UWThPh-1993-10
March 31, 1993

AT9300112

Introduction to Left-Right Symmetric Models*

W. Grimus
Institute for Theoretical Physics
Boltzmanngasse 5
A-1090 Vienna, Austria

Abstract

We motivate left-right symmetric models by the possibility of spontaneous parity breaking. Then we describe the multiplets and the Lagrangian of such models. Finally we discuss lower bounds on the right-handed scale.

*) Lecture given at the 4th Hellenic School on Elementary Particle Physics, Corfu, 2 – 20 September 1992.

1 Motivation and Basic Ingredients

1.1 Motivation

The standard model (SM) [1] is up to now in excellent agreement with all experiments and its radiative corrections beyond QED are about to being tested in the near future (see the contribution of C. Verzegnassi to these proceedings). Thus at the moment there is no compelling reason to go beyond the SM. However, to be able to judge the meaning and importance of tests of the SM it is necessary to have a more general framework and to have in mind in which directions the SM could be extended. Before motivating left-right (LR) symmetric models [2,3] it might be useful to point out two features of the SM which could be starting points for such extensions:

i) Parity (P) is explicitly broken in the SM by the asymmetry between left (L)- and right (R)-handed multiplets. Thus the breaking of P is not at the same footing as the breaking of the gauge group which is spontaneous.

ii) Neutrino masses are zero again by choice of the gauge multiplets. Dirac neutrino masses are zero because there is no right-handed neutrino in the SM. (Note that such multiplets would be trivial gauge singlets and taking them into the SM would introduce the problem of naturally having neutrino masses of order of their charged lepton counterparts.) Since in the SM there are also no gauge singlet or triplet Higgs scalars also Majorana masses cannot be generated.

Motivation For LR-Symmetric Models:

i) Their gauge group is a very simple extension of the SM gauge group and therefore the discussion of LR-symmetric models is interesting in itself without needing further motivation.

ii) Spontaneous breaking of P (and also CP) is possible but it requires R-handed neutrino degrees of freedom.

iii) These additional neutrino multiplets give rise to Majorana masses and allow for scenarios such that the neutrino masses are naturally light, i.e. $m_{\nu_\ell} \ll m_\ell$ ($\ell = e, \mu, \tau$).

Of course, the first point is the strongest argument for considering LR-symmetric models. However, in the following we want to take ii) as the starting-point. In doing this it is useful to have in mind a general notion of P and CP which we want to work out by using QED as an intuitive example.

1.2 P and CP in QED

Let ψ be the electron field. Then P and CP transformations are given by

$$\begin{aligned} P : \psi(x) &\to \gamma^0 \psi(\hat{x}) \\ & \qquad \text{with } \hat{x} = \begin{pmatrix} x^0 \\ -\vec{x} \end{pmatrix}. \\ CP : \psi(x) &\to -C\psi^*(\hat{x}) \end{aligned} \tag{1}$$

The unitary antisymmetric charge conjugation matrix C is defined by $C^{-1}\gamma_\mu C = -\gamma_\mu^T$. Defining the chiral projections

$$\psi_{L,R} \equiv \frac{1}{2}(1 \mp \gamma_5)\psi \tag{2}$$

we can write down two independent L-handed fields

$$\psi_{L1} \equiv \psi_L, \qquad \psi_{L2} \equiv C\gamma_0^T \psi_R^* \equiv (\psi_R)^c. \tag{3}$$

Now parity can be rewritten in terms of the chiral fields:

$$P: \psi_L \to \gamma_0 \psi_R, \qquad \psi_R \to \gamma_0 \psi_L \tag{4}$$

which shows that in this formulation P exchanges L and R fields. There is yet another formulation using ψ_{L1}, ψ_{L2} where an interesting comparison between P and CP is possible:

$$\begin{aligned} P: \begin{pmatrix} \psi_{L1} \\ \psi_{L2} \end{pmatrix} &\to \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} C \begin{pmatrix} \psi_{L1} \\ \psi_{L2} \end{pmatrix}^* \\ CP: \begin{pmatrix} \psi_{L1} \\ \psi_{L2} \end{pmatrix} &\to \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} C \begin{pmatrix} \psi_{L1} \\ \psi_{L2} \end{pmatrix}^* . \end{aligned} \tag{5}$$

Equ. (5) can easily be verified by using the properties of C. We have left out x and $\hat{x}$ for simplicity. What can we learn from Equ. (5)? First of all we see that in Equ. (5) P and CP look quite similar. The only difference is given by the matrix acting on the vector of L-handed fields ψ_L. Assuming that ψ_{L1} transforms according to the phase transformation $e^{i\alpha}$ with respect to the gauge group $U(1)_{em}$ we can phrase the action of the gauge group by ψ_{L1} being in the one-dimensional representation $\mathbf{1}$ and ψ_{L2} in the complex conjugate representation $\mathbf{1}^*$. This formulation allows the statement that CP acts within each of the representations, i.e. it does not change the electric charge of the fields, whereas P exchanges fields of opposite charge. In the case of (5) it exchanges the representations $\mathbf{1}$ and $\mathbf{1}^*$.

These features are quite general. Let us for the time being assume that in the model under consideration all fermion fields are given in L-handed form (this can always be achieved, see Equ. (3)). Then a transformation of P or CP type has the structure $\omega_L \to AC\omega_L^*$ where ω_L is the vector of L multiplets and A is a unitary matrix acting in the gauge group representation space (compare Equ. (5)). It can be shown that there is a canonical way of defining CP within each irreducible multiplet of ω_L such that a pure gauge theory (i.e. no Yukawa couplings and no Higgs potential) is always CP invariant [4,5]. This means that A always decays into blocks according to the irreducible parts of ω_L. Furthermore, A does not change the charges (the eigenvalues of the generators which are elements of the Cartan subalgebra [4] characterizing the states in ω_L apart from degeneracy). The CP transformation of Equ. (5) illustrates this statement.

On the other hand, in the case of parity the matrix A reverses the sign of a non-empty subset of these charges. This means that P requires the existence of sufficiently many states with opposite charges such that A transforms them into each other. The simplest situation is given by having for every irreducible multiplet in ω_L also its complex conjugate in ω_L and these representations are carried into each other by P. This is realized in QED where we only have the electric charge and A exchanges the representations $\mathbf{1}$ and $\mathbf{1}^*$. There is also the possibility that an irreducible representation of the gauge group is carried into itself under P. In connection with LR-symmetric models we will become acquainted with yet another type when the gauge group is a direct product $G = G' \times G'$. Of course, for parity to be a symmetry a corresponding transformation also has to be performed in the space of gauge bosons which must be consistent with the transformation in the fermion sector. To complete this sketchy discussion of P we have to add the physical requirement that among the subset of charges which are reversed by P contains at least the electric charge. If this were not the case then parity in the formulation (4) would in general connect fields with different electric charges or ψ (Equ. (1)) would be composed of chiral fields with different electric charges.

Now it is easily seen that in the SM parity cannot be defined. Let us e.g. consider the multiplets of the lepton sector. There no multiplet has a complex conjugate counterpart and there are no multiplets which contain opposite electric charges.

1.3 Enlarging the SM to Allow For P

There are two simple routes for doing this.

A. Keeping the gauge group $SU(2) \times U(1)$ and adding to every SM fermion multiplet ψ_X ($X = L$ or R denotes the chirality) a multiplet $\psi_{X'}$ with opposite chirality. ψ_X and $\psi_{X'}$ transform alike under the gauge group. This allows to define

$$P : \psi_X \to \gamma^0 \psi_{X'}, \qquad \psi_{X'} \to \gamma^0 \psi_X . \tag{6}$$

In this scheme of mirror fermions [6] the fermion sector is enlarged compared to the SM but the gauge boson sectors remain the same.

In Equ. (6) P has been formulated analogously to Equ. (3). If we use $(\psi_{X'})^c$ as the partner of ψ_X then both multiplets have the same chirality X and in this picture P looks like Equ. (5).

B. As an alternative to mirror fermions one can put the R-handed singlets of the SM together to form $SU(2)$ doublets. However, to do this in the lepton sector one has to add a neutrino singlet ν_R:

$$u_R, d_R \to \begin{pmatrix} u_R \\ d_R \end{pmatrix}; \qquad \nu_R, \ell_R \to \begin{pmatrix} \nu_R \\ \ell_R \end{pmatrix}. \tag{7}$$

In this scheme the R-handed $SU(2)$ cannot be $SU(2)_L$ since otherwise the charged current would be a vector current instead of V-A. This is the route which leads to LR-symmetric models. In these models the set of SM fermions is enlarged by R-handed neutrino fields ν_R for each lepton flavour, there are two sets of $SU(2)$ gauge fermions and the gauge group is $SU(2) \times SU(2) \times U(1)$ [2].

From now on we will discuss alternative B. To complete the characterization of the fermion multiplets in this scenario we have to evaluate their $U(1)$ quantum numbers $\tilde{Y}$. Since L and R doublet members have the same electric charges and the $U(1)$ acts on both of them we have the condition

$$\frac{1}{2}\tau_3 + \frac{1}{2}\tilde{Y}\,1 = Q \tag{8}$$

for the fermion doublets. Q is the electric charge matrix and $\tilde{Y}$ must be the same for L and R. We observe that $\tilde{Y} = -1$ for lepton doublets and $\tilde{Y} = 1/3$ for quark doublets to agree with Equ. (8). This can be written in a compact way by

$$\tilde{Y} = B - L \tag{9}$$

where B is the baryon number and L the overall lepton number. The definition of parity in LR-symmetric models is shifted to the next section.

2 Multiplets, LR Symmetry and the Lagrangian

2.1 Basic Multiplets and Interactions

In the last section we have established that LR-symmetric models are characterized by the gauge group

$$G = SU(2)_L \times SU(2)_R \times U(1)_{B-L}. \tag{10}$$

Thus irreducible representations of G can be denoted by the triplet $(d_L, d_R, \tilde{Y})$ where d_L, d_R denote the dimensions of representations of $SU(2)_L$ and $SU(2)_R$, respectively. The fermion doublets are given by

$$\begin{array}{lll} \text{quarks:} & Q_L(2,1,1/3), & Q_R(1,2,1/3) \\ \text{leptons:} & L_L(2,1,-1), & L_R(1,2,-1) \end{array} \tag{11}$$

as we have discussed before. Note that every doublet has a flavour index in addition. Furthermore we will assume in the following that apart from the gluons the only non-trivial colour multiplets in the Lagrangian are the quarks. The gluons will be left out in the formulation of the Lagrangian. Equs. (10) and (11) uniquely fix the fermionic gauge Lagrangian

$$\begin{aligned} \mathcal{L}_f &= \sum_{\psi=Q,L} \Big\{ \bar{\psi}_L i\gamma^\mu \left(\partial_\mu + ig_L \frac{\vec{\tau}}{2} \cdot \vec{W}_{L\mu} + ig' \frac{B-L}{2} B_\mu \right) \psi_L + \\ &+ \bar{\psi}_R i\gamma^\mu \left(\partial_\mu + ig_R \frac{\vec{\tau}}{2} \cdot \vec{W}_{R\mu} + ig' \frac{B-L}{2} B_\mu \right) \psi_R \Big\}. \end{aligned} \tag{12}$$

The matrices τ_a $(a = 1,2,3)$ are the Pauli matrices. To give masses to the fermions a Yukawa Lagrangian of the type $\mathcal{L}_Y \sim \bar{\psi}_L \phi \psi_R$ is necessary where ϕ is a 2×2 matrix of scalar fields. The gauge transformation properties of ψ_L, ψ_R require that ϕ and therefore also $\tilde{\phi} \equiv \tau_2 \phi^* \tau_2$ transform as

$$\phi,\ \tilde{\phi}(2,2,0). \tag{13}$$

Consequently, the most general coupling of the fermions to ϕ is given by

$$\mathcal{L}_Y(\phi) = -\sum_{\psi=Q,L} \{\bar{\psi}_{Li} \Gamma^\psi_{ij} \phi \psi_{Rj} + \bar{\psi}_{Li} \Delta^\psi_{ij} \tilde{\phi} \psi_{Rj} + h.c.\}. \tag{14}$$

For the sake of clearness we have also exhibited the flavour space indices i, j.

The gauge transformations performed at the multiplets which we have already introduced are summarized by

$$\psi_{L,R} \to e^{-i\tilde{Y}\alpha/2} U_{L,R} \psi_{L,R}.$$

$$\begin{aligned}
\phi &\to U_L \phi U_R^\dagger \\
\frac{\vec{\tau}}{2}\cdot \vec{W}_{L,R\mu} &\to U_{L,R}\frac{\vec{\tau}}{2}\cdot \vec{W}_{L,R\mu}U_{L,R}^\dagger + \frac{i}{g_{L,R}}(\partial_\mu U_{L,R})U_{L,R}^\dagger \\
B_\mu &\to B_\mu + \frac{1}{g'}\partial_\mu \alpha
\end{aligned} \tag{15}$$

where U_L, U_R are space-time dependent $SU(2)$ matrices. As for the electric charge we know from the fermion sector (Equs. (8), (9)) that it has the general form

$$Q = T_{3L} + T_{3R} + \frac{1}{2}(B-L) \tag{16}$$

for all multiplets where T_{3L} and T_{3R} are generators of $SU(3)_L$ and $SU(3)_R$, respectively. This allows to calculate the charges of the fields in

$$\phi = \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix} \quad \text{by} \quad Q\phi = \left[\frac{1}{2}\tau_3, \phi\right] = \begin{pmatrix} 0\cdot\phi_{11} & +\phi_{12} \\ -\phi_{21} & 0\cdot\phi_{22} \end{pmatrix} \tag{17}$$

by using Equ. (15) and $\tilde{Y}(\phi) = 0$. Therefore ϕ_{11}, ϕ_{22} are neutral scalars which can acquire vacuum expectation values (VEV) without violating $U(1)_{em}$.

2.2 LR Symmetry as a Parity Transformation

In LR-symmetric models every L field in the fermion sector has an R counterpart and also $\vec{W}_L$ and $\vec{W}_R$ correspond to each other. This allows the definition of a symmetry which in the sense of Equ. (4) can be interpreted as a parity transformation:

$$\begin{aligned}
\vec{W}^\mu_{L,R}(x) &\to \varepsilon(\mu)\vec{W}^\mu_{R,L}(\hat{x}), \\
B^\mu(x) &\to \varepsilon(\mu)B^\mu(\hat{x}), \\
\psi_{L,R}(x) &\to V^\psi_{R,L}\gamma^0\psi_{R,L}(\hat{x}), \\
\phi(x) &\to \phi^\dagger(\hat{x})
\end{aligned} \tag{18}$$

with $\varepsilon(\mu) = 1$ for $\mu = 0$ and -1 for $\mu = 1,2,3$. The unitary matrices $V^\psi_{L,R}$ act in flavour space. The parts of the Lagrangian density which we have already discussed are not automatically invariant under (18) but P requires

$$g_L = g_R, \qquad (V^\psi_R)^\dagger \Gamma_\psi V^\psi_L = \Gamma^\dagger_\psi, \qquad (V^\psi_R)^\dagger \Delta_\psi V^\psi_L = \Delta^\dagger_\psi. \tag{19}$$

The condition on the gauge coupling constants comes from $\mathcal{L}_f$ and the other ones from $\mathcal{L}_Y(\phi)$. It is possible to get interesting models through special choices of $V^\psi_{L,R}$ [7] but the simplest case is

$$V^\psi_{L,R} = 1 \Rightarrow \Gamma^\dagger_\psi = \Gamma_\psi, \qquad \Delta^\dagger_\psi = \Delta_\psi. \tag{20}$$

Note that rewriting (18) analogously to the P transformation (5) leads to a picture which looks superficially like P in the mirror fermion case. But now P does not connect a representation and its complex conjugate but $(2,1,\tilde{Y})$ and $(1,2,-\tilde{Y})$.

2.3 Enlarging the Higgs Sector to Incorporate $SU(2)_R$ and Parity Breaking

It is easy to check that the VEVs of ϕ neither break G to $SU(2)_L \times U(1)_Y$ nor to $U(1)_{em}$. Thus the Higgs sector has to be enlarged to get a mechanism for breaking G to the SM group. This procedure is not unique but interesting models are obtained by the introduction of scalar triplets [3]

$$\Delta_L(3,1,2), \qquad \Delta_R(1,3,2). \tag{21}$$

(For another mechanism see Ref. [8].) LR-symmetric models which contain not more than the three scalar multiplets ϕ, Δ_L, Δ_R are often called minimal LR-symmetric models. The Yukawa couplings of the triplets are given by

$$\mathcal{L}_Y(\Delta) = L^T_{Li} G_{Lij} C^{-1} i\tau_2 \Delta_L L_{Lj} + L^T_{Ri} G_{Rij} C^{-1} i\tau_2 \Delta_R L_{Rj} + h.c. \tag{22}$$

and they transform under the gauge group as

$$\Delta_L \to e^{-i\alpha} U_L \Delta_L U_L^\dagger, \qquad \Delta_R \to e^{-i\alpha} U_R \Delta_R U_R^\dagger. \tag{23}$$

The $\Delta_{L,R}$ are complex $SU(2)$ triplets written as

$$\Delta = \frac{1}{\sqrt{2}}\tau_a \delta_a = \frac{1}{\sqrt{2}} \begin{pmatrix} \delta_3 & \delta_1 - i\delta_2 \\ \delta_1 + i\delta_2 & -\delta_3 \end{pmatrix} \tag{24}$$

with lepton number $L = -2$. Since they have no colour they cannot couple to quarks. The charges of the components of Δ are easily calculated by

$$Q\Delta = \left[\frac{1}{2}\tau_3, \Delta\right] + 1 \cdot \Delta = \begin{pmatrix} \Delta_{11} & 2 \cdot \Delta_{12} \\ 0 \cdot \Delta_{21} & \Delta_{22} \end{pmatrix} \tag{25}$$

thus leading to the charge assignments

$$\Delta = \begin{pmatrix} \frac{1}{\sqrt{2}}\delta^+ & \delta^{++} \\ \delta^0 & -\frac{1}{\sqrt{2}}\delta^+ \end{pmatrix}. \tag{26}$$

Parity in Δ space is given by

$$\Delta_{L,R}(x) \to -\Delta_{R,L}(\hat{x}). \tag{27}$$

As a consequence one obtains conditions on the coupling matrices $G_{L,R}$:

$$G_{L,R} = (V^L_{L,R})^T G_{R,L} V^L_{L,R}$$

which in the simplest case reduces to

$$V^L_{L,R} = 1 \Rightarrow G_L = G_R. \tag{28}$$

Now we have specified already all the multiplets of the minimal LR-symmetric model which together with gauge invariance and P invariance determines the total Lagrangian

$$\mathcal{L} = \mathcal{L}_g + \mathcal{L}_f + \mathcal{L}_s + \mathcal{L}_Y(\phi, \Delta) + \mathcal{L}_{HP} \tag{29}$$

where $\mathcal{L}_g$ is the gauge field part (kinetic energy and self-interactions), $\mathcal{L}_{HP}$ denotes the Higgs potential and $\mathcal{L}_s$ the scalar Lagrangian

$$\mathcal{L}_s = \mathrm{Tr}\ \{(D_\mu\phi)^\dagger(D^\mu\phi) + (D_\mu\Delta_L)^\dagger(D^\mu\Delta_L) + (D_\mu\Delta_R)^\dagger(D^\mu\Delta_R)\}$$

with

$$\begin{aligned} D_\mu\phi &= \partial_\mu\phi + ig\left(\frac{\vec{\tau}}{2}\cdot\vec{W}_L\phi - \phi\frac{1}{2}\vec{\tau}\cdot\vec{W}_R\right) \\ D_\mu\Delta_{L,R} &= \partial_\mu\Delta_{L,R} + ig\left[\frac{\vec{\tau}}{2}\cdot\vec{W}_{L,R}, \Delta_{L,R}\right] + ig'B_\mu\Delta_{L,R}. \end{aligned} \tag{30}$$

The trace Tr affects the products of 2×2 matrices occurring in (30). For a discussion of the Higgs potential see Ref. [3].

3 Spontaneous Symmetry Breaking

Assuming that the Higgs potential has an electric charge conserving minimum we obtain the VEVs (see (17) and (26))

$$\langle\phi\rangle_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} v & 0 \\ 0 & w \end{pmatrix}, \qquad \langle\Delta_{L,R}\rangle_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 \\ u_{L,R} & 0 \end{pmatrix}. \tag{31}$$

In the following we will always assume the order of magnitude relations

$$|u_L|^2 \ll |v|^2 + |w|^2 \ll |u_R|^2. \tag{32}$$

Taking into account only the effects of u_R one easily checks that u_R breaks $SU(2)_R \times U(1)_{B-L}$ to $U(1)_Y$ generated by the SM hypercharge

$$Y = 2T_{3R} + B - L. \tag{33}$$

Thus we obtain the breaking scheme

$$SU(2)_L \times SU(2)_R \times U(1)_{L-B} \times P \overset{u_R}{\to} SU(2)_L \times U(1)_Y \overset{v,w}{\to} U(1)_{e.m.}. \tag{34}$$

At the intermediate stage one has an extended SM with the two Higgs doublets ϕ, ν_R, δ's and effects of the R-handed scale. It is interesting to note that through the VEVs of $\Delta_{L,R}$ the lepton number is broken. However, this does not give rise to a Goldstone boson (majoron) because $B-L$ is associated with the gauge boson B_μ. The scheme (34) justifies the assumption about $|u_R|^2$. One can show that in this case the smallness of $|u_L|^2$ in (32) follows from the Higgs potential [3].

4 Masses and Mixings

4.1 $W_L W_R$ Mixing

Masses and mixings of fermions and gauge bosons are calculated at the tree level by considering $\mathcal{L}_Y(\langle\phi\rangle_0, \langle\Delta\rangle_0)$ and $\mathcal{L}_s(\langle\phi\rangle_0, \langle\Delta\rangle_0)$, respectively. The LR-mixing of the gauge bosons is thereby determined by $\langle\phi\rangle_0$ and its explicit form is

$$-g^2\{(v^* w W_L{}^+{}_\mu W_R^{-\mu} + h.c.) + \frac{1}{2}(|v|^2 + |w|^2)W_{L3\mu}W_{R3}{}^\mu\} \tag{35}$$

with $W^{\pm}_{L,R} = (W_{L,R1} \mp iW_{L,R2})/\sqrt{2}$. One can check in a similar way as it was done for the Higgs multiplets that $\pm$ is the correct charge assignment. The mixing of the charged vector bosons is described by

$$\begin{pmatrix} W_1^+ \\ W_2^+ \end{pmatrix} = \begin{pmatrix} \cos\xi & -\sin\xi e^{i\lambda} \\ \sin\xi e^{-i\lambda} & \cos\xi \end{pmatrix} \begin{pmatrix} W_L^+ \\ W_R^+ \end{pmatrix} \tag{36}$$

where $W^+_{1,2}$ are the mass eigenstates. Examining $\mathcal{L}_s$ in more detail and taking into account (32) then after some algebra one obtains [10,11,12,13]

$$e^{i\lambda} = -\frac{vw^*}{|vw|}, \qquad \xi \simeq \frac{2|vw|}{|v|^2+|w|^2}\left(\frac{M_1}{M_2}\right)^2, \qquad |v|^2+|w|^2 \simeq (\sqrt{2}G_F)^{-1}$$

and

$$M_1^2 \simeq \frac{1}{4}g^2(|v|^2+|w|^2), \qquad M_2^2 \simeq \frac{1}{4}g^2(2|u_R|^2+|v|^2+|w|^2). \tag{37}$$

The mass M_1 is essentially the SM W mass whereas the mass M_2 of the heavy charged gauge boson W_2 is determined by the breaking scale u_R of $SU(2)_R$. The mixing of the neutral gauge bosons is more involved because LR mixing adds to $W_{L3}B$ mixing [10,12,13].

4.2 Quark Mixing

The quark mass matrices are determined by $\mathcal{L}_Y(\langle\phi\rangle_0)$ and they are given by

$$M_u = \frac{1}{\sqrt{2}}(v\Gamma_Q + w^*\Delta_Q), \qquad M_d = \frac{1}{\sqrt{2}}(w\Gamma_Q + v^*\Delta_Q). \tag{38}$$

Defining mass eigenstates $u_{L,R}$, $d_{L,R}$ by

$$u^0_{L,R} = U^u_{L,R}u_{L,R}, \qquad d^0_{L,R} = U^d_{L,R}d_{L,R}$$

with

$$Q_{L,R} = \begin{pmatrix} u^0 \\ d^0 \end{pmatrix}_{L,R} \tag{39}$$

and unitary matrices $U^{u,d}_{L,R}$ the mass matrices are diagonalized by

$$\begin{aligned} \widehat{M}_u &= \text{diag}\,(m_u, m_c, m_t) = U_L^{u\dagger} M_u U_R^u \\ \widehat{M}_d &= \text{diag}\,(m_d, m_s, m_b) = U_L^{d\dagger} M_d U_R^d. \end{aligned} \tag{40}$$

Since in LR-symmetric models there are L and R sectors there are also two mixing (KM) matrices

$$K_{L,R} = U^{u\dagger}_{L,R}U^d_{L,R}. \tag{41}$$

In general K_L and K_R are independent but it exists an interesting special case where the mixing angles (not the phases) are the same in both sectors. This is the case of "manifest CP invariance" [14] where P symmetry (20) (Γ_Q, Δ_Q hermitean) together with the usual CP invariance (Γ_Q, Δ_Q real) implies

$$\Gamma_Q, \Delta_Q \text{ real and symmetric.} \tag{42}$$

Therefore in this case the quark mass matrices are symmetric and one can choose $U_R^{u,d} = U_L^{u,d*}$ [15]. Consequently we have

$$|K_{Lij}| = |K_{Rij}| \tag{43}$$

which are phase convention independent relations and therefore the physical consequence of manifest CP invariance.[1]

4.3 The Charged Current Lagrangian

Putting the results of the previous subsections together we can write down the charged current interaction of the quarks

$$\begin{aligned} -\mathcal{L}_{cc}^{Q} &= \frac{g}{\sqrt{2}}(\bar{u}_L^0\gamma^\mu d_L^0 W_{L\mu}^+ + \bar{u}_R^0\gamma^\mu d_R^0 W_{R\mu}^+) + h.c. \\ &= \frac{g}{2\sqrt{2}}[\cos\xi\bar{u}\gamma^\mu(1-\gamma_5)K_L d - e^{-i\lambda}\sin\xi\bar{u}\gamma^\mu(1+\gamma_5)K_R d]W_{1\mu}^+ + \\ &\quad + \frac{g}{2\sqrt{2}}[e^{i\lambda}\sin\xi\bar{u}\gamma^\mu(1-\gamma_5)K_L d + \cos\xi\bar{u}\gamma^\mu(1+\gamma_5)K_R d]W_{2\mu}^+ + h.c. \end{aligned} \tag{44}$$

Lagrangian (44) is the starting-point for discussions of CP violation in LR-symmetric models (see J.-M. Frère in these proceedings, [15,16] and Refs. therein and also [11,17] for the electric dipole moment).

5 Lepton Sector

Neutrino masses and mixing have special features in LR-symmetric models which warrant a discussion in a separate section. The charged lepton mass matrix

$$M_\ell = \frac{1}{\sqrt{2}}(w\Gamma_L + v^*\Delta_L) \tag{45}$$

is given in analogy to the down quarks but the corresponding neutrino matrix

$$M_D = \frac{1}{\sqrt{2}}(v\Gamma_L + w^*\Delta_L) \tag{46}$$

is only the 3×3 "Dirac part" of the 6×6 Majorana mass matrix in the three family case (for reviews on neutrino physics see e.g. [18]). The reason is that the triplet scalars $\Delta_{L,R}$ generate Majorana mass terms for ν_L and ν_R, respectively, via $\mathcal{L}_Y(\Delta)$ (Equ. (22)) because of

$$i\tau_2\langle\Delta_{L,R}\rangle_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} u_{L,R} & 0 \\ 0 & 0 \end{pmatrix}. \tag{47}$$

Putting the 6 L-handed degrees of freedom into the vector

$$\Omega_L = \begin{pmatrix} \nu_L \\ (\nu_R)^c \end{pmatrix} \tag{48}$$

[1] Note that the so-called "manifest LR symmetry" defined by $K_L = K_R$ does not follow from a symmetry and is thus an arbitrary stipulation.

the Majorana neutrino mass term is given by

$$\mathcal{L}_\nu^m = \frac{1}{2}\Omega_L^T C^{-1} \mathcal{M}_\nu \Omega_L + h.c.$$

with

$$\mathcal{M}_\nu = \begin{pmatrix} \sqrt{2}u_L G_L & M_D^* \\ M_D^\dagger & \sqrt{2}u_R^* G_R^* \end{pmatrix}. \tag{49}$$

Matrix (49) is ideally suited for the seesaw mechanism [19]. Assuming that u_L can be neglected [3,9] one gets the order of magnitude estimate

$$\mathcal{M}_\nu \sim \begin{pmatrix} 0 & m_\ell \\ m_\ell & u_R \end{pmatrix} \tag{50}$$

for every generation. It has been taken into account that M_D is naturally of order of the charged lepton masses m_ℓ. Since u_R is a scale higher than the SM W mass we get three heavy Majorana neutrinos N and three light neutrinos with masses

$$m_N \sim u_R \sim M_2, \qquad m_{\nu_\ell} \sim m_\ell^2/m_N. \tag{51}$$

Thus LR-symmetric models offer a possibility to explain the smallness of the standard neutrino masses by naturally incorporating the seesaw mechanism. Finally we want to remark that mass relations like $m_{\nu_e} = m_{\nu_\tau} \sim m_e m_\tau/m_N$ can also be obtained by invoking special symmetries [7].

6 Lower Bounds on the R-Handed Scale

Since no effects beyond the SM have been seen so far one can only derive lower bounds on the R-handed scale. In doing this it is appropriate to carefully point out the conditions under which such bounds are obtained. There are three main areas which are important in this connection:

A. $K_L K_S$ mass difference: In LR-symmetric models one has a $W_1 W_2$ box diagram in addition to the SM box diagram. Taking L and R mixing angles equal (e.g. manifest LR symmetry) then barring accidental cancellations one gets a lower bound on the W_2 mass of order 1 - 3 TeV [20,21]. In the same way a bound of order 10 TeV on the mass of the flavour changing neutral scalar in the minimal LR-symmetric model has been obtained [21]. In the general case with V_R arbitrary and possible cancellations no bound on M_2 can be derived without any further qualifications. In Ref. [22] a bound $M_2 \gtrsim 300$ GeV has been calculated in the general case by requiring "avoiding of extreme fine-tuning". For a definition of this notion consult Ref. [22].

B. Neutrinoless $\beta\beta$ decay: Since Majorana neutrinos are their own antiparticles their existence in LR-symmetric models allows neutrinoless $\beta\beta$ decay by joining the two neutrino lines. For the decay $^{76}Ge \to {}^{76}Se + 2e^-$ an experimental lower bound on the halflife of order $\tau_{1/2} > 10^{24}y$ exists. This can be converted into a bound on a rather complicated function of masses and mixing matrix elements. With the assumptions $M_2 \simeq m_N$, $V_{Rud} \simeq 1$ and $\tilde{V}_{Rei} \simeq \delta_{ei}$ ($\tilde{V}_R$ is the R-handed mixing matrix in the lepton sector) the bound $M_2 \gtrsim 800$ GeV has been derived [22].

C. Production of W_2 and heavy neutrinos in colliders: Cross sections for such processes can be found in Refs. [12,23]. In $p\bar{p}$ collisions search for $W_R \to \ell\nu_{R\ell}$ ($\ell = e, \mu$) (i.e. $\xi = 0$ and $\tilde{V}_{Re.} = \delta_{e\iota}$) has given $M_2 > 520$ GeV (95% c.l.) for $m_{\nu_R} < 15$ GeV [24]. There is also a bound on the mass of the predominantly R-handed Z boson of 800 GeV derived from LEP data in the minimal LR-symmetric model [25]. This can be transformed into a bound on the W_2 mass of 300 GeV [12].

Upper bounds on the LR mixing angle ξ can also be derived from different assumptions. They are of order 10^{-3} (see Ref. [22] and Refs. therein).

In conclusion we see that lower bounds on the R-handed scale are not very stringent in the general case. But with plausible assumptions like manifest CP invariance or others such bounds are around 1 TeV.

Acknowledgement

The author thanks the organizers for the hospitable and relaxed atmosphere at this meeting.

References

[1] S.L. Glashow, Nucl. Phys. *22* (1961) 579;
S. Weinberg, Phys. Rev. Lett. *19* (1967) 1264;
A. Salam, in proc. 8th Nobel Symposium, Aspenäsgården, 1968, ed. N. Svartholm (Almqvist and Wiksell, Stockholm, 1968).

[2] J.C. Pati and A. Salam, Phys. Rev. *D10* (1975) 275;
R.N. Mohapatra and J.C. Pati, Phys. Rev. *D11* (1975) 566 and 2558.

[3] R.N. Mohapatra and G. Senjanović, Phys. Rev. Lett. *44* (1980) 912; Phys. Rev. *D23* (1981) 165.

[4] R. Slansky, Phys. Rep. *79* (1981) 1.

[5] N.V. Smolyakov, Theor. and Math. Phys. *50* (1982) 225.

[6] J. Maalampi and M. Roos, Phys. Rep. *186* (1990) 53.

[7] G. Ecker, W. Grimus and M. Gronau, Nucl. Phys. *B279* (1987) 429.

[8] R.N. Mohapatra and G. Senjanović, Phys. Rev. *D12* (1975) 1502.

[9] D. Chang, R.N. Mohapatra and M.K. Parida, Phys. Rev. Lett. *52* (1984) 1072; Phys. Rev. *D30* (1984) 1052;
D. Chang and R.N. Mohapatra, Phys. Rev. *D32* (1985) 1248.

[10] G. Senjanović, Nucl. Phys. *B153* (1979) 334.

[11] G. Ecker, W. Grimus and H. Neufeld, Nucl. Phys. *B229* (1983) 421.

[12] W. Buchmüller and C. Greub, Nucl. Phys. *B381* (1992) 109.

[13] R.N. Mohapatra, in CP Violation, ed. C. Jarlskog (World Scientific, Singapore, 1989), p. 384.

[14] D. Chang, Nucl. Phys. *B214* (1983) 435.

[15] I. Schur, Am. J. Math. *67* (1945) 472;
B. Zumino, J. Math. Phys. *3* (1962) 1055.

[16] J.-M. Frère et al., Phys. Rev. *D46* (1992) 337.

[17] J.-M. Frère et al., Phys. Rev. *D45* (1992) 259.

[18] S.M. Bilenky and S.T. Petcov, Rev. Mod. Phys. *59* (1987) 671;
R.D. Peccei, in proc. of Symposium on Heavy Flavour Physics, 1988, Beijing, PR China, p. 235;
R.N. Mohapatra and P.B. Pal, Massive Neutrinos in Physics and Astrophysics, World Scientific, Singapore, (1991).

[19] M. Gell-Mann, P. Ramond and R. Slansky, in Supergravity, eds P. van Nieuwenhuizen and D. Freedman, North-Holland, Amsterdam (1979);
T. Yanagida, in proc. of Workshop on Unified Theory and Baryon Number in the Universe, eds. O. Sawada and A. Sugamoto, KEK (1979).

[20] G. Beall, M. Bender and A. Soni, Phys. Rev. Lett. *48* (1982) 848.

[21] G. Ecker and W. Grimus, Nucl. Phys. *B258* (1985) 328.

[22] P. Langacker and S. Uma Sankar, Phys. Rev. *D40* (1989) 1569.

[23] J. Maalampi, A. Pietilä and J. Vuori, Phys. Lett. *B297* (1992) 327.

[24] F. Abe et al., Phys. Rev. Lett. *67* (1991) 2609.

[25] G. Altarelli et al., Phys. Lett. *B263* (1991) 459.