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EFFECT OF DETECTOR SIZE AND POSITION ON MEASURED VIBRATION SPECTRA OF STRINGS AND RODS

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REPORT

Effect of detector size and position on measured vibration spectra of strings and rods

Submitted to Progress in Nuclear Energy

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ABSTRACT

The effect of detector size and position on the measured spectra has been investigated and the main characteristics of the transfer function were calculated by using a simple model of standing waves. The theoretical estimation was checked in laboratory rod vibration experiments, and the main features were also found in pressure fluctuation spectra measured at Paks Nuclear Power Station. Fundamental result have been achieved using the theory of the superposition of running waves and their reflection on the clamped ends of the rod.

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KIVONAT

A detektorok mérete és elhelyezkedése jelentősen kihat a mérés során kialakuló spektrumokra. Első közelítésben állóhullámok segítségével modellezzük a teljesítményspektrumokban és az átviteli függvényekben jelentkező általános tendenciákat és elnyeléseket. Elméleti leírásunkat laboratóriumi rúdrezgési kísérletekkel igazoljuk. A vizsgált jelenségek a Paksi Atomerőműben mért nyomásspektrumokban is megtalálhatók voltak. A tanulmány második részében tárgyalt modell, ami a gerjesztés támadáspontjából kiinduló és a megfogásoknál visszaverődő haladó hullámok szuperpozícióján alapul, a detektor véges méretéből és elhelyezkedéséből származó jelenségeken túl a húr és rúdrezgés sajátfrekvenciáit is visszaadja.

Effect of detector size and position on measured vibration spectra of strings and rods

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Abstract. The effect of detector size and position on the measured spectra has been investigated and the main characteristics of the transfer function were calculated by using a simple model of standing waves. The theoretical estimation was checked in laboratory rod vibration experiments, and the main features were also found in pressure fluctuation spectra measured at Paks Nuclear Power Station. Fundamental result have been achieved using the theory of the superposition of running waves and their reflection on the clamped ends of the rod.

1 Introduction

Vibrations are usually measured by sensors which have finite mass and size and which are connected at a certain position of the vibrating object. Due to the different motion of different points of the object, change of detector position can have strong effect on measured frequency spectra. This position may not generally be chosen arbitrarily. However it is important to know the effect of the chosen position on the spectra. This effect can be described by a transfer function which depends on the position and length of the detector and on frequency. Let us call this function the weight function to distinguish it from the transfer function given by the manufacturer; this latter, transfer, function depends only on the type of detector. It is emphasized that the weight function depends not only on position and size of the detectors but also depends on the type of vibration.

In this paper, weight functions of string and rod vibrations are described by standing and running wave models. Dependence on position and size of detectors are investigated with calculations; the calculated results are compared with laboratory and operating NPP measurements.

2 Detection of standing waves

During detection of standing waves the signal of current from the detector depends on the transfer function of the detector, but it is also influenced by the position of the detector. The transfer function is mainly dependent on frequency and it is generally given by manufacturer. However the position dependent factor, the weight function, depends on the features of the standing waves required to be measured. Let the weight function be $g(x)$ where x stands for the position. $g(x)$ describes that feature of the measurement that the same standing wave $f(x)$ is sensed by the detector in different ways in different positions of the string or rod. Thus the measured signal is proportional to the absolute value of the standing wave. Let the proportional factor be equal to one, then

$$g(x) = |f(x)|,$$

where $f(x)$ is a function describing the standing wave. Let us investigate the simplest case when the standing wave has a sinusoidal shape. The weight function $g(x)$ can be described as

$$g(x,\lambda) = 2\left|\sin(2\pi\frac{x}{\lambda})\right|,$$

where λ stands for the wavelength and x is the distance of the detector from the starting point (from a node) of the standing wave.

In practice, detectors have finite length[1] so an integrated value must be used along the detector:

$$g(x_0,\lambda,l) = \frac{1}{l}\int_{x_0-\frac{l}{2}}^{x_0+\frac{l}{2}} |g(x,\lambda)|\,dx = \frac{1}{l}\int_{x_0-\frac{l}{2}}^{x_0+\frac{l}{2}} \left|2\sin(2\pi\frac{x}{\lambda})\right|\,dx,$$

where l denotes the length of the detectors and x_0 stands for the central point of the detector. After integration the weight function will be

$$g(x_0,\lambda,l) = \frac{2\lambda}{l\pi}\left|\sin(2\pi\frac{x_0}{\lambda})\sin(l\frac{\pi}{\lambda})\right|.$$

For string vibration (and for other vibrations which can be described with analogous equations, e.g. pressure waves) after substituting expression

$$\lambda = c/\nu \tag{1}$$

— where ν stands for the frequency, c is constant (generally a propagating velocity) —, the weight function

$$g(x,\nu,l) = \frac{2c}{l\pi\nu}\left|\sin(\frac{2\pi x\nu}{c})\sin(\frac{\pi l\nu}{c})\right| \tag{2}$$

[1] For example SPNDs measuring neutron signals have an approximate length of 20 cm.

is obtained.

For rod vibration (1) is not valid. To get an analogous expression let us start from the differential equation of rod vibration.

$$\frac{\partial^2 y}{\partial t^2} + \frac{IE}{A\varrho}\frac{\partial^4 y}{\partial x^4} = 0.$$

A particular solution of the above differential equation is the following equation of a harmonic plane wave

$$\psi = K \cdot \sin\left[\omega\left(t - \frac{x}{\sqrt{r_i a \omega}}\right) + \alpha\right], \tag{3}$$

where $r_i = \sqrt{I/A}$ is the radius of inertia of the cross section of the rod, $a = \sqrt{E/\varrho}$ stands for the propagating velocity of a longitudinal wave in the rod, and $\omega = 2\pi\nu$ denotes the angular frequency. The following connection is valid between the wavelength and frequency of the plane wave (3).

$$\nu\lambda^2 = 2\pi r_i a. \tag{4}$$

However real problems with different boundary conditions are only rarely satisfied by these kinds of waves because the field of solutions of the fourth order differential equation of rod vibration cannot be stretched out by harmonic functions only. This means that different constraints have deforming effect on waves, especially at relatively long wavelengths (when the wavelength is comparable to the length of the rod). At shorter wavelengths (higher frequency range) harmonic components are dominant.

Let us see what difference it makes. Equation (4) is the exact connection between frequency and wavelength of standing waves developing on rods with pin connection at both ends. For other constraints it is an approximation and there is no closed formulae (like (4)). However error can be estimated from comparing eigenfrequency series of rods with different constraints. These eigenfrequencies can be calculated from the following formula, see Lipcsei et al. (1992):

$$\nu_n = \frac{1}{2\pi}\Gamma_n\frac{1}{L^2}\sqrt{\frac{IE}{A\varrho}} = \frac{1}{2\pi}\Gamma_n\frac{1}{L^2}r_i a, \tag{5}$$

where values of Γ_n are dimensionless constants depending on the type of constraints only, and L is the length of the rod. For small n-s Γ_n can be calculated from transcendent equations, for n larger than 4 the following approximation with small errors (less then 1 %) can be given:

$$\Gamma_n \approx \begin{cases} \pi^2(n-\frac{1}{2})^2 & \text{for built in one end – free other end} \\ \pi^2 n^2 & \text{for pin connections at both ends} \\ \pi^2(n+\frac{1}{4})^2 & \text{for built in one end — pin connection at other end} \\ \pi^2(n+\frac{1}{2})^2 & \text{for both ends built in.} \end{cases}$$

Thus expression

$$\nu_n = \frac{2\pi r_i a}{\lambda_n^2}$$

is valid for rods with infinite length (or with finite length and pin connections at both ends). The above expression is a good approximation at higher frequencies in other cases too. Then the weight function of rod vibration is

$$g(x,\nu,l) = \frac{2\sqrt{c_r}}{l\pi\sqrt{\nu}}\left|\sin\left(\frac{2\pi x\sqrt{\nu}}{\sqrt{c_r}}\right)\sin\left(\frac{\pi l\sqrt{\nu}}{\sqrt{c_r}}\right)\right|, \tag{6}$$

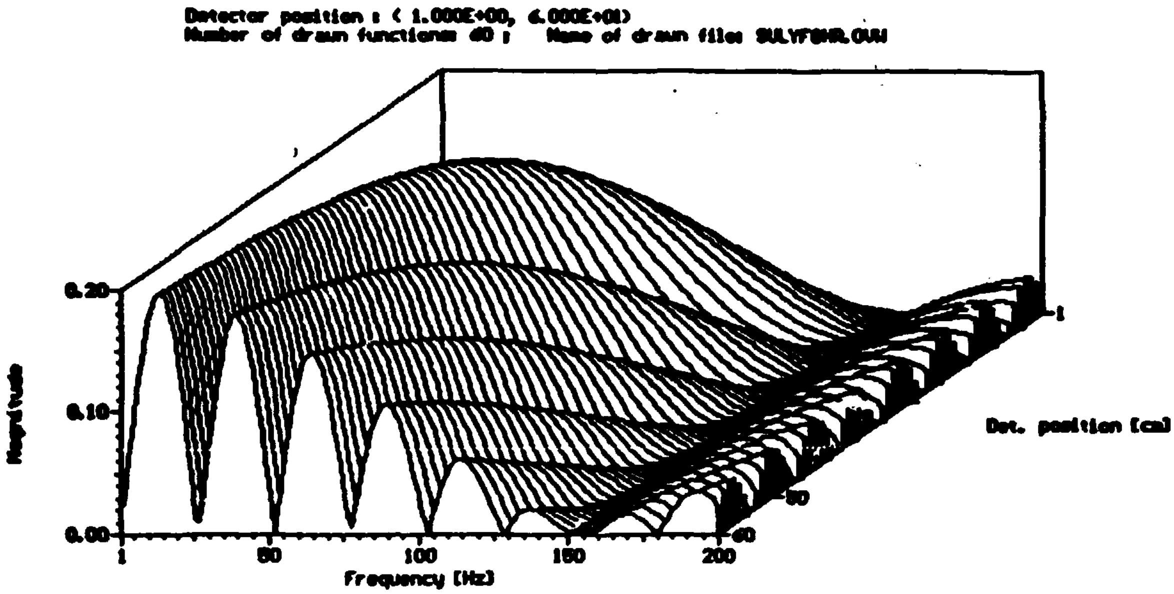

Fig. 1. Weight function of a 20 cm long detector on a half infinite string ($c = 30$ m/s)

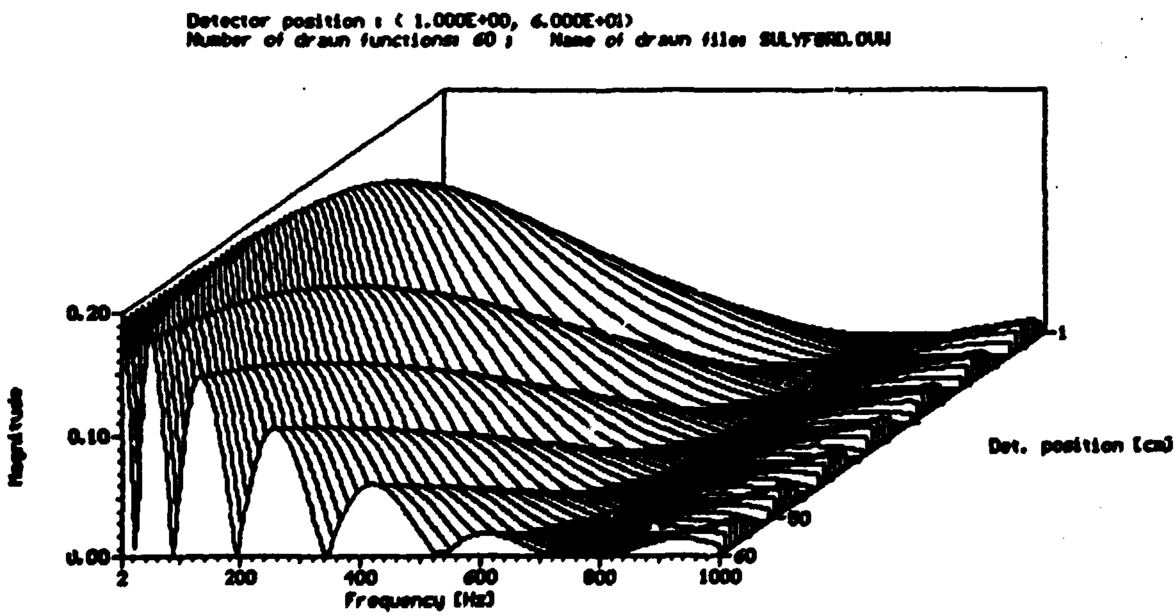

Fig. 2. Weight function of a 20 cm long detector on a half infinite rod ($c = 30$ m/s)

where $c_r = \nu\lambda^2 = 2\pi r_i a$. It can be seen directly from (2) and (6) that the sensitivity is lower in the higher frequency range.

Weight functions of a half infinite string and rod are shown with a detector length of 20 cm in Figs. 1 and 2, respectively. The frequency of standing waves is assumed to be continuous.

Figures 1 and 2 help to analyse effects which can be seen from weight functions (2) and (6). The most important feature is that weight functions have many zero points, which are called sinks based on the view of these zero points in spectra.

Let us interpret the zero points of (2) and (6). In the spectra there are two kinds of sinks at a given (and fixed) detector position and size.

One kind of sink depends on the position of the detector (see the argument of the first sinusoidal term of (2) or (6)). The longer the distance of the detector from the constraint, the more sinks can be found in a given frequency range. At any given frequency there are positions of the string or rod — these positions are equidistant — where the detector is absolutely insensitive (!) to this standing wave frequency.

The other kind of sinks (determined by the second sinusoid term of (2) or (6)) — which is like a puddle in Figs. 1 and 2 — depends on the size of the detector. The longer the detector, the more sinks can be found in a given frequency range. In an ideal case, with zero length detector, there are no sinks like this: this type of behaviour disappears!

The following statement is valid for both kinds of sinks. Along the frequency sinks are equidistant for strings and get rarer quadratically $(1, 4, 9, 16, \ldots)$ for rods.

3 Experimental results

The shape and features of the weight functions analysed above can be seen in measured spectra. Equations (2) and (6) mean weighting factors by the absolute value of the product of two sinusoidal functions. The finite size of real detectors has a strong effect on weight functions. Its integrating effect causes decreasing sensitivity at higher frequencies (see factors $1/\nu$ and $1/\sqrt{\nu}$ in (2) and (6), respectively). Otherwise finite size has a similar effect to position: it causes insensitivity at certain frequencies: sinks appear at these points of spectra.

3.1 Deforming effect of weight function on measured spectra in laboratory experiments

Laboratory experiments have been carried out to determine the vibration response of a single fuel rod to white noise excitation (Fig. 3). The detectors utilized were B&K 4375. Pin connection was used at both ends of the rod and white noise excitation through one of the constraints.

Vibration spectra measured at different detector positions and weight functions belonging to the measurements are shown in Figs. 4, 5 and 6. This kind of weight function gives information only on sinks coming from the detector position and on the main characteristics of spectra.

It can be seen that the structure of weight functions appears in measured spectra. There are sinks in the spectra at each zero point of the weight function and weight functions are followed by the global form of spectra.

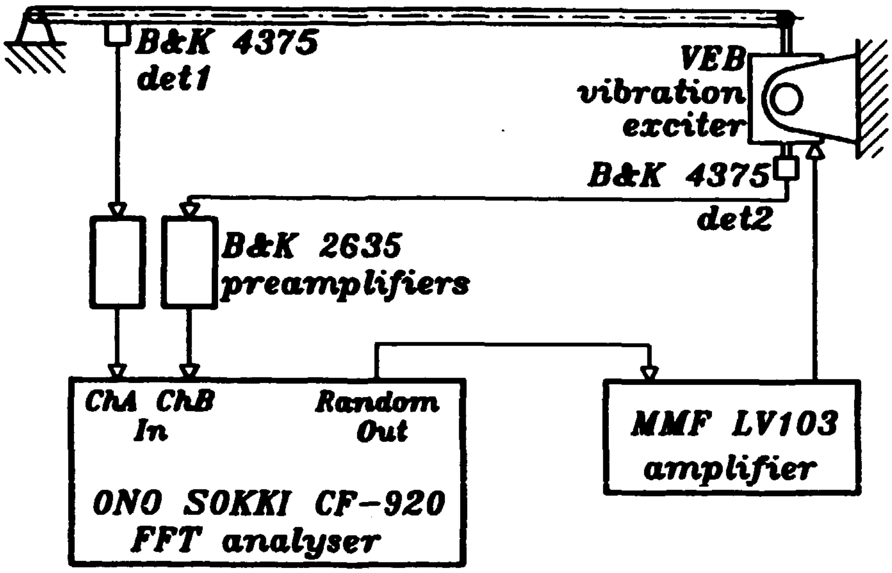

Fig. 3. Scheme of measuring equipment

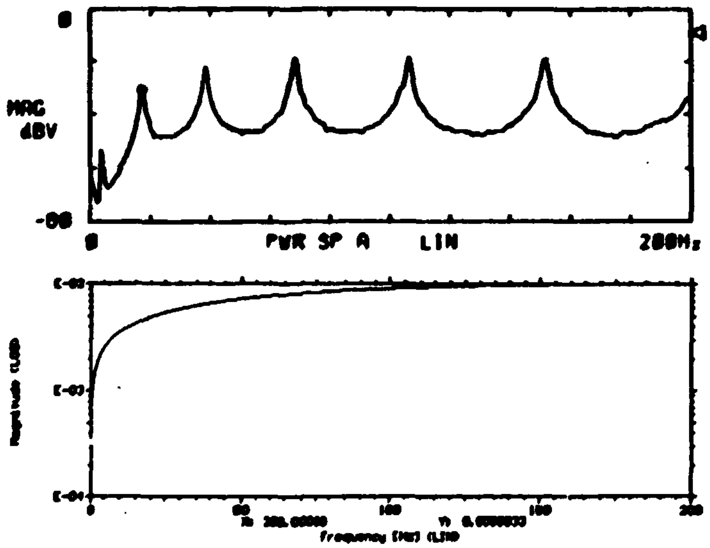

Fig. 4. Vibration spectrum (above) and weight function (below)
of a 130 cm long rod with pin connection at both ends
Detector position is 10 cm from one of the constraints

3.2 Effects described by weight function in measured pressure fluctuation spectra in working PWR

It is well known that standing waves developing in closed vessels or pipelines can be described by equations analogous to those of string vibration (Grunwald et al. 1982, 1985; Mullens et al. 1982; Turkcan 1982). Thus, weight functions are similar to those of string vibration.

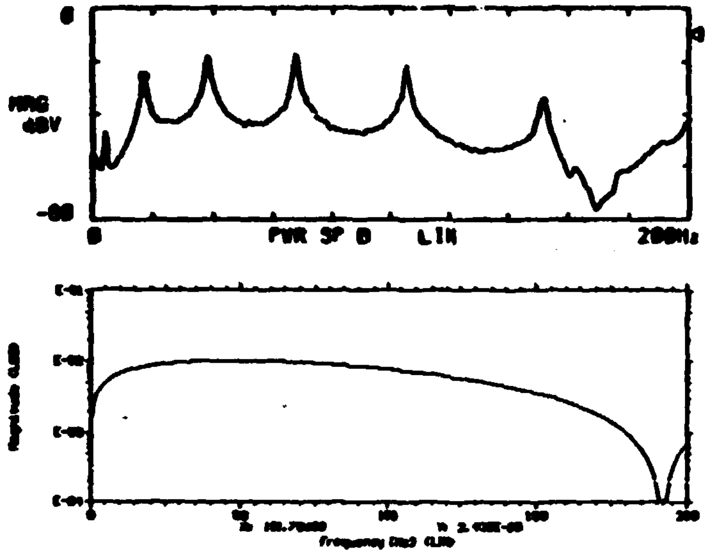

Fig. 5. Vibration spectrum (above) and weight function (below) of a *130 cm long rod with pin connection* at both ends
Detector position is 20 cm from one of the constraints

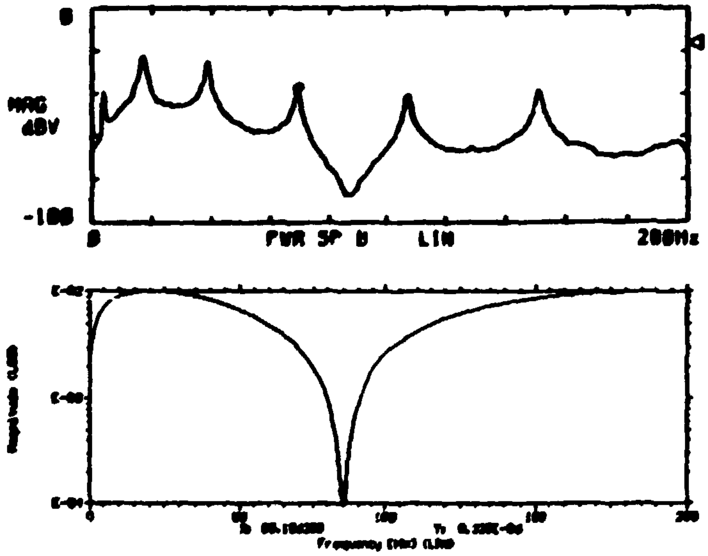

Fig. 6. Vibration spectrum (above) and weight function (below) of a 130 cm long rod *with pin connection* at both ends
Detector position is 30 cm from one of the constraints

APSDs of pressure fluctuation measured in two different units of Paks NPP and the corresponding weight functions are shown in Figs. 7 and 8, respectively. The sensing lines of the two units are not of the same length: this causes a difference between the two weight functions.

In Unit 1 the pressure transducer is attached near to the end of the impulse line, in Unit 2 the pressure transducer is attached at a position two-thirds of the way along the 50 m sensing line. It is well known that the nodes of longitudinal waves are positioned where the density fluctuation is the highest, while bellying occurs where it is the smallest. Consequently there is always a node at the opening (entrance) of the sensing line, while bellying occurs at its very end. Pressure fluctuation spectra and their weight functions are shown for Unit 1 and 2 in Figs. 6 and 7, respectively. When analysing these measured spectra one must not forget that their structure is first of all due to the physical system itself and not due to the weight function since the system itself has no white spectrum. Nevertheless, sinks coming from weight functions can very easily be identified. In our case the exact location of the sensor has not been known in Unit 2 but it can be estimated from the position of the sinks in the spectra: it must be at about 13 m from the closed end of the sensing line. It can also be seen from the spectra measured in Unit 2 that the standing wave has a smaller amplitude in comparison to the measurements in Unit 1 because the pressure transducer is nearer to a node.

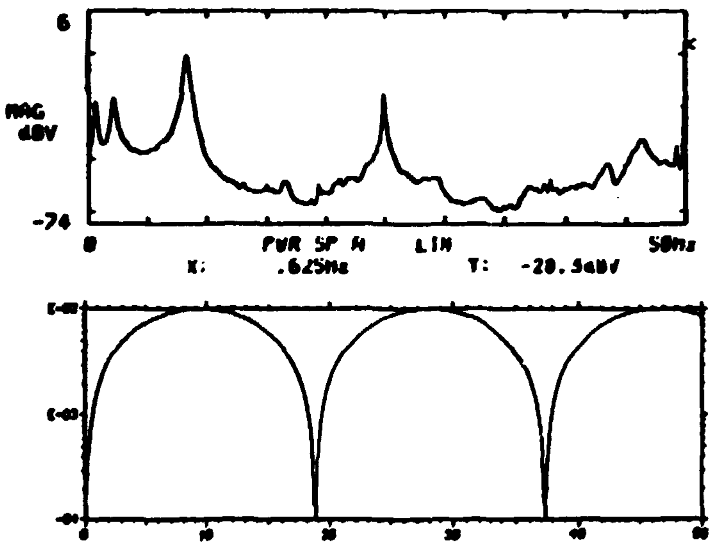

Fig. 7. Pressure fluctuation spectrum measured in Unit 1 (above) and the accompanying weight function (below) (c = 1300 m/s, T = 80°C, p = 130 bar, detector position is 4 m from the end of the 40 m long sensing line)

In each case the expected spectrum can be estimated by multiplying the weight function by the noise spectrum. It can easily be calculated that when a peak of the noise source spectrum nearly coincides with a sink of the weight function, a shift of the peak from the sink frequency can be observed.

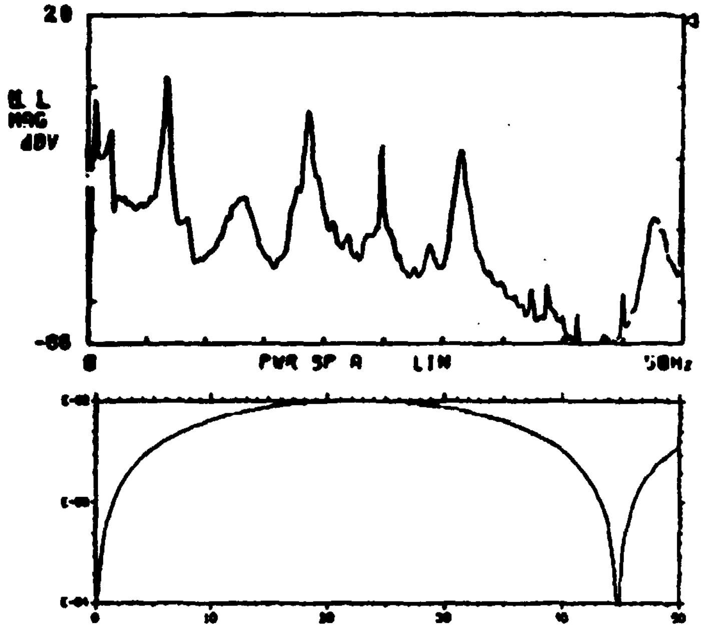

Fig. 8. Pressure fluctuation spectrum measured in Unit 2 (above) and the accompanying weight function (below) (c = 1300 m/s, T = 80°C, p = 130 bar, detector position is 13 m from the end of the 40 m long sensing line)

4 Modelling of vibrating systems with running waves

4.1 Running waves with a single reflection

For finite length strings and rods there are standing waves at discrete frequencies only, so amplitude can be described as interfer[illegible] running waves. Let us investigate a rod (or string or liquid in a closed pipeline) of length d which is excited at one of the ends with frequency ν. Then waves are running along the rod and are reflected at the other rod end with phase π. At distance x' from the excited end the starting wave can be described as

$$\Psi_0 = Ce^{-\kappa_1 t - \kappa_2 x'} \sin\left[2\pi\left(\nu t - \frac{x'}{\lambda}\right)\right],$$

where ν is the frequency of the wave, λ stands for the wavelength, κ_1 and κ_2 denote damping factors depending on time and position, respectively. The reflected wave is

$$\Psi_1 = -Ce^{-\kappa_1 t - \kappa_2(2d - x') - \kappa_3} \sin\left[2\pi\left(\nu t - \frac{2d - x'}{\lambda}\right)\right],$$

where κ_3 stands for the damping at the reflection. Thus amplitude at x' can be described as the sum of the two above waves. At first approximation let us investigate an ideal case: let κ_i ($i = 1, 2, 3$) be equal to zero. Then the amplitude is

$$\Psi = \Psi_0 + \Psi_1 = 2C \sin\left(2\pi \frac{d - x'}{\lambda}\right) \cos\left[2\pi\left(\nu t - \frac{d}{\lambda}\right)\right].$$

The weight function can be obtained from the amplitude, as earlier (see Section 2):

$$g(x, \lambda, l, t) = \frac{2}{l}\left| \int_{x-\frac{l}{2}}^{x+\frac{l}{2}} 2C \sin\left(2\pi \frac{x}{\lambda}\right) \cos\left[2\pi\left(\nu t - \frac{d}{\lambda}\right)\right] dx \right|,$$

where $x = d - x'$ is the detector position (the distance from the non-excited end) and l stands for the length of the detector. After integration

$$g(x, \lambda, l, t) = \frac{2}{l}\left| \frac{2C\lambda}{\pi} \sin\left(\frac{2\pi x}{\lambda}\right) \sin\left(\frac{l\pi}{\lambda}\right) \cos\left[2\pi\left(\nu t - \frac{\lambda}{l}\right)\right]\right|.$$

Measurement gives time averages, so the weight function can be described as

$$g(x, \lambda, l, t) = \frac{2}{l}\left| \frac{2C\lambda}{\pi} \sin\left(\frac{2\pi x}{\lambda}\right) \sin\left(\frac{l\pi}{\lambda}\right)\right| \frac{1}{t}\int_0^t \left|\cos\left[2\pi\left(\nu t' - \frac{\lambda}{l}\right)\right]\right| dt'.$$

For large t the following approximation is valid,

$$\frac{1}{t}\int_0^t \left|\cos\left[2\pi\left(\nu t' - \frac{\lambda}{l}\right)\right]\right| dt' \approx \frac{2}{\pi}.$$

For $t \to \infty$ (for measurements with good statistics) the weight function is

$$g(x, \lambda, l) = \left| C_0 \frac{\lambda}{l} \sin\left(\frac{2\pi x}{\lambda}\right) \sin\left(\frac{l\pi}{\lambda}\right)\right|,$$

where all constants are collected in C_0. This formula agrees with (2) and (6) with substitutions $\lambda = c/\nu$ and $\lambda = \sqrt{c_r/\nu}$, respectively.

Numerically calculating the weight function we found that increasing damping factor κ_3 leads to smaller depth of the sink (see Fig. 9).

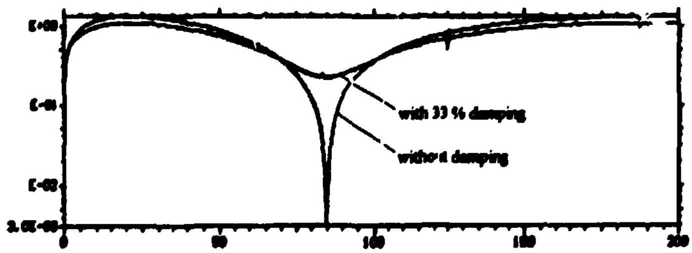

Fig. 9. Weight function of waves reflected without damping and with 33 % damping $c_r = 32 \text{ m}^2/\text{s}$

4.2 Running waves with multiple reflections

Accuracy of the estimation of calculated weight functions can be increased by using multiple reflection. Multiple reflections give rise to running waves on the rod.

$$\Psi_0 = Ce^{-\kappa_1 t-\kappa_2 x'} \sin\left[2\pi\left(\nu t - \frac{x'}{\lambda}\right)\right],$$

$$\Psi_{2i+1} = -Ce^{-\kappa_1 t-\kappa_2(2id-x')-\kappa_3(2i-1)} \sin\left[2\pi\left(\nu t - \frac{2id-x'}{\lambda}\right)\right],$$

$$\Psi_{2i} = Ce^{-\kappa_1 t-\kappa_2(2id+x')-\kappa_3 2i} \sin\left[2\pi\left(\nu t - \frac{2id+x'}{\lambda}\right)\right],$$

where Ψ_0 stands for the starting wave and Ψ_k-s are the same waves after the k^{th} reflection. Measurable vibrations can be estimated as the sum of these waves.

$$\Psi = \sum_{i=0}^{n} (\Psi_{2i} + \Psi_{2i+1}).$$

For one of the laboratory measurements the weight function was calculated with data $\kappa_1 = \kappa_2 = 0$, $\kappa_3 = 0.1$, $n = 50$ and formulae $\lambda = \sqrt{c/\nu}$. In these calculations, with a very small detector, we limited ourselves to a point detector approximation, therefore the weight function was estimated simply as the time average of $|\Psi|$:

$$g(x,\lambda) = \frac{1}{t}\int_0^t |\Psi(t')|\,dt'.$$

The measured vibration spectrum and calculated weight function are shown in Fig. 10.

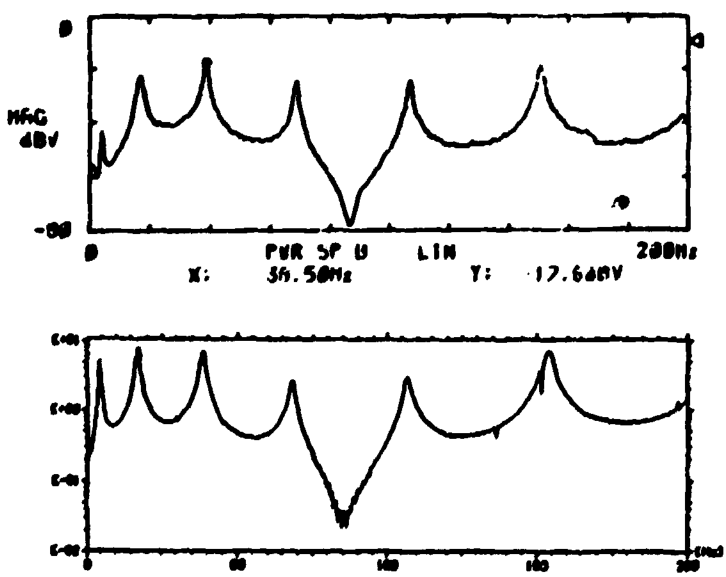

Fig. 10. Vibration spectrum (above) and weight function (below) of a 130 cm long rod subjected to white noise excitation with pin connection at both ends
Detector position is 30 cm from one of the constraints

Figure 10 shows a strong similarity between the measured vibration spectrum and calculated weight function. The main difference is at the low frequency range; this is caused by the transfer of the accelerometer itself.

It is important to note that measured vibration spectra come from acceleration whereas weight functions are calculated from amplitude values. Thus calculated spectra should be multiplied by ω^2. In spite of this, there is good agreement between the two functions because the frequency dependence of damping was neglected in the calculated model (it is well known that the power of viscous friction (damping) is proportional to the square of frequency).

6 Conclusions

A formalism has been derived for estimating the effects of detector position and length when measuring oscillation spectra of strings, rods or in tubes. Estimated weight functions (2) and (6) can help very much in interpreting measured spectra—especially in understanding missing peaks or other elements of the spectra.

These formulae explain the appearance of sinks in vibration spectra. It has been shown by numerical calculation that the depth of a sink strongly depends on the damping factor of reflections (Fig. 8). As a consequence one can claim that for an ideal sensing line the closed end of that should have strong damping in reflection. Therefore we suggest that some damping material be put at the end of sensing lines.

In the case of single reflections of running waves we could get back the weight function derived by the standing wave method. From the formulae estimated by multiple reflections of running waves one can see that the first term of the sum gives back the excitation, the second term is responsible for the sinks, while all other structural components of spectra are from higher order terms.

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References

Grunwald, G., G. Hessel, P. Liewers and W. Schmitt (1982) *Progr. Nucl. Energy* **9**, 569–579.

Grunwald, G., K. Junghanss and P. Liewers (1985) *Progr. Nucl. Energy* **15**, 651–659.

Lipcsei S., Kiss S. and Pór G. (1992) *Progr. Nucl. Energy* **27**, 11–24.

Mullens, J. A. and J. A. Thie (1982) Understanding Pressure Dynamic Phenomena in PWRs for Surveillance and Diagnostic Application. *5th Power Plant Dynamics, Controls and Testing Symposium,* Knoxville, Tennessee

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