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3D-Ising Model as a String Theory in Three Dimensional Euclidean Space

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Abstract

A three dimensional string model is analyzed in the strong coupling regime. The contribution of surfaces with different topology to the partition function is essential. A set of corresponding models is discovered. Their critical indices,which depend on two integers (m, n), are calculated analytically. The critical indices of the three dimensional Ising model should belong to this set. A possible connection with the chain of three dimensional lattice Pott's models is pointed out.

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1 Introduction

The understanding and construction of strings in noncritical dimensions is presently attracting interest of string theorists. For example, the string theory in four dimensional space-time is probably a good framework for a theory of strong interactions of the elementary particles. This framework first appeared at the end of seventies,motivated by the dual resonance approach [1]. Moreover, the three dimensional Ising model (3DIM), being a gauge theory with the group Z_2, is, in fact, a theory of random surfaces. Polyakov has put forward the idea [2] of its equivalence with a fermionic string near the point of the second order phase transition. An important element in understanding of the 3DIM as a string theory is the fact that closed surfaces with different topologies contribute to the partition function, and that the module of the string coupling constant is one. Thus, we have a string in the strong coupling regime.

There have been many attempts to construct a fermionic string on the lattice [2]-[9] which corresponds to the 3DIM, but the construction suggested in [8] differs essentially from the others. There, a naive continuum limit (lattice spacing $a \to 0$) of the lattice action exists and three dimentional Dirac fermions appear in the action quadratically. This fact allowed the author to develop in [10] the idea of equivalence of 3DIM with the theory of some kind of matter fields which interact with 2D quantum gravity.

The essential ingredient of the approach considered in [10] is the evaluation of the contribution of the two dimensional manifolds, immersed into 3D-euclidean space and singular at the end points of selfintersection lines, to the functional integral over all surfaces (partition function of the 3DIM). The singular surfaces are essential for the 3DIM, because some of them appear in the partition function with the weight -1, ensuring cancellation of the contribution of a part of surfaces. On the other hand, some of singular surfaces are nothing else but unoriented surfaces immersed into 3D-euclidean space (sphere with Möbius cups).

It happens that the spins of Dirac fermions in the presence of singularities are modified and, also, that the original vacuum must be changed by filling it with them. The change of the spin of the fermions diminishes the central charge of the matter fields and that is why the KPZ equation [11] for the $SL(2.R)$ coupling constant of the 2D quantum gravity, which is the condition of restoration of the reparametrization invariance has a solution for $D = 3$.

In this article I shall further develop the approach presented in ref [10] correcting at the same time the errors made in that article. It will be shown that the matter sector of the model, consists of three scalars (surface coordinates $\vec{X}$) and a spin (0,1) fermionic (b, c) system (corresponding to singularities) with their total central charge equal to one. In conformal gauge the 2D-gravity represented by Liouville acton and the ghost fields has to be introduced. The sum over all possible singular surfaces (and at the same time over all topologies) in the partition function induces a new term in effective action of the model while keeping the vacuum unchanged, namely, inteaction of Liouville field with the spin (0,1) bosonized fermionic (b, c) system. The true vacuum now differs from the original one which corresponded to a simple topology. The set of models is discovered, for which the contribution of the singular surfaces in partition function is essential. Then the scaling behaviour will be analyzed and the critical indices, which depend on two integers (m, n), analitically calculated. The critical indices of the 3DIM should belong to this set.

It is known [11],[22], that $c = 1$ string theory lies on the boundary between the "weak" and "strong" interacting phases of 2D quantum gravity. Perhaps the approach presented here may open a way to cross the $c = 1$ barrier.

2 2D-gravity structure of the 3DIM

In order to reveal the structure of 3DIM near the critical point we shall work with the lattice formulation of the fermionic string, where the three dimensional Dirac fermions ψ_L and ψ_R are placed in the middle points of links of the lattice as explained in ref [8]. The general ideology is that of ref. [10].

As it was shown in ref. [8], this action reproduces exactly the 3DIM partition function at arbitrary temperature, along with the correct sign-factor for the surfaces. The classical continuum limit (lattice spacing $a \to 0$) is

$$S_0 = \lambda \cdot area + \frac{i}{2}\int d^2\xi\sqrt{g}\bar{\psi}(\xi)(\gamma^\alpha\vec{\partial}_\alpha - \vec{\partial}_\alpha\gamma^\alpha)\psi(\xi). \qquad (2.1)$$

In eq.(2.1) $\bar{\psi}, \psi$ are 3D Dirac fields living on the two-dimensional surface $\vec{X}(\xi)$; $g_{\alpha\beta} = \partial_\alpha\vec{X} \cdot \partial_\beta\vec{X}$ is an induced metric , $\gamma_\alpha = \partial_\alpha\vec{X} \cdot \vec{\sigma}$ ($\vec{\sigma}$ are the Pauli matrices) and $\lambda = \frac{1}{a^2}\ln\ th\ J/KT$, where J is the 3DIM coupling constant and T is the temperature.

Here, the following remark is in order. In ref. [8], taking classical continuum limit of the lattice action, we did not distinguish left (or right) fermions, placed at the opposite links of the plaquettes (see fig.3 in [8]) and therefore translational invariance of the theory was broken. This distinction should be made, as one can see by making diagonalization of the action in the momentum space of a flat manifold. However, we will then have a complicated expression for the action in the continuum limit even in a flat case, which is free of anomalies, but like the Nambu action has some other problems. By identifying some of the fields we obtain a simple massless action (2.1) which now has gravitational (or $SO(3)$ rotational) anomalies, absent in 3DIM. Following the general procedure we can correct the situation by adding the 2D-gravity action induced (due to anomalies) by quantum fluctuations of ψ (and also of $\vec{X}$) and restore the original symmetries of the theory [1]. In principle, one is doing the same, when replacing the Nambu action for bosonic string by the Polyakov's action.

[1] I thank A. Polyakov for a criticism at this point.

Therefore, the candidate for the action of 3DIM near the critical point is

$$S_1 = \lambda \cdot area + \frac{i}{2}\int d^2\xi\sqrt{g}\bar{\psi}\Omega^{-1}\sigma^a e_a^\alpha(\partial_\alpha + \Gamma_\alpha)\Omega\psi + W_{grav}(\vec{X}) + S(ghost) \quad (2.2)$$

In (2.2) we have made an algebraic transformation of the fermionic action (2.1) (see [12] for details).Here, $e_a{}^\alpha$ and Γ_α are the zweibeins and $SO(2)$ spinor connection, corresponding to induced metric, Ω is the element of Spin(3) and defines the matrix which rotates ortonormalized Frene basic vectors of the surface into the flat ones.The 2D-gravity action $W_{grav}(\vec{X})$ with the $SL(2,R)$ coupling constant k (or with the charge Q_L in conformal gauge) should be added, ensuring restoration of all anomalous symmetries of the theory. This corresponds to the KPZ [11] equation $C_{tot} = 0$.

Now we should investigate the contribution of Whitney singularities of the two dimensional surfaces, immersed in 3D euclidean space in the string functional integral. Let us represent Ω as

$$\Omega = \Omega_s \cdot \Omega_r, \quad (2.3)$$

where Ω_r is a regular rotation matrix, making a surface flat, but not affecting singularities and Ω_s is the singular part of Ω.

Since the rotation and reparametrization invariances are restored by addition of W_{grav}, we can represent the fermionic action in the form (Ω_r dissapears and metric becomes flat)

$$\frac{i}{2}\int d^2\xi\bar{\psi}\Omega_s^{-1}\sigma^a\partial_a\Omega_s\psi. \quad (2.4)$$

It is easy to calculate Ω_s. In the case of flat surface the Whitney singularity looks like in Fig.1.

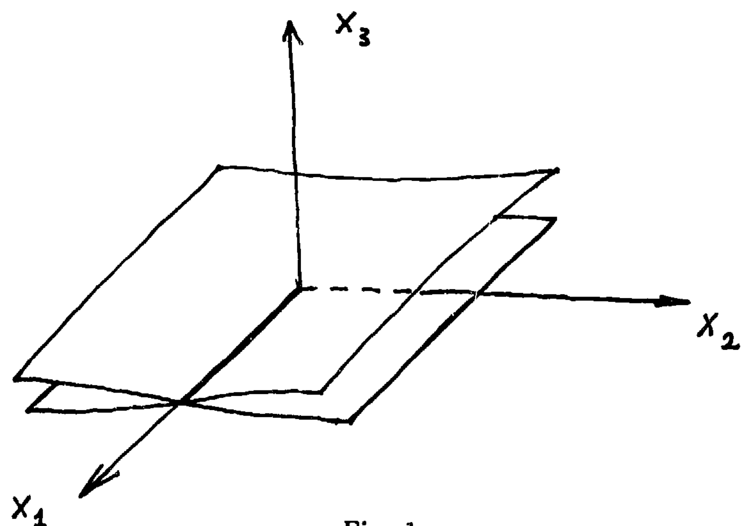

Fig. 1

One can parametrize the surface as follows

$$X_3 = 0, \qquad w = z^2,$$
$$w = X_1 + iX_2, \quad z = \xi_1 + i\xi_2.$$

The straightforward calculations, by use of formula (2.7) in [10], gives

$$\Omega_s^{-1}\partial\Omega_s = \frac{1}{4z}\sigma_3$$
$$\Omega_s^{-1}\bar{\partial}\Omega_s = -\frac{1}{4\bar{z}}\sigma_3 \tag{2.5}$$

and

$$\Omega_s = \left(\frac{z}{|z|}\right)^{-1/2}, \tag{2.6}$$

which means that there is a magnetic flux in the point.

Then, redefining the fields

$$b = \bar{\psi}_L\Omega_s^{-1}, c = \Omega_s\psi_L \tag{2.7}$$

one obtains a free action

$$\frac{1}{\pi}\int d^2z b\partial c + c.c. \tag{2.8}$$

The spins of b, c fields are equal to $\frac{1}{2}(1 \pm Q)$, where $Q = 1$.[2]

Thus, in the presence of vortices (singularities) the conformal spin of fermions changes and the correspodning central charge of the conformal theory becomes $1 - 3Q^2$.

Besides modifying the spin of the fermions, as was shown in [10], one should fill the vacuum state of the system with the (b, c) pairs and the corresponding correlator will appear in the expression for free energy.

This means that when acting by (b, c) operators on the vacuum state we create the singularities on the sphere transforming it into the surface with a different topology.

Next, following ref. [10], instead of induced metric $g_{\alpha\beta} = \partial_\alpha\vec{X}\partial_\beta\vec{X}$, we introduce the independent zweibeins $e_a{}^a$ and fix the gauge. The ghost fields will appear. For our purpose, to calculate the critical indices of the system, the conformal gauge $g_{\alpha\beta} = e^\varphi\delta_{\alpha\beta}$ and the technic, developed by F. David [14] and J. Distler, H. Kawai [15] (DDK), seem to be most appropriate.

[2] Here, I would like to mention that in ref. [10] a mistake of taking $Q = k/2$ has been made. Along with a renormalization of the coupling constant k the field h has also to be renormalized. This should be done consistently with the fact, that the topological origin of the sign-factor of 3DIM (which takes values ± 1) prohibits its renormalization. This imposes the condition $\sqrt{\frac{k}{2}}dh^{ren} \simeq \delta^{(2)}(\xi)$, which implies that $Q = 1$, instead of $Q = k/2$. Besides, there is no need to introduce an additional fermion.

Finally, the free energy F of the 3DIM near the critical point is

$$F = \sum_{N=0}^{\infty} \int \mathcal{D}\vec{X}\mathcal{D}(b,c)\mathcal{D}ghost\mathcal{D}\varphi \cdot \frac{1}{(N!)^2} \cdot \left(\int d^2 z b(z) \int d^2 z' c(z')\right)^N e^{-S}, where \tag{2.9}$$

$$S = \frac{1}{2\pi}\int d^2 z \partial\vec{X}\bar{\partial}\vec{X} + \frac{1}{\pi}\int d^2 z b\bar{\partial}c + c.c. + S(ghost) + \frac{1}{2\pi}\int d^2 z(\partial\varphi\bar{\partial}\varphi - \frac{1}{4}Q_L\sqrt{g}\hat{R}\varphi) + \mu\int d^2 z e^{A\varphi} \tag{2.10}$$

$$\mu = \lambda_0 - \lambda, \quad A = -\frac{Q_L}{2} + \sqrt{Q_L^2 - 8},$$

In eq.(2.9), N is the number of pairs of singularities of the surface, the presence of the factor $(N!)^2$ is a consequence of having N identical b and N identical c fields, λ_0 is bare cosmological constant and Q_L is the Liouville background charge. The expression for Q_L, in terms of $SL(2,R)$ couling cosntant k, is

$$Q_L{}^2 = \frac{k}{k+2} - 2k - 1. \tag{2.11}$$

The KPZ equation [11]

$$c_{tot} = 3 + (1 - 3Q^2) - 26 + (1 + 3Q_L^2) = 0, \tag{2.12}$$

which is the condition for restoration of all anomalous symmetries, can be solved. For $Q = 1$ we obtain $k = -3$ and $Q_L = 2\sqrt{2}$. The corresponding central charge of the matter fields of the theory is equal to one.

3 Scaling properties. Specific heat index α

In order to calculate the critical indices consider the bosonized anticommuting (b, c) system [16]. The action is

$$\frac{1}{4}\int d^2 z b\bar{\partial}c + c.c. = \frac{1}{2\pi}\int d^2 z(\partial\phi\bar{\partial}\phi - \frac{i}{4}Q\hat{R}\phi) \tag{3.1}$$

where $b = e^{-i\phi(z)}$ and $c = e^{i\phi(z)}$.

As they appear in vacuum (see (2.9)), the (b, c) fields are polarized by the gravitational filed φ and also by ϕ (selfpolarization),so that the ground state is reparametrization invariant. After this dressing,

$$b = e^{B\varphi}e^{iD\phi}, \quad c = e^{B\varphi}e^{iD\phi}, \tag{3.2}$$

the conformal dimensions of both b and c field should be (1,1):

$$-\frac{B(B+Q_L)}{2}+\frac{D(D+Q)}{2}=1$$
$$-\frac{\bar{B}(\bar{B}+Q_L)}{2}+\frac{\bar{D}(\bar{D}+Q)}{2}=1 \tag{3.3}$$

Let us introduce the string coupling constant Λ, by writing Λ^{2N} in front of the exponent in (2.9) and take the sum in F over (b,c) pairs. In order to simplify the expression for the effective action S_{eff} we change the sum over N of the series in (2.9) into the product of two independent exponential series for b and c. One can do that, because the condition of mutual cancellation of the (b,c), cosmological and background charges (see (3.5)) will select the equal number of b and c fields in the expression for the path integral (2.9). Then, from eq. (2.9), we obtain:

$$S_{eff}=S+\Lambda\int d^2ze^{B\varphi}e^{iD\phi}+\Lambda\int d^2ze^{\bar{B}\varphi}e^{i\bar{D}\phi} \tag{3.4}$$

This action describes a Liouville theory interacting with a bosonized fermionic (b,c) system. What is the geometrical meaning of the presence of b and c fields in the vacuum ? Appearence of one pair of (b,c) in front of exp in (2.9) is equivalent to a change of vacuum charges $Q(1-g)$ and $Q_L(1-g)$ (g is the genus of Riemann surface) of the fields ϕ and φ to $Q(1-g+(D+\bar{D})/Q)$ and $Q_L(1-g+(B+\bar{B})/Q_L)$ respectively, and can be interpreted as a change of g.

There are two types of singularities of Riemann surfaces in 3D-space. One changes the topology (Fig.2a) and the other (Fig. 2b) does not.

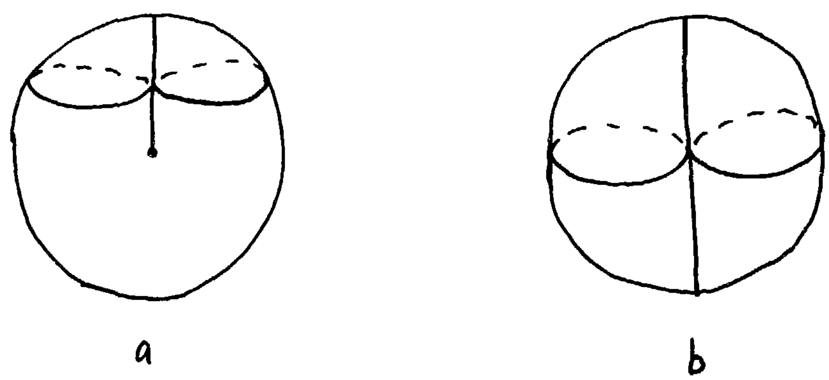

Fig.2

In 3DIM the surfaces with all topologies are present in the partition function. Therefore, the expression (3.4) should contain the dressed (b,c) system, whose appeareance in the vacuum is equivalent to a change of genus by $\frac{1}{2}$ (as in Fig.2a).

Let us take n pairs of (b,c) fields, which, together with m cosmological terms, create the Möbius cap on the manifold, as in Fig. 2a, which corresponds to the following equations

$$\begin{aligned} n(D+\bar{D}) &= -Q/2 \\ n(B+\bar{B})+mA &= -Q_L/2. \end{aligned} \tag{3.5}$$

Equations (3.3) and (3.5) determine $B, \bar{B}, D, \bar{D}$ as functions of integers m and n.

Following DDK [14, 15], we can evaluate specific heat index α by a simple scaling argument. Consider the scaling transformations

$$\phi \to \phi + \phi_0, \quad \varphi \to \varphi + \frac{\varphi_0}{A}, \quad \mu \to \mu e^{-\varphi_0} \quad \text{and} \quad \Lambda \to \Lambda e^{-x} \tag{3.6}$$

with constant ϕ_0, φ_0. In order to find the scaling behaviour we should demand the invariance of the interacting terms in eq. (3.4), which implies

$$\begin{aligned} -x + \frac{B}{A}\varphi_0 + iD\phi_0 &= 0 \\ -x + \frac{\bar{B}}{A}\varphi_0 + i\bar{D}\phi_0 &= 0. \end{aligned} \tag{3.7}$$

Solving these equations, along with (3.3) and (3.5), one finds

$$i\phi_0/\varphi_0 = \frac{Q}{A^2(-\frac{Q_L}{A} + \frac{m}{n-1/2})} = \frac{1}{2(2+\frac{m}{n-1/2})} \tag{3.8}$$

and

$$\frac{x}{\varphi_0} = p = \frac{1}{2n}\left(1 - m - \frac{1}{4(2+\frac{m}{n-1/2})}\right). \tag{3.9}$$

Then, using the scaling transformations, it is not hard to see that

$$S_{eff} \to S_{eff} - \frac{Q}{A}\varphi_0 - iQ\phi_0. \tag{3.10}$$

Equivalently

$$F(\mu e^{-\varphi_0}, \Lambda e^{-x}) = e^{(\frac{Q}{A} + Q\frac{i\phi_0}{\varphi_0})\varphi_0} F(\mu, \Lambda), \tag{3.11}$$

i.e.

$$F(\mu,\Lambda) = \mu^{-(\frac{Q}{A}+Q\frac{i\phi_0}{\varphi_0})} F\left(1, \frac{\Lambda}{\mu^{x/\varphi^0}}\right) = \mu^{2-\alpha}\tilde{F}\left(\frac{\Lambda}{\mu^p}\right). \tag{3.12}$$

Thus, the specific heat (or string susceptibility) index is (use also eq. (3.8))[3]:

$$\alpha = \frac{1}{2(2+\frac{m}{n-1/2})} \tag{3.13}$$

[3] For a review of the definition and old numerical calculations see, for example, [17].

and $\Lambda \sim \mu^p$. Since $m \geq 1$ implies $p < 0$, we are in the strong coupling regime.

If we introduce dressed (b, c) operators, which do not create topology (case of Fig.2b) and which depend on integers (m', n'), their scaling equations (eq. like (3.7)) should be consistent with eq. (3.7). It is easy to find that the condition of consistency is $\frac{m'}{n'} = \frac{m}{n-1/2}$ and to see that the specific heat index α does not change.

4 Correlation length index ν

In order to calculate ν one should investigate long distance behaviour of the correlator of some operators $\Phi(\vec{X})$, placed at distance $\vec{X}$. Then the correlation length and its index ν are defined as follows

$$\frac{\int d\vec{X}(\vec{X})^2 < \Phi(\vec{X})\Phi(0) >}{\int d\vec{X} < \Phi(\vec{X})\Phi(0) >} \equiv \xi^2 \sim \mu^{-2\nu}. \tag{4.1}$$

This expression shows that we need to find the anomalous scaling dimension of $\vec{X}$. In the string picture the correlation length of operators in an external space is calculated by insertion of the operators of the type

$$\Phi(\vec{X}, \cdot) = \int d^2\xi \Phi(\cdot)\delta^{(D)}(\vec{X} - \vec{X}(\xi)) \tag{4.2}$$

into the partition function integral.

To find the anomalous scaling dimension of $\vec{X}$ and then to calculate the correlation length index ν it is enough to consider the world-sheet operator $\mathbf{1}$, with the conformal dimension 0. It seems to us, that in the presence of background Liouville and (b, c) charges Q_L and Q that is

$$\mathbf{1} = e^{-Q_L\varphi}e^{-iQ\phi} \tag{4.3}$$

Imposing the scale invariance of the

$$e^{-Q_L\varphi}e^{-iQ\phi} \cdot \delta^{(3)}(\vec{X} - \vec{X}(\xi)) \tag{4.4}$$

under the transformations (3.6) along with $\vec{X} \to \rho\vec{X}$ and using the scaling arguments employed in the previous section we obtain:

$$\nu = \frac{2 - \alpha}{3}. \tag{4.5}$$

5 Discussion of results

We have obtained sets of critical indices which depend on two integers, m and n. The common origin of all of them is the contribution of singular surfaces to the partition function of random surfaces. Among them one can find the critical indices of 3DIM.

An enormous and diverse work has been done on numerical calculations of 3DIM critical indices. A summary can be found in an article by J.-C. Le Guillou and J. Zinn-Justin [18]. The results quoted in this reference are:

$$\alpha = 0.11, \quad \gamma = 1.239, \quad \nu = 0.631, \quad \beta = 0.327, \quad \eta = 0.0375 \tag{5.1}$$

We can pick a choice of integers m and n, for example $m = 4$, $n = 2$, for which $\alpha \sim 0.107, \nu \sim 0.631$ and obtain the results within experimental errors. However, it seems to us that the case $m = 1, n = 1$, for which

$$\alpha = 1/8, \quad \nu = 0.625 \tag{5.2}$$

is more natural. These values coincide with some old estimates [17]. They also agree with a recent Monte Carlo renormalization group study of indices [19], where $\nu = 0.624(2)$ and $\eta = 0.026(3)$ (we see, that the hardest index η becomes essentially smaller than one in (5.1)).

The case $m = 0, n = 1$ with the indices

$$\alpha = 1/4, \nu = 7/12 \tag{5.3}$$

is in good agreement with the three dimensional indices for a self-avoiding walk(SAW) problem [20]. In the limit, where $n = 1, m \to \infty$ the indices get the values

$$\alpha = 0 \quad \text{and} \quad \nu = 2/3, \tag{5.4}$$

which are in a good agreement with the 3D $U(1)$-model [13, 21]. One can conjecture that the series $n = 1$ and arbitrary m corresponds to 3D Pott's models.

At the end of this analysis I would like to add a few words about magnetic susceptibility index γ. In order to calculate the magnetic susceptibility γ we need to analize the response of the system to the presence of the magnetic field h, i.e. to the presence of the additional term, $h \sum_i \sigma_i$, in the original lattice action of the 3DIM. Then γ is defined by the anomalous scaling dimension of spin field σ. It is possible to develope a rough arguments (see also [17]) which gives the result $\gamma = 2\nu$ (equivalently $\eta = 0$), but, to obtain a precise answer, one should represent the original spin variable σ in terms of stringy fields and calculate complicated correlators, as suggested in ref. [2]. In principle, the direct and more precise numerical calculation of η can be decisive for final determination of the critical indices.

Here we have calculated the indices starting from the spherical topology, but all nonorientable surfaces are taken into account by use of singularities. As for orientable ones, the carefull analysis of operator algebra is needed. The first impression is that the consistent introduction of operators which increase the genus for one, does not change the scaling behaviour.

Presumably, like two dimensional minimal models interacting with gravity, this set of models also corresponds to some topological field theory. This is an interesting question and may be relevant to finding a correct topological definition of the sign-factor of the 3DIM.

In conclusion, I would like to make the last remark. It seems to me that the picture presented here can be interpreted in the following way. We have a matter field $\vec{X}$ interacting with the ordinary Liouville field (representing the fluctuations of metric of 2D gravity) and also interacting with another "Liouville" field originating from the bosonized fermionic (b, c) system and representing the "fluctuations" of topology. (There is also an interaction between two Liouville fields). It is tempting to conjecture that this interpretation of these two Liouville fields may be valid for a general noncritical strings.

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References

[1] J. Schwarz, Phys. Reports 8 (1973) 269.
S. Mandelstam, Phys. Reports **C13** (1974) 259.

[2] A. Polyakov, Phys. Lett. **82B** (1979) 247, Phys. Lett. **103B** (1981) 211
V. Dotsenko, thesis Landau Institute (1981), Nucl. Phys. **285B** (1987) 45.

[3] E. Fradkin, M. Srednicki, L. Susskind, Phys. Rev. **D21** (1980) 2885.

[4] S. Samuel, J. Math. Phys. **21** (1980) 2806, 2815, 2820.

[5] A. Casher, D. Foerster, P. Widney, Nucl. Phys. **251B** [FS13] (1985) 29.

[6] C. Itzykson, Nucl. Phys. **B210** [FS6] (9182) 477.

[7] A. Sedrakyan, Phys. Lett. **137B** (1984) 397.

[8] A. Kavalov, A. Sedrakyan, Nucl. Phys. **B285** [FS19] (1987) 264.

[9] P. Orland, Phys. Rev. Lett. **59** (1987) 2393.

[10] A. Sedrakyan, Phys. Lett. **260B** (1991) 45.

[11] V. Knizhnik, A. Polyakov, A. Zamolodchikov, Mod. Phys. Lett. **A3** (1988) 819.

[12] A. Kavalov, I. Kostov, A. Sedrakyan, Phys. Lett. **175B** (1986) 33.

[13] G. Ahlers, Phase transition, Proceedings of the 1980 Cargèse Summer Institute, edited by M. Levy, J.-C. Le Guillou, J. Zinn-Justin, 1982.

[14] D. David, Mod. Phys. Lett. **A3** (1988) 1651

[15] J. Distler, H. Kawai, Nucl. Phys. **B321** (91989) 509.

[16] D. Friedan, E. Martinec, S. Shenker, Nucl. Phys. **B271** (1986) 93.

[17] R. Brout, Phys. Reports **C10** (1974) 1-61.

[18] J.-C. Le Guillou, J. Zinn-Justin, J. Physique **48** (1987) 19.

[19] C.F. Baillie, R. Gupta, K. Hawick, G. Pawley, Phys. Rev. **B45** (1992) 10438.

[20] J.P. Cotton, J. de Physique Lett **41** (L-23 (1980).

[21] T.C.P. Chiu, J.A Lipa, Phys. Rev. Lett. **51** (1983) 2291.

[22] E. Brezin, V. Kazakov, Al.B. Zamolodchikov Nucl. Phys. **B338** (1990) 673
D. Gross, N. Miljkovic, Phys. Lett. **238B** (1990) 217 P. Ginsparg, J. Zinn-Justin, Phys. Lett. **240B** (1990) 333 G. Parisi, Phys. Lett. **238B** (1990) 209