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EFFECTIVE NUMBER OF INELASTICALLY INTERACTING NUCLEONS IN RARE NUCLEUS-NUCLEUS PRODUCTION PROCESSES

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МОСКОВСКИЙ ОРДЕНА ЛЕНИНА, ОРДЕНА ОКТЯБРЬСКОЙ РЕВОЛЮЦИИ И
ОРДЕНА ТРУДОВОГО КРАСНОГО ЗНАМЕНИ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ
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НАУЧНО-ИССЛЕДОВАТЕЛЬСКИЙ ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ

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Abstract

A model of nucleus-nucleus interaction using one inelastic NN-interaction is suggested for the exclusive production processes with small cross-section. A-dependence nuclear coherent and incoherent production cross-section are predicted.

1 Introduction

The various problems are suggested in connection with planed heavy ions collider experiments on *RHIC* of *BNL* and *LHC* of *CERN* at present time. We should like to pick on such problems in which nucleus-nucleus processes are considered on the analogy of nucleon-nucleon processes. These reactions have a small cross-sections and we call them rare processes. A nucleon diffraction dissociation, double-pomeron exchange for central hadroproduction, jet production and others are related to such kind of processes. It was suggested in ref./1,2/ to investigate the double-pomeron nucleus-nucleus process with glueballs production by the help of Quark-Gluon Spectrometer on RHIC.

The present work devotes to a development of the process theory in nucleus-nucleus interactions. We use the famous Glauber-Sitenko approach and an approximation of one inelastic interaction (OII) of nucleons resulting in production of new particles /3/.

Two experiments are considered. In the first one the produced hadrons and both nuclei in the ground state are detected. We call such process the coherent production process (CPP) in according to the hadron-nucleus interaction theory /3/. In the second one two protons must be registered instead of nuclei. They must be detected in the kinematical regions corresponding to quasi-free interaction of two nucleons from colliding nuclei. The nuclei may be exited or broken up, i.e. to be any final state except ground state. These processes is named the incoherent production processes (IPP).

A ratio of nucleus-nucleus and nucleon-nucleon cross-sections integrated over momentum transfers gives us an effective number $< N(A_1A_2) >$ of inelastically interacting nucleons of nuclei A_1 and A_2. It shows in what times the cross-section of large in compare with cross-section of NN-process. Note that our number $< N(A_1A_2) >$ is not a number of inelastic collisions $< \nu(A_1A_2) >$ used usually when the total cross-sections and particle multiplicity are discussed /4/. For hadron-nucleus (hA) interaction this number is $< \nu(hA) >= A\sigma_{\rm in}(hN)/\sigma_{\rm in}(hA)$.

The main aim of present work is an examination of A-dependency of the effective nucleon number $< N(A_1A_2) >$ for coherent and incoherent processes.

2 Theory

The elastic scattering amplitude of two nuclei, $A_1A_2 \to A_1A_2$ in Glauber-Sitenko approach is equal to

$$\mathcal{F}_{A_1A_2}(\vec{q}) = \frac{ik}{2\pi}\int d^2\vec{b}\exp(i\vec{q}\vec{b}) < A_1A_2 \mid 1 - \hat{S}(\vec{b}) \mid A_1A_2 >, \qquad (1)$$

where k is the nucleon momentum in the NN c.m.system, q -momentum transfers. $\hat{S}(\vec{b})$ is the operator in nucleon coordinate space $\vec{r}_{i,j} = (\vec{s}_{i,j}, z_{i,j})$, given by

$$\hat{S}(\vec{b}) = \prod_{i=1}^{A_1}\prod_{j=1}^{A_2}(1 - \Gamma(\vec{b} - \vec{s}_i + \vec{u}_j)). \qquad (2)$$

Here $\Gamma(\vec{a})$ is the nucleon profile function and Fourier-Bessel image of the NN-elastic scattering amplitude $f(\vec{q}) = (ik/4\pi)\sigma_t \exp(-\beta\vec{q}^2/2)$:

$$\Gamma(\vec{a}) = \frac{1}{2\pi ik}\int d^2\vec{q}\exp(-i\vec{q}\vec{a})f(\vec{q}) = (\sigma_t/4\pi\beta)\exp(-\vec{a}^2/2\beta) \qquad (3)$$

where $\sigma_t = \sigma_t(NN)$ is the total cross-section of NN-interaction, β is the slope parameter. Define an nucleus-nucleus scattering phase $\chi(\vec{b}) \equiv \chi_{A_1A_2}(\vec{b})$ by the equation

$$e^{-\chi(\vec{b})} = < A_1A_2 \mid \hat{S}(\vec{b}) \mid A_1A_2 > . \qquad (4)$$

The well-known rigid nuclear approximation (RNA) /5/ and the optical approximation (OA) /6/ give us

$$\chi^{(RNA)}(\vec{b}) = \int d^2\vec{u}\, T_{A_2}(\vec{u})(1 - \exp(-\int d^2\vec{s}\, T_{A_1}(\vec{s})\Gamma(\vec{b} - \vec{s} + \vec{u})), \qquad (5)$$

$$\chi_0(\vec{b}) \equiv \chi^{(OA)}(\vec{b}) = \int d^2\vec{s} d^2\vec{u}\;\; T_{A_1}(\vec{s})\, T_{A_2}(\vec{u})\Gamma(\vec{b} - \vec{s} + \vec{u}), \qquad (6)$$

$$T_A(\vec{b}) = A\int dz\; \rho_A(\vec{b}, z) \qquad (7)$$

is the function of nuclear thickness.

If we know the phase $\chi(\vec{b})$ we can calculate an integral total, elastic and inelastic A_1A_2-cross-sections:

$$\sigma_t(A_1A_2) = 2\int d^2\vec{b}(1 - Re(e^{-\chi(\vec{b})})), \qquad (8)$$

$$\sigma_{el}(A_1A_2) = \int d^2\vec{b} \mid 1 - e^{-\chi(\vec{b})} \quad |^2, \tag{9}$$

$$\sigma_{in}(A_1A_2) = \sigma_t(A_1A_2) - \sigma_{el}(A_1A_2). \tag{10}$$

The production cross-section of new particles can be expressed following to /7/ as

$$\sigma_{pr}(A_1A_2) = \int d^2\vec{b}(1 - exp(-\sigma_{pr} \int d^2\vec{s}\ T_{A_1}(\vec{s})\, T_{A_2}(\vec{b} - \vec{s}))), \tag{11}$$

where $\sigma_{pr} = \sigma_{pr}(NN)$ is the cross-section of particle production in NN-interaction.

A total A_1A_2-cross-section is a sum

$$\sigma_t(A_1A_2) = \sigma_{el}(A_1A_2) + \sigma_{\rm inc}(A_1^*A_2) + \sigma_{\rm inc}(A_1A_2^*)$$
$$+\sigma_{\rm inc}(A_1^*A_2^*) + \sigma_{pr}(A_1A_2). \tag{12}$$

Here the separate parts $\sigma_{\rm inc}$ give a contribution of the incoherent cross-section of excitation and disintegration of nucleus A_1 or A_2 or both nuclei. They are about some percents from $\sigma_{\rm in}(A_1A_2)$.

Let's use the multiple scattering decomposition suggested in ref. /9/ as

$$\hat{S}(\vec{b}) = (1- < \Gamma(\vec{b}) >)^{A_1A_2} \sum_{r=0}^{A_1A_2} (-1)^r \hat{G}_r(\vec{b})$$
$$= (1- < \Gamma(\vec{b}) >)^{A_1A_2}(1 - \hat{G}_1(\vec{b}) + \ldots), \tag{13}$$

where

$$< \Gamma(\vec{b}) > \equiv < A_1A_2 \mid \Gamma(\vec{b} - \vec{s} + \vec{u}) \mid A_1A_2 >$$
$$= \frac{1}{A_1A_2} \int d^2\vec{s} d^2\vec{u}\ T_{A_1}(\vec{s})\, T_{A_2}(\vec{u}) \Gamma(\vec{b} - \vec{s} + \vec{u}). \tag{14}$$

There is a recurrent relation for $\hat{G}_r(\vec{b})$ in ref. /9/. Here we need only two first members of sum

$$\hat{G}_0(\vec{b}) = 1,$$
$$\hat{G}_1(\vec{b}) = \sum_{i=1}^{A_1} \sum_{j=1}^{A_2} \gamma_{ij}(\vec{b} - \vec{s}_i + \vec{u}_j), \tag{15}$$

where

$$\gamma_{ij}(\vec{b}) = \frac{\Gamma(\vec{b} - \vec{s}_i + \vec{u}_j) - < \Gamma(\vec{b}) >}{1 - < \Gamma(\vec{b}) >}. \tag{16}$$

It is easy to be convinced of

$$< \Gamma(\vec{b}) > = \frac{1}{A_1 A_2} \chi_0(\vec{b})$$
$$< A_1 A_2 \mid \gamma_{ij} \mid A_1 A_2 > = 0. \tag{17}$$

The optical approximation corresponds to the neglect of all members in sum (13) except a member with $r = 0$. In that case

$$\hat{S}^{(OA)}(\vec{b}) = (1 - < \gamma(\vec{b}) >)^{A_1 A_2} |_{A_{1,2} \gg 1} = e^{-\chi_0(\vec{b})}. \tag{18}$$

Generalise the previous formalism to the case of particle production. We mark a system of particles as $\{M\}$. The amplitude of nucleon-nucleon particle production in process

$$N_1 + N_2 \rightarrow N_1 + N_2 + \{M\} \tag{19}$$

is

$$f_{NN}^{(M)}(\vec{q}, V_M) = \frac{ik}{2\pi} \int d^2\vec{b} \exp(i\vec{q}\vec{b}) \Gamma_{NN}^{(M)}(\vec{b}), \tag{20}$$

and the differential cross-section can be expressed as

$$\frac{d\sigma_{NN}}{d\tau} \equiv \frac{d^3\sigma_{NN}}{d\Omega_1 d\Omega_2 dV_M}(\vec{q}, V_M) = \mid f_{NN}^{(M)}(\vec{q}, V_M) \mid^2. \tag{21}$$

Here $d\Omega_1$ and $d\Omega_2$ are the elements of solid angle of nucleon N_1 and N_2 momentum, dV_M is the phase volume element of particles $\{M\}$, V_M is their set of kinematical variables, $\vec{q} = \vec{q}_1 + \vec{q}_2$ is the total momentum transfers to the system $\{M\}$ and $\tau = (\Omega_1, \Omega_2, V_M)$ is the total set of variables.

An exclusive process of production in nucleus-nucleus collision

$$A_1 + A_2 \rightarrow A_1 + A_2 + \{M\} \tag{22}$$

is described by the amplitude

$$\mathcal{F}_{A_1 A_2}^{(M)}(\tau) = \frac{ik}{2\pi} \int d^2\vec{b} \exp(i\vec{q}\vec{b}) < A_1 A_2, M \mid 1 - \hat{S}(\vec{b}) \mid A_1 A_2, 0 >. \tag{23}$$

The coherent cross-section (A_1 and A_2 remains in the ground state) is equal to

$$\frac{d\sigma_{\underline{coh}}}{d\tau}(\tau) = | \mathcal{F}^{(M)}_{A_1A_2}(\vec{q}) |^2 . \tag{24}$$

The ortohonalisation of initial $| 0 >$ and final $| M >$ hadron states leads to

$$< M | \gamma_{ij} | 0 >= \Gamma^{(M)}_{NN}(\vec{b} - \vec{s}_i + \vec{u}_j). \tag{25}$$

We take into account here that $< \Gamma(\vec{b}) > << 1$.

Substitute Eq.(13) in Eq.(23) and limit oneself by members with $r = 0$ and 1. We remain $< M | \gamma_{ij} | 0 >$ only in the first degree. It corresponds to OII approximation. Then we get

$$\mathcal{F}^{(M)}_{A_1A_2}(\tau) = \frac{ik}{2\pi} \int d^2b\exp(i\vec{q}\vec{b})e^{-\chi_0(\vec{b})} < A_1A_2 | \sum_{ij}^{A_1A_2} \Gamma^{(M)}_{NN}(\vec{b} - \vec{s}_i + \vec{u}_j) | A_1A_2 > . \tag{26}$$

The region of NN-interaction $d \sim \sqrt{(\beta)}$ is more less than the nuclear radius, $(d/R_{A_1,A_2})^2 << 1$. So the function $\chi_0(b)$ is a smooth function in compare with $\Gamma_{NN}(\vec{b} - \vec{s}_i + \vec{u}_j)$. On such condition we get the coherent production amplitude as

$$\mathcal{F}^{(M)}_{A_1A_2}(\tau) = f^{(M)}_{NN}(\tau) < A_1A_2 | \sum_{ij} \exp(iq(\vec{s}_i - \vec{u}_j) - \chi_0(\vec{s}_i - \vec{u}_j)) | A_1A_2 > . \tag{27}$$

Using the factorised form of nuclear density /3/ for A_1 and A_2 one obtains (27) as

$$\mathcal{F}^{(M)}_{A_1A_2}(\tau) = f^{(M)}_{NN}(\tau) F_{A_1A_2}(\vec{q}), \tag{28}$$

where

$$F_{A_1A_2}(\vec{q}) = \int d^2\vec{s}d^2\vec{u} \exp(iq(\vec{s}_i - \vec{u}_j) - \chi_0(\vec{s}_i - \vec{u}_j))T_{A_1}(\vec{s})T_{A_2}(\vec{u}) \tag{29}$$

is a nucleus-nucleus part of the amplitude for process (22). It's not dependent from the hadron system characteristics.

The coherent nucleus-nucleus cross-section is proportional to the production cross-section on nucleon (21):

$$\frac{d\sigma_{\underline{coh}}}{d\tau}(\tau) = \frac{d\sigma_{NN}}{d\tau}(\tau) | F_{A_1A_2}(\vec{q}) |^2 . \tag{30}$$

It is a straight consequence of OII approximation.

If one puts the phase χ_0 is equal to zero in Eq.(29), i.e. takes the distorted factor as $e^{-\chi_0(\vec{s}-\vec{d})} \equiv 1$, then he gets

$$\mathcal{F}^{(M)}_{A_1 A_2}(\tau)|_{\chi_0=0} = f^{(M)}_{NN}(\vec{q}) A_1 A_2 S_{A_1}(\vec{q}) S_{A_2}(-\vec{q}), \quad (31)$$

where $S_A(\vec{q})$ is a nuclear form-factor. Such assumption is very rough. In the theory of hadron-nucleus interaction it is called the impulse approximation sometimes.

In a certain case $A_1 = 1$ $(T_{A_1}(s) = A_1 \delta^{(2)}(s))$, $A_2 = A$ i.e. for the process

$$N + A \rightarrow N + A + \{M\}, \quad (32)$$

one obtains from Eq.(28) and (29) the result of Kolbig and Margolis for equal cross-section in input and output channels:

$$\mathcal{F}^{(M)}_{NA}(\tau) = f^{(M)}_{NN}(\tau) \int d^2\vec{b} \exp(i\vec{q}\vec{b})\, T_A(\vec{b}) \exp(-\sigma_t/2 T_A(\vec{b})). \quad (33)$$

Let the nucleon-nucleon amplitude (20) has the factorised form and it's $\vec{q}$-dependence is the same for any state of $\{M\}$:

$$f^{(M)}_{NN}(\vec{q}, V_M) = \varphi_{NN}(\vec{q}) \omega^{(M)}_{NN}(V_M). \quad (34)$$

Then a ratio of nucleus-nucleus and nucleon-nucleon cross-sections integrated in the region of solid angle Ω_0 for each nucleus is a coefficient of the particle production strengthening in nucleus-nucleus collision, which is equal to

$$R_{\text{coh}}(A_1 A_2/pp, \Omega_0) = \frac{\int_{\Omega_0} d\Omega_1 \int_{\Omega_0} d\Omega_2 d\sigma_{\text{coh}}/d\tau}{\int_{\Omega_0} d\Omega_1 \int_{\Omega_0} d\Omega_2 d\sigma_{NN}/d\tau} \quad (35)$$

$$= \frac{\int_{\Omega_0} d\Omega_1 \int_{\Omega_0} d\Omega_2 \mid \varphi_{NN}(\vec{q}) F_{A_1 A_2}(\vec{q}) \mid^2}{\int_{\Omega_0} d\Omega_1 \int_{\Omega_0} d\Omega_2 \mid \varphi_{NN}(\vec{q}) \mid^2}.$$

At $\Omega_0 = 4\pi$ the ratio (35) gives us a number which we call a coherent effective number of interacting nucleons:

$$R_{\text{coh}}(A_1 A_2/pp, \Omega_0)|_{\Omega_0=4\pi} = < N(A_1 A_2) >_{\text{coh}}. \quad (36)$$

Consider the process when the nuclei A_1 and A_2 are broken up:

$$A_1 + A_2 \rightarrow p_1 + p_2 + \{M\} + X. \quad (37)$$

Here p_1 and p_2 are protons and X is the other fragments of nuclei. The experimental set up detects hadron system $\{M\}$ and nucleons (protons) in the kinematical region of quasi-free NN-collision. The measurable proton momentums $\vec{p}_1$ and $\vec{p}_2$ define the momentums transfers $\vec{q}_1$ and $\vec{q}_2$.

Let's take the amplitude (27) in OII approximation. Rewrite it for any final states of nuclei A_1 and A_2 as

$$\mathcal{F}^{(M)}_{A_1'A_2'}(\tau) = f^{(M)}_{NN}(\tau) < A_1'A_2' \mid \sum_{ij} \exp(i\vec{q}(\vec{s}_i - \vec{u}_j) - \chi_0(\vec{s}_i - \vec{u}_j)) \mid_{A_1A_2} > . \quad (38)$$

The sum over all final nuclear states is

$$\frac{d\sigma_{\mathrm{SUM}}}{d\tau}(\tau) = \sum_{A_1'A_2'} \mid \mathcal{F}^{(M)}_{A_1'A_2'}(\tau) \mid^2 . \quad (39)$$

Substitute Eq.(38) in Eq.(39) and use a closure of nuclear states. Then one finds diagonal $(i = i', j = j')$ and nondiagonal $(i \neq i', j \neq j')$ members in the sum over i, j, i', j'/10, 11/.

The nondiagonal members contain the coherent cross-section, which we must subtract from Eq.(39), and a small contribution in compare with incoherent cross-section. This contribution depends on q_k and vanishes at $q_k > 1/R_k$. Here R_k is a nuclear radius, $k = 1, 2$.

The diagonal members give us an incoherent nucleus-nucleus cross-section

$$\frac{d\sigma_{\mathrm{inc}}}{d\tau}(\tau) = \frac{d\sigma_{NN}}{d\tau}(\tau) < N(A_1A_2) >_{\mathrm{inc}}, \quad (40)$$

where $< N >_{\mathrm{inc}}$ is an incoherent effective number of interacting nucleons in the production process (37):

$$< N(A_1A_2) >_{\mathrm{inc}} = < A_1A_2 \mid \sum_{ij} \exp(-2\chi_0(\vec{s}_i - \vec{u}_j)) \mid A_1A_2 >$$

$$= \int d^2\vec{s}\, d^2\vec{u}\ \ T_{A_1}(\vec{s})\, T_{A_2}(\vec{u}) \exp(-2\chi_0(\vec{s}_i - \vec{u}_j)). \quad (41)$$

In the certain case $A_1 = 1, A_2 = A$ one gets the known result for hadron-nucleus interaction /3/ at equal cross-sections in input and output channels:

$$< N(1, A) >_{\mathrm{inc}} = \int d^2\vec{u}\ \ T_A(\vec{u}) \exp(-\sigma_t T_A(\vec{u})) \equiv N_1(A, \sigma_t). \quad (42)$$

Note that in the impulse approximation ($\chi_0 = 0, \sigma_t = 0$), i.e. without distortion, we see

$$< N(A_1A_2) >_{\text{inc}} = A_1A_2,$$
$$N_1(A, \sigma_t) = \text{A}. \tag{43}$$

If we detect the protons in the reaction (37) then we must compare with experimental data the cross-section

$$\frac{d\sigma_{\text{inc}}}{d\tau}(\tau) = \frac{d\sigma_{pp}}{d\tau}(\tau)\frac{Z_1Z_2}{A_1A_2} < N(A_1A_2) >_{\text{inc}}, \tag{44}$$

where Z_1 and Z_2 are the charges of nuclei A_1 and A_2.

It follows from Eq.(40) that the coefficient of strengthening in nucleus-nucleus collision for incoherent production process

$$R_{\text{inc}}(A_1A_2/NN) = \frac{\int_{\Omega_0} d\Omega_1 \int_{\Omega_0} d\Omega_2 d\sigma_{\text{inc}}/d\tau}{\int_{\Omega_0} d\Omega_1 \int_{\Omega_0} d\Omega_2 d\sigma_{NN}/d\tau}$$
$$=< N(A_1A_2) >_{\text{inc}} \tag{45}$$

doesn't depend on the solid angle Ω_0 and coincides the incoherent effective number.

Ending this section we should like to emphasize that the present model is the straight generalisation of Kolbig-Margolis approach on the nucleus-nucleus production process and only the first step in the development of theory of rare nucleus-nucleus processes. We don't take into account here the effects of longitudinal momentum transfers /10/, the inelastic shadowing /12/, the color transparency /13/ and others. The large attention was given to these effects in the theory of hadron-nucleus interactions. We don't consider also the effects of tree and loop diagrams appearing in nucleus-nucleus collisions. They are important, for example, in the description of elastic nucleus-nucleus scattering at large $q(qR > 1)/14/$.

3 Calculations

It is well-known from the hadron-nucleus interaction physics that A-dependence of processes proceeding in the surface region of nuclei is very sensitive to the shape of nuclear density. One of example is the incoherent hadron-nucleus cross-section /15/. A nuclear radius is rather well determined by A-dependence of total and inelastic integral cross-section.

We haven't the experimental data on incoherent nucleus-nucleus cross-sections until now. There are not enough complete and not always clear data on the total (8), elastic (9), inelastic (10) and production (11) cross-sections. We discuss here available and reliable data at the energy region from some GeV per nucleon onwards. The more precise calculation of the integral cross-sections are fulfilled in ref./8/. Experimental data on $\sigma_{\text{in}}(A_1A_2)$ for different couple of A_1 and A_2 are satisfactorily described by Fermi-distribution of nuclear density

$$\rho_F(r) = \frac{\rho_0}{1+\exp((r-R(A))/d)}, \tag{46}$$

$$R(A) = r_0 A^{1/3},\ r_0 = 1.12 fm,\ d = 0.4 fm \tag{47}$$

for nuclei with $A_{1,2} > 12$. For the light nuclei with $A_{1,2} \leq 12$ the Gaussian density was used.

In ref./16/ it was suggested to use the dependence

$$R(A) = r_0 A^{1/3} - r_1 A^{-1/3}, \tag{48}$$

which approximates the calculation of the density-dependent Hartree-Fock theory and improves the data description in the light nuclei region. The comparison with data on $\sigma_{\text{in}}(pA)$ gives the next parameters

$$r_0 = 1.19 fm,\ r_1 = 1.51 fm,\ d = 0.54 fm, \tag{49}$$

We take an symmetrisable Fermi-distribution of nucleus density in our calculations

$$\begin{aligned} \rho_{SF}(r) = n_0\left(\frac{1}{1+\exp(r-R/d)} + \frac{1}{1+\exp(-r-R)/d)} - 1\right) = \\ = n_0 \frac{sh(R/d)}{ch(R/d)+ch(r/d)} \\ n_0 = 3/(4\pi R^3(1+(\pi d/R)^2)), \qquad R \equiv R(A). \end{aligned} \tag{50}$$

It's very closed to Fermi-distribution (46) and permit us to take some integral in a closed form.

Fig.1 shows the comparison of results with two dependence (47) and (48) for $R(A)$ and the density distribution (50). The parameterization (48) leads to visible bend of cross-sections with decreasing of A at $A < 27$(see also the data description with the help of $(A_1^{1/3} + A_2^{1/3} - \delta)$/4/). We know the only experimental data at GeV-region for σ_{tot} and σ_{el} with equal nuclei (^{4}He and ^{4}He /17/). They are good described by the curve with two parameters (r_0, r_1) of $R(A)$ on fig.1. At $A > 27$ the behavior of $\sigma_{tot}(AA)$ and $\sigma_{el}(AA)$ is like to $A^{2/3}$-dependence. Further we fulfill all our calculations with parameterization (48) and (50).

Authors of ref./18/ have analysed a nucleus-nucleus elastic amplitude and shown that the contribution of loop diagrams is small. They've statemented that rigid nucleus approximation (RNA) is good. In fig.2a we compare RNA and optical approximation (OA) for $\sigma_{in}(A_1A_2)$. The curves of these two approximations draw visible together with increasing of A. The value of parameter $\sigma_t(pp)$ corresponds to the necessary energy of pp-collisions.

The JIRN data /19/ for $^{22}Ne + A$ at $4.1A$ GeV are neither exact inelastic or production cross-section. The elastic scattering and $\sigma_{inc}(A_1A_2^*)$, when a nucleus ^{22}Ne is in the ground state, are excluded. So the data include the cross-sections $\sigma_{inc}(A_1^*A_2), \sigma_{inc}(A_1^*A_2^*)$ and $\sigma_{pr}(A_1A_2)$. The first two ones put together $10 - 14\%$ from inelastic cross-section /8/. That is why the experimental points in fig.2a are low the curves $\sigma_{in}(A_1A_2)$.

And let's show the comparison with another data (fig.2b) for $\sigma_{pr}(A_1A_2)$. There are two points ^{16}O + Al and $^{16}O + Pb$ at $200A$ GeV in the experiment $NA36$/20/. The comparison data and our results is satisfactory. Summery, we supposed that the new experiments are necessary for more careful comparison theory and experiment.

Let's calculate the coherent effective number $< N >_{coh}$ for process (22). Let the detectors of nucleus A_1 and A_2 are along the axis of colliding nuclei and register nuclei in all region of asimuthal angles $0 \leq \varphi_1, \varphi_2 \leq 2\pi$ and in limited and equal region of polar angles $0 \leq \Theta_1, \Theta_2 \leq \Theta_0$. For certainty we take the next $\vec{q}$-dependence of NN-interaction amplitude (34):

$$\varphi_{NN}(\vec{q}) = \exp(-b(t_1 + t_2)/2), \quad t_k = q_k^2. \tag{51}$$

For double pomeron exchange a parameter b is equal $b = 11(\mathrm{GeV}/c)^{-2}$. Then the

Fig.1

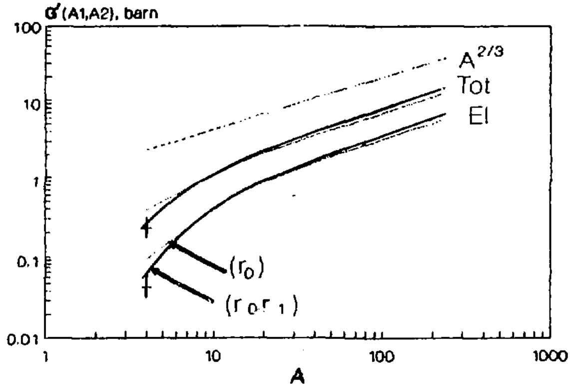


Fig. 1: Total $\sigma_t(A_1, A_2)$ and elastic $\sigma_{el}(A_1, A_2)$ nucleus-nucleus cross-sections for $A_1 = A_2 = A$ and $\sigma_t(pp) = 39$ mbarn. Data for $\alpha\alpha$ - interactions at $\sqrt{s} = 126$ GeV /17/.

Fig.2

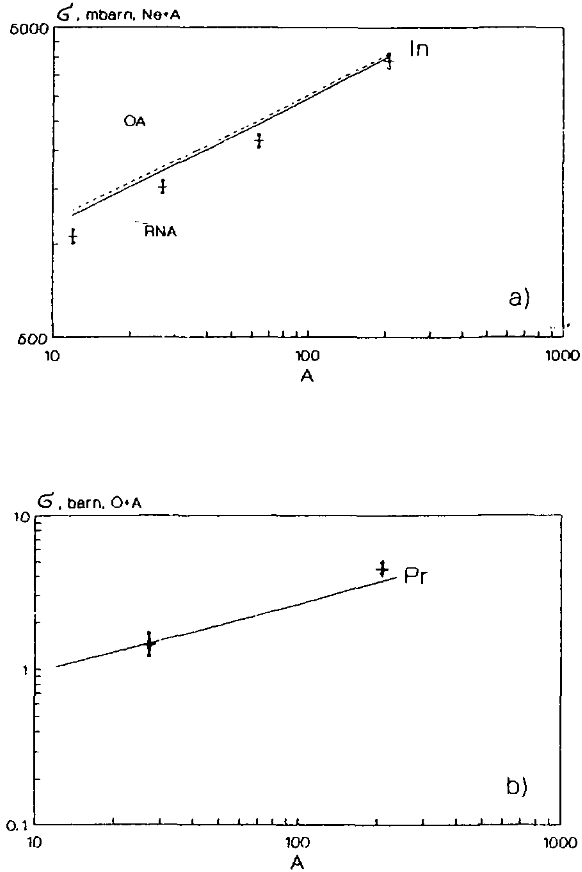


Fig.2: Inelastic $\sigma_{in}(A_1, A_2)$ and production $\sigma_{pr}(A_1, A_2)$ cross-sections (sol[illegible] - RNA, dashed line $-OA$ approximations) plotted versus A [illegible] Proje[illegible] nucleus is ^{22}Ne at 4.1A GeV /19/, $\sigma_t(pp) = 41$ mbarn [illegible] at 200A GeV /20/, $\sigma_t(pp) = 39$ mbarn.

coefficient of strengthening (35) can be calculated by formula

$$R_{coh}(A_1A_2/pp, t_o) = (b^2/8\pi) I_{A_1A_2}(t_o),$$

$$I_{A_1A_2}(t_o) = (1 - e^{-bt_o/2}) \int_0^{t_o} dt_1 \int_0^{t_o} dt_2 \int_0^{2\pi} d\varphi \, e^{-b(t_1+t_2)} \mid F_{A_1A_2}(\vec{q}) \mid^2, \quad (52)$$

where $t_0 \simeq (k_0\Theta_0)^2$ and $q = (t_1 + t_2 + 2\sqrt{t_1t_2}\cos\varphi)^{1/2}$. At $t_0 \to \infty$ we get coherent effective number $< N(A_1A_2) >_{coh}$. The results are obtained at energy of colliding nuclei $\sqrt{s} = 100A$ GeV setting $\sigma_t(pp) = 39$mbarn. The value of nucleon momentum k_0 is equal $k_0 = 100$GeV/c. The $\Theta_0 = 0.025$mrad taken corresponds to the position of first diffractive minimum of differential cross-section for very large nuclei $A > 200$ in according to the plans of RHIC experiment with Quark-Gluon Spectrometer /1/. For that value of Θ_0 we get $t_0 = 0.0006(\mathrm{GeV}/c)^2$.

Fig.3 shows the calculation of effective number of nucleons participating in production particle in AA-collisions. The coherent effective number $< N(AA) >_{coh}$ is comparable with incoherent number $(Z/A)^2 < N(AA) >_{inc}$ and 3-4 times as less than $< N(AA) >_{inc}$. Ratio of incoherent and coherent numbers is represented in fig.4.

The small quantities of the coherent effective number $< N(AA) >_{coh} = 1 \div 5$ for $A = 4 \div 238$ is caused by a little probability of process (22) when both nuclei are neither broken out or excited. This is a consequence of the strong effects of nucleon interactions in nucleus before and after an act of particle production ($\sigma_t \neq 0$). So the integral of coherent cross-section over momentum transfers is comparable and less than the integral incoherent cross-section which is proportional to a sum of probability over all final states of A_1 and A_2 different from their ground states.

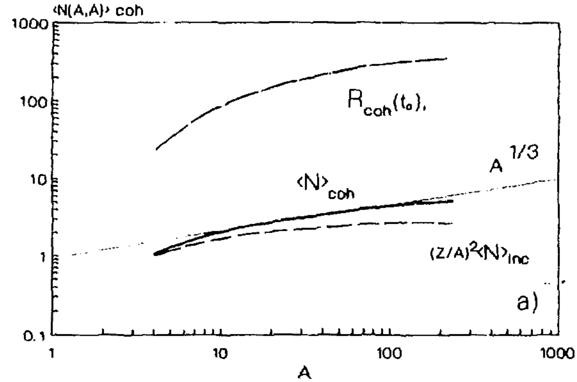
The A-dependence effective numbers is well approximated by the function $A^{1/3}$:

$$< N(AA) >_{coh} = 1.0A^{1/3}, \quad < N(AA) >_{inc} = 3.1A^{1/3}. \quad (53)$$

This is a consequence of surface feature of interaction as in coherent so incoherent processes. As the effective number calculations are sensitive to the nuclear density distribution and to its parameters so they must be determined more precisely later. For illustration we show in fig.3*b* the calculation with Gaussian nuclear density with the parameter $R(A) = 1.12A^{1/3}$fm. This result is distinguished very strong from one with Fermi-density (49).

The coherent differential cross-section (30) has a sharp peak at small q, $q(R_1 + R_2) < 1$. So the coefficient of strengthening $R_{coh}(A_1A_2/pp, \Omega_0)$ depends very

Fig.3



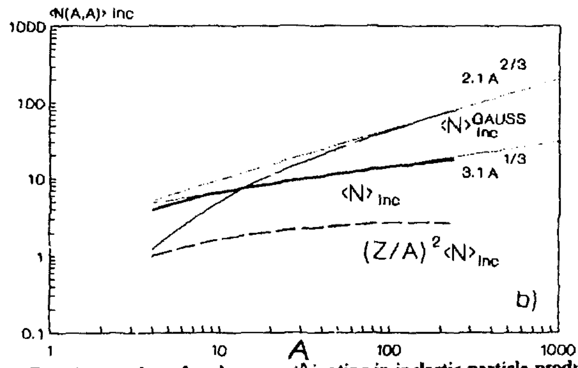


Fig.3: Effective number of nucleons participating in inelastic particle production in nucleus-nucleus interactions ($A_1 = A_2 = A$) - solid lines: a) for cohere b) for incoherent processes. A^{α} - approximations are dashed curves. Ratio cross-sections $R_{coh}(A_1A_2/pp, t_0)$ at $t_0 = 0.0006 (GeV/c)^2$ is on Fig.3a. Inco number with Gaussian nuclear density is on Fig.3b.

strong from solid angle Ω_0 of nucleus detection after interaction. At $t_0 = 0.0006(\mathrm{GeV}/c)^2$ $R_{coh}(t_0)$ is two orders large than $< N(AA) >_{coh}$ (fig.3a). In that case of course the absolute value of cross-sections decreases but the coherent cross-section decreases weakener than pp-interaction cross-section with decreasing of t_0. By the same reason the ratio of incoherent and coherent events which is equal to quantity $(Z/A)^2 R_{inc}/R_{coh}(t_0)$ is small for $t_0 = 0.0006(\mathrm{GeV}/c)^2$. It changes from 5% to 1% for nuclei from A=4 to A=238 (fig.4).

4 Conclusion

The present work suggested a model of nucleus-nucleus interaction for the exclusive production processes. The model uses the approximation of one inelastic interaction of nucleons in colliding nuclei and may be applied for the description of the processes with small cross-section.

The ratio of differential nucleus-nucleus and nucleon-nucleon cross-sections is calculated. That ratio for incoherent process is the effective number of participating nucleons $< N(A_1A_2) >_{inc}$. The ratio for coherent process is a strong depending function from momentum transfers. If we take the ratio of cross-sections integrated over momentum transfer we get the coherent effective number $< N(A_1A_2) >_{coh}$.

Our calculations show that A-dependence of both incoherent and coherent effective number has a form $A^{1/3}$ and their values are comparable. The detection of nuclei near beam direction in a small restricted region results in the sharp rise of the coherent events, over incoherent events.

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Fig.4

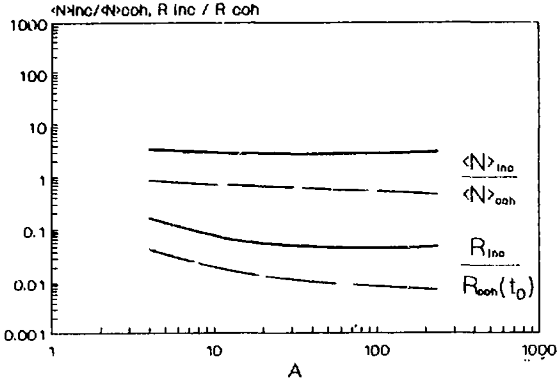

Fig. 4: Ratio of incoherent and coherent effective numbers and cross-sections for $A_1 = A_2$=A. Ratio of cross-sections is calculated at $t_0 = 0.0006(\mathrm{GeV}/c)^2$. Solid curves - without, dashed curves - with a factor $(Z/A)^2$.

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