within differential operator technique[34].The effects of bond dilution on the magnetic properties of a transverse spin-1 Ising model with random longitudinal field and the pure case of a transverse spin-1 BEG model are studied within an expansion technique for cluster identities of spin-1 localized spin systems[31,35]. But nobody, up to now, has studied the case of the bond dilution transverse spin-1 BEG model.

Since the transverse Ising model was introduced by de Gennes[36] as a pseudo-spin model of hydrogen-bonded ferroelectrics, our aim is to study the effects of bond dilution on the magnetic properties of a spin-1 Blume-Emery-Griffiths model in a transverse magnetic field using the Finite cluster approximation [37,38]with an expansion technique for cluster identities of spin -1 localized spin systems established by Ez-Zahraony et al.[39]. The dependence of longitudinal and transverse magnetizations on the temperature for several values of crystal-field interactions, biquadratic exchange interactions and transverse field, are calculated for coordination number $N=6$. General formula of the magnetizations and quadrupolar moments are given for an arbitrary coordination number.

The present paper is organized as follows. In section 2 we give the model and the method. The results and discussion are given in section 3.

2.Model and method

The spin-1 BEG model in a transverse magnetic field is described by the following Hamiltonian

$$H = - \sum_{\langle ij \rangle} J_{ij} S_{i\alpha} S_{j\alpha} - D \sum_{\langle ij \rangle} (S_{ij})^2 (S_{ij})^2 - \Delta \sum_i (S_{iz})^2 - \Omega \sum_i S_{ix} \quad (1)$$

where S_{ix} and S_{iz} are the component of a spin-1 operator at site i , namely

$$S_{ix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad S_{iz} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad (S_{ij})^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where D represent the biquadratic nearest-neighbor exchange couplings between sites i and j respectively, D , Ω are respectively the crystal and transverse fields. The first and second summations run over all neighboring pairs of spins. While J_{ij} is the bilinear exchange interactions between spins at site i and j , which is given by the following distribution law

$$P(J_{ij}) = b\delta(J_{ij}-1) + (1-b)\delta(J_{ij})$$

where b is the bond concentration and the function $\delta(x)$ is given as follows:

$$\delta(x) = 1, \text{ for } x=0 \quad \text{and} \quad \delta(x) = 0, \text{ otherwise}$$

Using a single cluster approximation in which attention is focused on a cluster comprising just a single selected spin labelled 0, and the neighboring spins with which it directly interacts, then the Hamiltonian containing 0, namely

$$H_0 = AS_0z + (\Delta + C)(S_0z)^2 + BS_0x \quad (2)$$

where

$$A = -\theta \quad ; \quad B = -\Omega \quad ; \quad C = -D\xi \quad (3)$$

with

$$\theta = \sum_{i=1}^N J_{ij} S_{i\alpha} \quad ; \quad \zeta = \sum_{i=1}^N (S_{i\alpha})^2$$

This single-site Hamiltonian can readily be diagonalized and its eigenvalues and eigenvectors are easily obtained. The three eigenvectors corresponding to the eigenvalues

$$\lambda_k = 2(\Delta + C + \sqrt{1 - \rho} \cos \phi_k) / 3 \quad ; \quad \text{for } k=1,2,3 \quad (4)$$

with

$$\phi_k = \frac{1}{3} \text{Arccos}(-27q/2\rho) + 2(k-1)\pi/3 \quad (5)$$

and

$$\rho = \frac{3\sqrt{3}}{2} (27q^2 + 14p^3 + 27q^2)^{1/2} \quad (6)$$

$$p = -(A^2 + B^2) - (\Delta + C)^2 / 3 \quad ; \quad q = -(\Delta + C)(2A^2 + 2/9(\Delta + C)^2 + B^2) / 3 \quad (7)$$

are

$$|\Psi_k\rangle = \alpha_k |+\rangle + \beta_k |-\rangle + \gamma_k |0\rangle \quad (8)$$

with

$$\alpha_k = \frac{|B(\lambda_k - \Delta - C + A)|}{\sqrt{2[B^2((\lambda_k - \Delta - C)^2 + A^2) + ((\lambda_k - \Delta - C)^2 - A^2)^2]}^{1/2}} \quad (9)$$

$$\beta_k = \frac{\lambda_k - \Delta - C - A}{\lambda_k - \Delta - C + A} \alpha_k \quad ; \quad \gamma_k = \frac{\sqrt{2}}{B} (\lambda_k - \Delta - C - A) \alpha_k \quad (10)$$

In a representation in which $S_{0\alpha}$ is diagonal. The starting point of single-site cluster approximation is a set of formal identities of the type

$$\langle\langle (S_{0\alpha})^p \rangle\rangle_c = \langle \frac{\text{tr}_0[(S_{0\alpha})^p \exp(-\beta H_0)]}{\text{tr}_0[\exp(-\beta H_0)]} \rangle \quad (11)$$

where $(S_{0\alpha})^p$ is the α -component of the spin operator S_0 raised to the power p , $\langle (S_{0\alpha})^p \rangle_c$ denotes the mean value of $(S_{0\alpha})^p$ for a given configuration c of all other spins, i.e when all other spins $S_i(i \neq 0)$ have fixed values. $\langle \dots \rangle$ denotes the average over all spin configurations. tr_0 means the trace performed over $S_{0\alpha}$ only. $\beta = 1/K_B T$, T the absolute temperature and K_B is the Boltzmann constant.

The equations (11) are not exact for an Ising system in a transverse field, they have nevertheless, been accepted as a reasonable starting point in many studies of that system[40].

Let $\langle S_{0\alpha} \rangle_c$ and $\langle (S_{0\alpha})^2 \rangle_c$ denotes respectively the mean value of $S_{0\alpha}$ and $(S_{0\alpha})^2$ for a fixed configuration c for all other spins.

To calculate, $\langle S_{0\alpha} \rangle_c$ and $\langle (S_{0\alpha})^2 \rangle_c$, one has to effect the inner traces in eqs.(11) over the states of the spin 0 and this is the most easily performed using the eigenstates of eqs.(8) as the basic states. In this way, it follows on setting $p=1$ and 2 it turn in eqs.(11) that

$$\langle S_{0\alpha} \rangle_c = \frac{\sum_{k=1}^3 (\alpha_k^2 - \beta_k^2) \exp(-\beta \lambda_k)}{\sum_{k=1}^3 \exp(-\beta \lambda_k)} \quad (12)$$

$$\langle S_{0\alpha} \rangle_c = \sqrt{2} \frac{\sum_{k=1}^3 (\alpha_k + \beta_k) \gamma_k \exp(-\beta \lambda_k)}{\sum_{k=1}^3 \exp(-\beta \lambda_k)} \quad (13)$$

$$\langle (S_{0\alpha})^2 \rangle_c = \frac{\sum_{k=1}^3 (\alpha_k^2 + \beta_k^2) \exp(-\beta \lambda_k)}{\sum_{k=1}^3 \exp(-\beta \lambda_k)} \quad (14)$$

$$\langle (S_{0\alpha})^2 \rangle_c = \frac{\sum_{k=1}^3 ((\alpha_k + \beta_k)^2 / 2 + \gamma_k^2) \exp(-\beta \lambda_k)}{\sum_{k=1}^3 \exp(-\beta \lambda_k)} \quad (15)$$

The magnetizations m_α ($\alpha = z, x$) and the quadrupolar moments q_α ($\alpha = z, x$) are given

by:

$$m_\alpha = \langle \langle f_\alpha(\theta, \zeta) \rangle_D \rangle \quad ; \quad q_\alpha = \langle \langle g_\alpha(\theta, \zeta) \rangle_D \rangle$$

with

$$f_\alpha(\theta, \zeta) = \langle S_{0\alpha} \rangle_c \quad (16)$$

$$g_\alpha(\theta, \zeta) = \langle (S_{0\alpha})^2 \rangle_c \quad (17)$$

where $\langle \dots \rangle$ denotes the average over all configurations of the spins $S_{j\alpha}(j \neq 0)$, and

$\langle \dots \rangle_D$ denotes the average over all configurations of the disorder of the exchange interactions J_{0j} . Using the distribution of J_{0j} mentioned above, the average over the disorder of J_{0j} of f_α and g_α are given by

$$\langle f_\alpha(\theta, \zeta) \rangle_D = \int f_\alpha \left(\sum_{j=1}^N J_{0j} S_{j\alpha} \right) \prod_{j=1}^N [(b \delta(J_{0j} - J) + (1-b) \delta(J_{0j}))] dJ_{0j} \quad (18)$$

$$\langle g_\alpha(\theta, \zeta) \rangle_D = \int g_\alpha \left(\sum_{j=1}^N J_{0j} S_{j\alpha} \right) \prod_{j=1}^N [(b \delta(J_{0j} - J) + (1-b) \delta(J_{0j}))] dJ_{0j} \quad (19)$$

Replacing the function $\delta(x)$ by its expression in equations (18) and (19), the expression of these equations become

$$\langle f_\alpha(\theta) \rangle_D = \sum_{l=0}^N b^l (1-b)^{N-l} \binom{N}{l} f_\alpha(\eta_l) \quad (20)$$

$$\langle g_\alpha(\theta) \rangle_D = \sum_{l=0}^N b^l (1-b)^{N-l} \binom{N}{l} g_\alpha(\eta_l) \quad (21)$$

With

$$\eta_l = J \sum_{i=1}^L S_{i\alpha} \quad (22)$$

and L is the number of bonds such that $J_{0i} = J$, and the $\binom{N}{l}$ are the binomial coefficients given by $m!n!(m-n)!$.

Then the equations of m_α and q_α become

$$m_\alpha = \sum_{l=0}^N b^l (1-b)^{N-l} \binom{N}{l} \langle f_\alpha(\eta_l) \rangle \quad (23)$$

$$q_\alpha = \sum_{l=0}^N b^l (1-b)^{N-l} \binom{N}{l} \langle g_\alpha(\eta_l) \rangle \quad (24)$$

To calculate $\langle f_{\alpha}(\eta_{\parallel}) \rangle$ and $\langle g_{\alpha}(\eta_{\parallel}) \rangle$ we have used the expansion technique for spin-1

Ising systems as follows[39]: Suppose one considers the general product

$\prod_{i=1}^N (\sum_{p_i=0}^2 (S_{iz})^{p_i})$ that containing 3^N terms. From these terms one may collect

together all those terms containing p factors of $(S_{iz})^2$, out of a maximum of N and q

factors of S_{iz} , out of a maximum of L . Such a group is to be denoted by

$\{(S_z)^2, S_z\}_{N,L,p,q}$. Our aim is to expand the functions $f_{\alpha}(\eta_{\parallel})$ and $g_{\alpha}(\eta_{\parallel})$ in terms of

these $\{(S_z)^2, S_z\}_{N,L,p,q}$. Thus, if one writes

$$f_{\alpha}(J \sum_{i=1}^L S_{iz} \sum_{i=1}^N (S_{iz})^2) = \sum_{q=0}^{L-N} \sum_{p=0}^{L-N-q} A_{pq}^{(\alpha)}(L,N) \{(S_z)^2, S_z\}_{N,L,p,q} \quad (25)$$

$$g_{\alpha}(J \sum_{i=1}^L S_{iz} \sum_{i=1}^N (S_{iz})^2) = \sum_{q=0}^{L-N} \sum_{p=0}^{L-N-q} B_{pq}^{(\alpha)}(L,N) \{(S_z)^2, S_z\}_{N,L,p,q} \quad (26)$$

the Problem is to find the coefficients $A_{pq}^{(\alpha)}(L,N)$ and $B_{pq}^{(\alpha)}(L,N)$. To achieve this

it is advantageous to transform the spin-1 system to a spin-1/2 representation

containing the Pauli operators $\sigma_{iz} = \pm 1$. This may be accomplished by setting

$S_{iz} = \tau_{iz} \sigma_{iz}$ with $\tau_{iz} = 0, 1$. In this representation, with $(\sigma_{iz})^2 = 1$ and $(\tau_{iz})^2 = \tau_{iz}$,

eqs.(25) and (26) become

$$f_{\alpha}(J \sum_{i=1}^L \tau_{iz} \sigma_{iz} \sum_{i=1}^N \tau_{iz}) = \sum_{q=0}^{L-N} \sum_{p=0}^{L-N-q} A_{pq}^{(\alpha)}(L,N) \{\tau_{iz}, \sigma_{iz}\}_{N,L,p,q} \quad (27)$$

$$g_{\alpha}(J \sum_{i=1}^L \tau_{iz} \sigma_{iz} \sum_{i=1}^N \tau_{iz}) = \sum_{q=0}^{L-N} \sum_{p=0}^{L-N-q} B_{pq}^{(\alpha)}(L,N) \{\tau_{iz}, \sigma_{iz}\}_{N,L,p,q} \quad (28)$$

and must hold for arbitrary choices of τ_{iz} . Suppose one now chooses the first r out

of the N operators τ_{iz} to be unity, and the remainder zero. Then eqs.(27) and (28)

give

$$f_{\alpha}(J \sum_{i=1}^r \sigma_{iz}, r) = \sum_{q=0}^{L-r} \sum_{p=0}^{L-r-q} A_{pq}^{(\alpha)}(L,N) i^{N-q} \{\sigma_{iz}\}_{r,q} \quad (29)$$

$$g_{\alpha}(J \sum_{i=1}^r \sigma_{iz}, r) = \sum_{q=0}^{L-r} \sum_{p=0}^{L-r-q} B_{pq}^{(\alpha)}(L,N) i^{N-q} \{\sigma_{iz}\}_{r,q} \quad (30)$$

where $\{\sigma_{iz}\}_{r,q}$ is the sum of all possible products of q spin operators, σ_{iz} , out of a

maximum of r , and the $\binom{m}{n}$ are the binomial coefficients $m!/(n!(m-n)!)!$. That is

$$f_{\alpha}(J \sum_{i=1}^r \sigma_{iz}, r) = \sum_{q=0}^r b_q^{(\alpha)}(r) \{\sigma_{iz}\}_{r,q} \quad (31)$$

$$g_{\alpha}(J \sum_{i=1}^r \sigma_{iz}, r) = \sum_{q=0}^r d_q^{(\alpha)}(r) \{\sigma_{iz}\}_{r,q} \quad (32)$$

with

$$b_q^{(\alpha)}(r) = \sum_{p=0}^{r-q} A_{pq}^{(\alpha)}(L,N) i^{r-q} \quad (33)$$

$$d_q^{(\alpha)}(r) = \sum_{p=0}^{r-q} B_{pq}^{(\alpha)}(L,N) i^{r-q} \quad (34)$$

The spin-1 problem of eqs.(25) and (26) containing L variables of type S_{iz} and N variables of type $(S_{ix})^2$ has thus been transformed to a spin-1/2 problem containing r variables of type S_{iz} only. The advantage of doing this is that it now enables one to use directly the results already established in Ref.[41] for the spin-1/2 system. It may also be noted that whereas the coefficients $b_{pq}^{(\alpha)}(r)$ and $d_{pq}^{(\alpha)}(r)$ for the spin-1/2 problem depend on the total number of spins present, the coefficients $A_{pq}^{(\alpha)}(L,N)$ and $B_{pq}^{(\alpha)}(L,N)$ are in fact independent of L and N , as is clear from eqs.(33) and (34). Thus the labels L and N are superfluous and may henceforth be dropped. This could, of course, have been inferred directly from eqs.(25) and (26) by setting one of S_i spins equal to its zero value throughout. Specializing the results of Ref.[41] to a single group of r spins, one has for the current problem

$${}^{(r)}_q b_{pq}^{(\alpha)}(r) = \frac{1}{2^r} \sum_{i=0}^r {}^{(r)}_i \varepsilon(r,q) f_{i,\alpha}(r) \quad (35)$$

$${}^{(r)}_q d_{pq}^{(\alpha)}(r) = \frac{1}{2^r} \sum_{i=0}^r {}^{(r)}_i \varepsilon(r,q) g_{i,\alpha}(r) \quad (36)$$

where

$$\varepsilon(r,q) = \sum_{\mu=0}^r (r-1)^\mu {}^{(r-1)}_{q-\mu} \quad (37)$$

and

$$f_{i,\alpha}(r) = f_{\alpha}(J(r-2i), r) \quad (38)$$

$$g_{i,\alpha}(r) = g_{\alpha}(J(r-2i), r) \quad (39)$$

3. Results and discussion.

In this part we give the results of the model (1) on a simple cubic lattice ($N=6$), especially, the typical behavior of the longitudinal and transverse magnetization as a function of temperature for several values of concentration, for a given values of the biquadratic exchange interactions coupling D , the crystal field Δ and the transverse field Ω .

Once the coefficients $b_{pq}^{(\alpha)}(r)$ and $d_{pq}^{(\alpha)}(r)$ have been calculated, the coefficients $A_{pq}^{(\alpha)}$ and $B_{pq}^{(\alpha)}$ may be found by the following procedure. First, $A_{0q}^{(\alpha)}$ and $B_{0q}^{(\alpha)}$ are got by setting $r=q$ in eqs.(35) and (36), that is

$$A_{0q}^{(\alpha)} = b_{qq}^{(\alpha)}(q) ; B_{0q}^{(\alpha)} = d_{qq}^{(\alpha)}(q) \quad (40)$$

Then, the other $A_{pq}^{(\alpha)}$ and $B_{pq}^{(\alpha)}$ may be obtained by expressing eqs.(35) and (36) as a recurrence relations, namely as

$$A_{r+q,q}^{(\alpha)} = b_{pq}^{(\alpha)}(r) \cdot \sum_{p=1}^{r-q-1} A_{pq}^{(\alpha)} ({}^{(r-q)}_p) \quad (41)$$

$$B_{r+q,q}^{(\alpha)} = d_{pq}^{(\alpha)}(r) \cdot \sum_{p=0}^{r-q-1} B_{pq}^{(\alpha)} ({}^{(r-q)}_p) \quad (42)$$

Then the magnetizations m_{α} ($\alpha=2,x$) and the quadrupolar moments q_{α} ($\alpha=2,x$) are given for an arbitrary coordination number N , by

$$m_{\alpha} = \sum_{l=0}^N b^l (1-b)^{N-l} \binom{N}{l} \sum_{q=0}^l \sum_{p=0}^{l-q} A_{pq}^{(\alpha)} \langle (S_x)^2 S_z^l \rangle_{N,l,p,q} \quad (43)$$

$$q_{\alpha} = \sum_{l=0}^N b^l (1-b)^{N-l} \binom{N}{l} \sum_{q=0}^l \sum_{p=0}^{l-q} B_{pq}^{(\alpha)} \langle (S_x)^2 S_z^l \rangle_{N,l,p,q} \quad (44)$$

Using the simplest approximation of the Zernike decoupling of the type

$$\langle S_x S_y \dots (S_x)^2 \dots \rangle \equiv \langle S_x \rangle \langle S_y \rangle \dots \langle (S_x)^2 \rangle \dots \text{ for } i \neq j+k, \dots$$

and seeing that the number of elements of the group $\{(S_x)^2 S_z^l\}_{N,l,p,q}$ is equal $\binom{l-q}{p} \binom{N-l}{p}$, the eqs (43) and (44) become

$$m_{\alpha} = \sum_{l=0}^N b^l (1-b)^{N-l} \binom{N}{l} \sum_{q=0}^l \sum_{p=0}^{l-q} A_{pq}^{(\alpha)} (m_x)^q (q_x)^p \binom{l-q}{p} \binom{N-l}{p} \quad (45)$$

$$q_{\alpha} = \sum_{l=0}^N b^l (1-b)^{N-l} \binom{N}{l} \sum_{q=0}^l \sum_{p=0}^{l-q} B_{pq}^{(\alpha)} (m_x)^q (q_x)^p \binom{l-q}{p} \binom{N-l}{p} \quad (46)$$

Let put $m = m_x = \langle S_x \rangle$ and $x = q_x = \langle (S_x)^2 \rangle$, and if we replace x in (45) by its expression taken from (46), we obtain an equation for m of the form

$$m = am + vm^3 + \dots \quad (47)$$

where

$$a = \sum_{l=0}^N b^l (1-b)^{N-l} \binom{N}{l} \sum_{p=0}^{N-l} A_{p0}^{(2)} \binom{N-l}{p} (x_0)^p \quad (48)$$

with x_0 the solution of the following equation

$$x_0 = \sum_{l=0}^N b^l (1-b)^{N-l} \binom{N}{l} \sum_{p=0}^{N-l} B_{p0}^{(2)} \binom{N-l}{p} (x_0)^p \quad (49)$$

The critical temperature of the second order transition is determined by $a=1$. In the vicinity of second order transition the magnetization m , is determined by:

$$(m_c)^2 = \frac{1-a}{v} \quad (50)$$

At this temperature the transverse magnetization is given by

$$m_x = \sum_{l=0}^N b^l (1-b)^{N-l} \binom{N}{l} \sum_{p=0}^{N-l} A_{p0}^{(2)} \binom{N-l}{p} (x_0)^p \quad (51)$$

and the quadrupolar moments q_{α} ($\alpha=2, N$) are given by

$$q_{\alpha} = \sum_{l=0}^N b^l (1-b)^{N-l} \binom{N}{l} \sum_{p=0}^{N-l} B_{p0}^{(\alpha)} \binom{N-l}{p} (x_0)^p \quad (52)$$

The right-hand side of eq.(50) must be positive. If this is not the case the transition is of first order. The point at which $a=1$ and $v=0$ is the tricritical point. To obtain the expression for v one has to solve (46) for small m . The solution is of the form

$$x = x_0 + x_1 m^2 \quad (53)$$

where x_1 is given by:

$$x_1 = \sum_{L=2}^N b^L (1-b)^{N-L} \binom{N}{L} \sum_{p=0}^{N-2} B^{(L)}_{p2} \binom{L-2}{p} (x_0)^p + \sum_{L=1}^N b^L (1-b)^{N-L} \binom{N}{L} \sum_{p=1}^N p B^{(L)}_{p0} \binom{L}{p} (x_0)^{p-1} x_1 \quad (54)$$

That is

$$x_1 = \frac{\sum_{L=2}^N b^L (1-b)^{N-L} \binom{N}{L} \sum_{p=0}^{N-2} B^{(L)}_{p2} \binom{L-2}{p} (x_0)^p}{1 - \sum_{L=1}^N b^L (1-b)^{N-L} \binom{N}{L} \sum_{p=1}^N p B^{(L)}_{p0} \binom{L}{p} (x_0)^{p-1}} \quad (55)$$

This yields

$$v = \sum_{L=3}^N b^L (1-b)^{N-L} \binom{N}{L} \sum_{p=0}^{N-3} A^{(L)}_{p3} \binom{L-3}{p} (x_0)^p + \sum_{L=1}^N b^L (1-b)^{N-L} \binom{N}{L} \sum_{p=1}^{N-1} p A^{(L)}_{p1} \binom{L-1}{p} (x_0)^{p-1} x_1 \quad (56)$$

The dependence of the magnetizations m_x and m_z , and the quadrupolar moments q_x and q_z , on the temperature for a fixed values of the biquadratic exchange interactions D , the crystal field Δ and the transverse field Ω are represented in Fig.1 and Fig.2 respectively, for several values of the bond concentration b . The

first order transition is characterized by the gap of the longitudinal magnetization m_z at the transition temperature (Fig.1, $b=0.98, 0.96, 0.94, 0.92$), while the magnetization m_x decreases continuously in the vicinity of the transition temperature and vanishes at $T=T_c$, this is the second-order transition. The temperature dependence of the transverse magnetization for several values of the bond concentration, b , is exhibited in Fig.2 where both first order and second order transition exist. Clearly, the greater the bond concentration the greater is the longitudinal magnetization but in contrast the transverse magnetization m_x decreases when increasing the bond concentration at low temperature. At greater concentrations the transverse magnetization increases with temperature at low temperatures, passes through a peak for the first order transition (Fig.2, $b=0.98, 0.96, 0.94, 0.92$) through a cusp for the second order transition (Fig.2, $b=1$) temperature of m_x and then falls off rapidly as determined by the relation (51). In Fig.3 and Fig.4, the temperature dependence of the longitudinal (q_x) and transverse (q_z) quadrupolar moments is displayed for the simple cubic lattice for a typical value of the bond concentration. At low temperature, the longitudinal q_x increases when increasing the bond concentration contrary to the transverse quadrupolar moment which decreases below the temperature transition, when increasing the bond concentration b . The temperature dependence of m_x and m_z for different values of the transverse field Ω , for a fixed values of biquadratic exchange D , crystal field Δ and bond concentration b , are given in Fig.5 and Fig.6 respectively. The longitudinal magnetization decreases when increasing the transverse field Ω , while the transverse magnetization increases when increasing Ω . From the above we conclude that the model(1) with bond dilution exhibit a tricritical behavior from which we pass from second order transition to the second order transition, and first order transition disappears for sufficiently large bond concentration b (see Fig.1) for a fixed value of the crystal field Δ , the biquadratic exchange D , and the

transverse field Ω , while first order transition is favored by increasing the transverse field Ω , for a given value of bond concentration b , crystal field Δ and biquadratic exchange D .

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Figures Captions

Fig.1 The temperature dependence of the longitudinal magnetization for $\Omega/J=0.1$, $D/J=0.1$ and $\Delta/J=3$. The number accompanying each curve denotes the value of the bond concentration b .

Fig.2 The temperature dependence of the transverse magnetization for $\Omega/J=0.1$, $D/J=0.1$ and $\Delta/J=3$. The number accompanying each curve denotes the value of b .

Fig.3 The temperature dependence of the longitudinal quadrupolar moment for $\Omega/J=0.1$, $D/J=0.1$ and $\Delta/J=3$. The number accompanying each curve denotes the value of the bond concentration b .

Fig.4 The temperature dependence of the transverse quadrupolar moment for $\Omega/J=0.1$, $D/J=0.1$ and $\Delta/J=3$. The number accompanying each curve denotes the value of b .

Fig.5 The temperature dependence of the longitudinal magnetization for $\Delta/J=0.5$, $D/J=0.5$ and $b=0.95$. The number accompanying each curve denotes the value of Ω/J .

Fig.6 The temperature dependence of the transverse magnetization for $\Delta/J=0.5$, $D/J=0.5$ and $b=0.95$. The number accompanying each curve denotes the value of Ω/J .

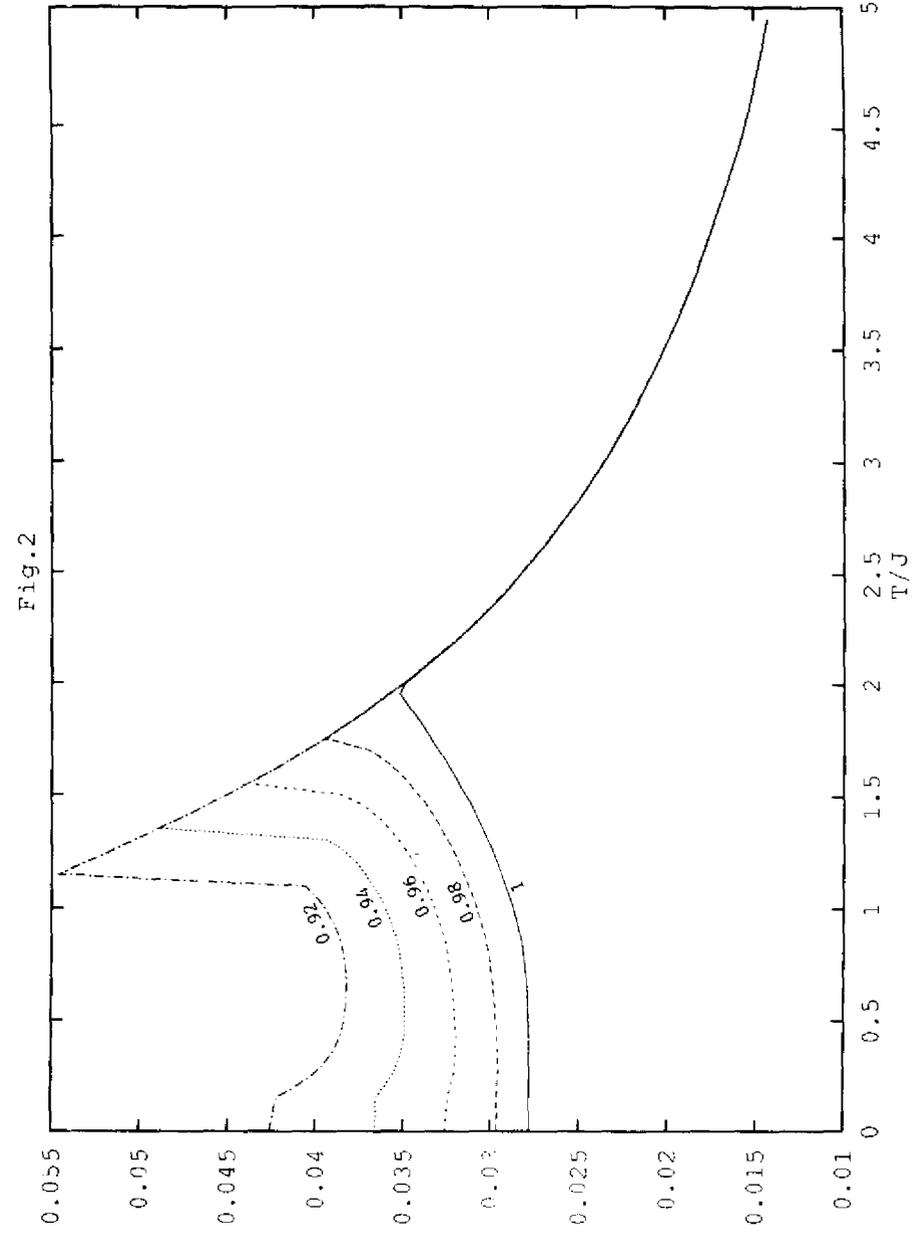
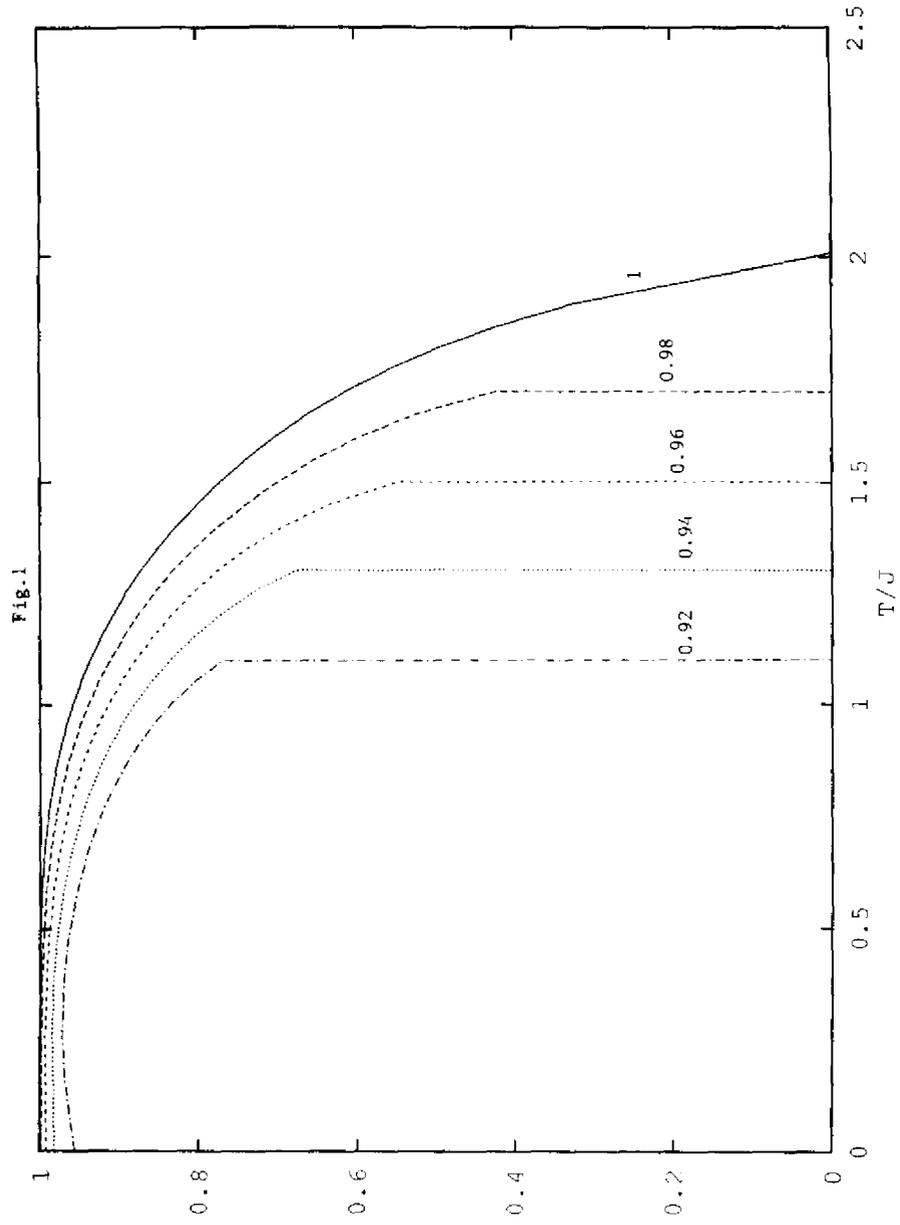


Fig. 3

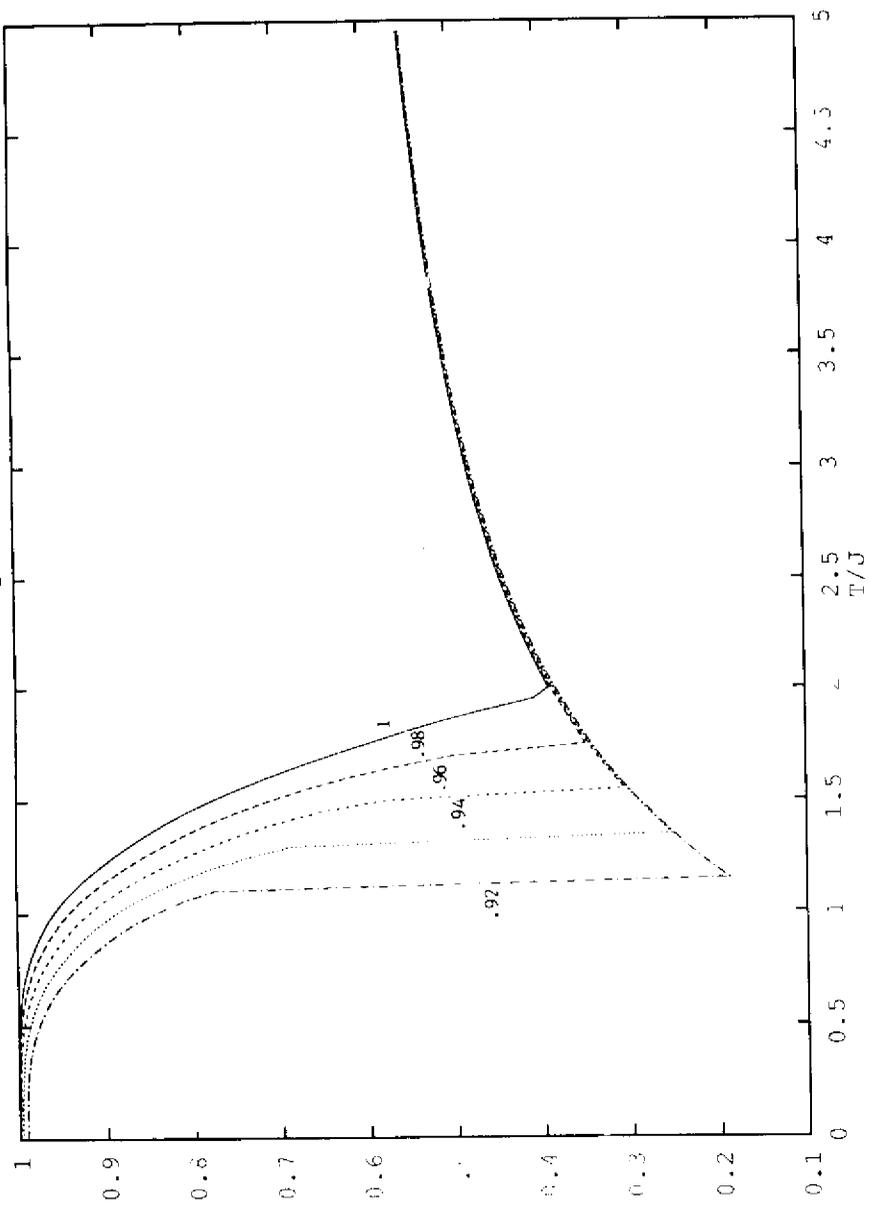


Fig. 4

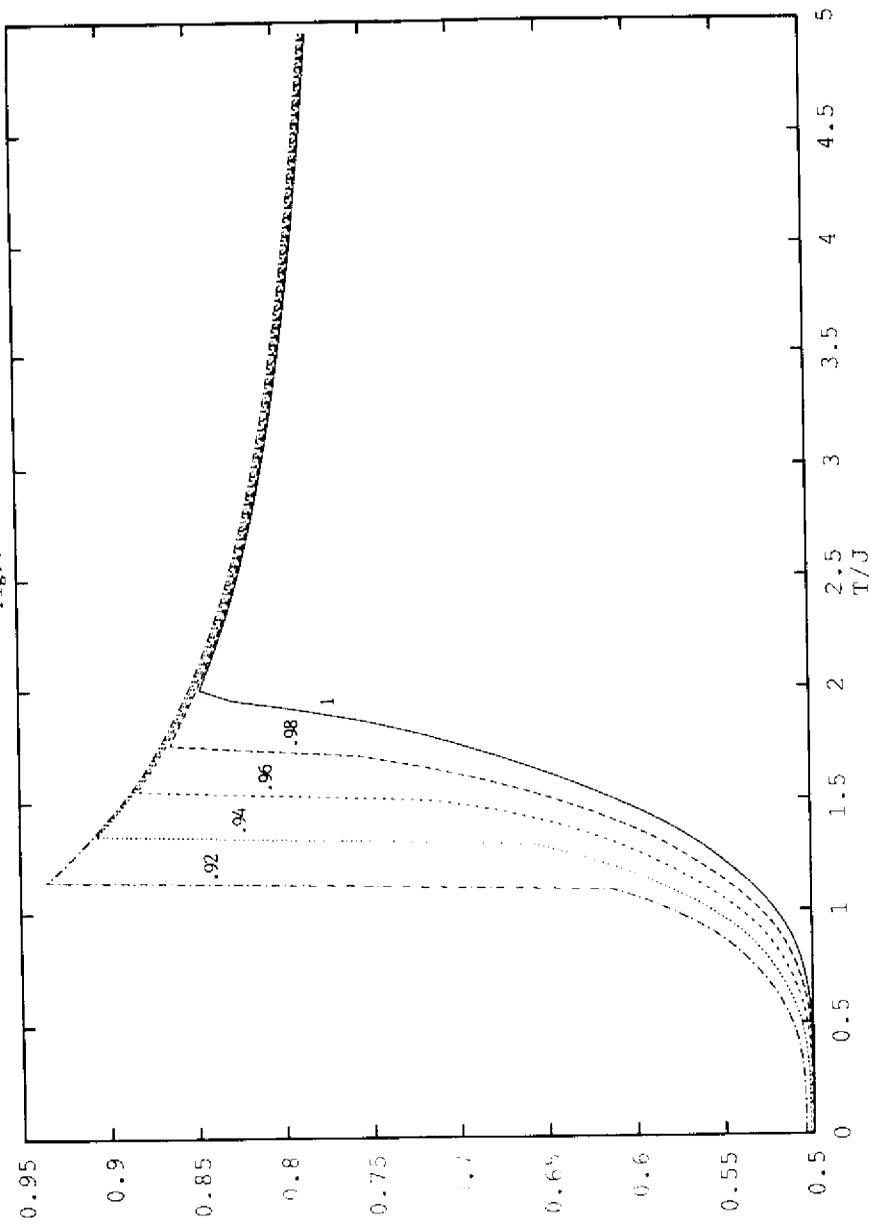


Fig.5

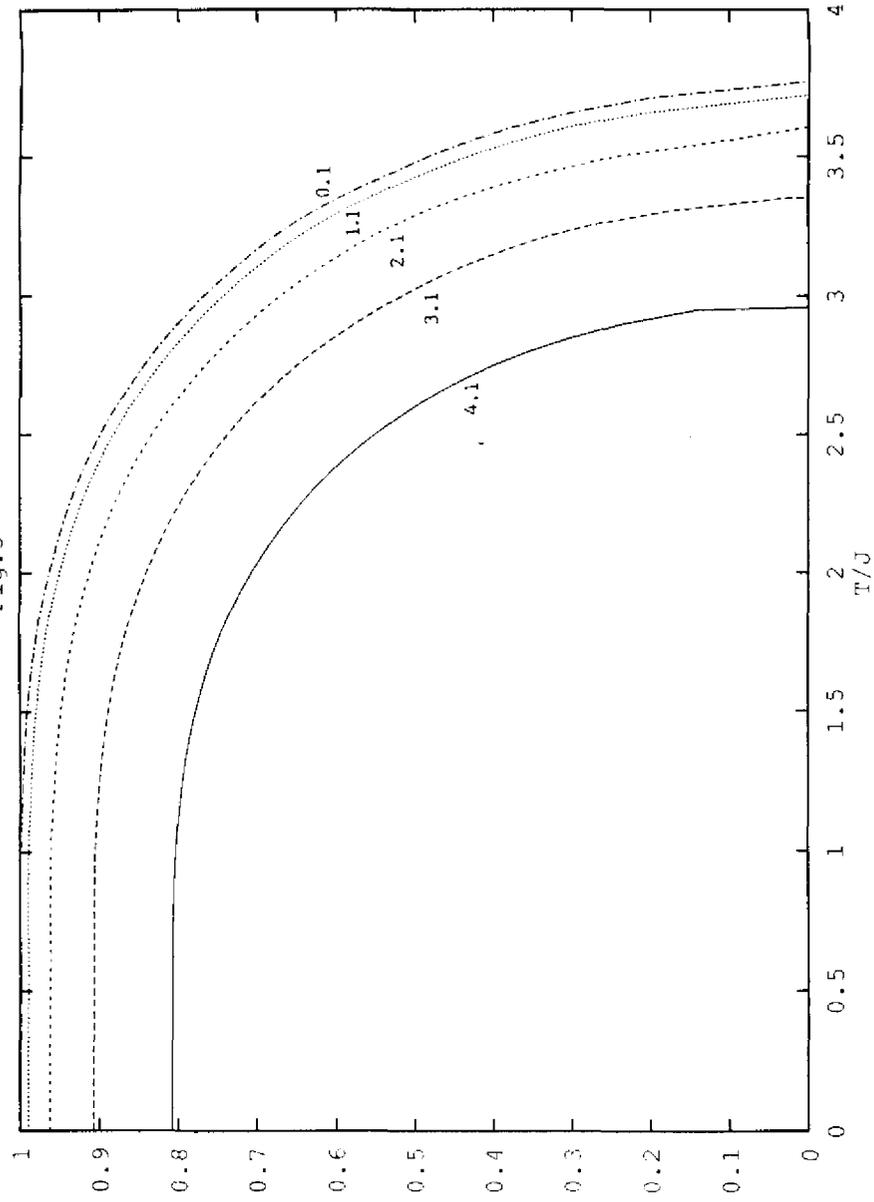


Fig.6

