BNL-49381

Submitted to Workshop on the Stability of Particle Motion in Storage Rings Brookhaven National Laboratory October 19-24, 1992

Summary for Working Group B on Long-Term Stability*

Stephen G. Peggs
BNL, Upton, NY 11973, USA

1 Introduction

A total of 36 workshop participants attended at least one session of the Long-Term Stability working group. We avoided turning these sessions into a specialized seminar series by meeting in two subgroups, loosely labeled Analysis and Diffusion & Tracking, so that working discussions among a reasonably small number of people were possible. Nonetheless, no attempt is made to categorize the 13 group B papers according to original subgroup.

A similar workshop, the Workshop on Accelerator Orbit and Particle Tracking Problems, was held almost exactly 10 years ago at Brookhaven. It is interesting to see how many of the participants in the photograph of that workshop appear again in the photograph at the front of these proceedings. Fortunately, it is not correct to infer that little progress has been made in the last decade, nor that the average age of the participants has increased significantly. Rather, the recent photograph has many more, younger, faces than its predecessor. This attests to the ongoing interest and vigorous activity in an area of central importance to accelerator physics.

2 Ten Years After

While a conclusive remark made during the workshop correctly states that "major breakthroughs have been made in the rapid improvement of computer capacity and speed, in the development of more sophisticated mathematical packages, and in the introduction of more powerful analytic approaches," it is easy to overlook the profound philosophical shift that has adiabatically accompanied these advances, and that underpins them. In the last decade, our attitude towards the legitimacy and role of numerical methods (computers) has changed from a rather grudging acceptance of the inevitable, to enthusiastic investigation of the possible. The average level of computer literacy has risen far. The following two examples illustrate this point.

The motion of particles in an accelerator is inherently discrete — first one magnet is traversed, and then an entirely different one. Although one may argue about reducing nonlinear magnets of interest to thin elements of vanishing length, it is clear that the underlying mathematical model is closer to one of **difference** equations, than of **differential** equations. Unfortunately, it is usually impossible to solve nonlinear difference equations analytically. For this reason, classical dynamicists, in this century and the last, developed differential Hamiltonian methods that are most appropriate for continuous systems. (Note, however, that Poincaré knew of the existence of chaos.) By contrast, it is natural to model difference systems such as accelerators using iterative computer codes. A decade or so ago, when computers began to be used intensively to investigate map dynamics, Hamiltonian descriptions of accelerators were attempted by incorporating time-dependent delta-function perturbations. Today, such Hamiltonian approaches are generally accepted as having limited applicability — especially to the problem of long term-stability — although they have had some important successes. Numerically

*Work performed under the auspices of the U.S. Department of Energy.

derived results are more rigorous, and they enjoy a higher degree of confidence than before. Ten years ago, we were still trying to understand round-off errors, and still learning to enforce symplecticity.

The second example is the change of paradigm that lies behind the advent of differential algebra. A decade ago, the unequivocal object of interest was the 4- or 6-dimensional phase-space vector. One wrote subroutines that input a phase-space vector in order to output it one turn later. In the interim, the propagation of high-order Taylor expansions of maps has gone from being a formal mathematical curiosity to being a routine numerical operation. Object-oriented languages now allow the manipulation of such large analytically unwieldy structures as pyramids of polynomial coefficients, using natural mathematical operators such as + and * in code, through the miracle of function overloading. The expansion variables need not be limited to 6-phase-space coordinates, but may also include lattice quantities such as sextupole corrector strengths, so that the **parametric** behavior of the map can now be investigated directly.

The study of modulation intrinsic to the beam, or caused by ripple in magnet power supplies, has been a growth industry. Having been mentioned in only 2 of the 20 papers in the 1982 proceedings, it is important in 8 of the 13 papers from this working group. One reason for this is that modulation effects are important, but another is that the time scales under study have vastly increased with the computing power available. Loosely speaking, we used to track for 10^3 turns, but now 10^6-turn studies are routine. Since the periods for most significant modulation sources are in the range of 10^2 to 10^3 turns, it could be said that we still track for 10^3 time units, but that now the natural unit of time is a modulation period, and not an accelerator turn.

Last but not least, it is crucial to note how much more operational experience we now have. Then, the SPS was the only hadron storage ring in routine operation. Now, not only have the Tevatron, HERA, and IUCF joined the SPS, but well thought out dynamics experiments have resulted in an important body of high quality data being made available. In ten years' time we hope and expect that RHIC, the SSC, and LHC will also join the ranks. And we expect it to be hard to remember how little we knew, and what our perspectives were, in 1992.

3 Contemporary themes

Without pretending to be comprehensive, this section outlines some of the themes that appear in the working group B papers.

Although almost all tracking codes are kick codes that represent nonlinear magnets by vanishingly thin elements, the opinion of Maletic and Ruggiero is that "... in order to predict the stability of the motion over very long periods of time it is mandatory that all kinematic terms be properly included in the model. Neglecting some of them may invalidate the results of very time-consuming exercises on the computer." Nonetheless, the other authors who report tracking results all use kick codes. They use a spectrum of accelerator tracking models, from a minimalist 1-D Henon map without tune modulation [Giovannozzi], to a maximalist model including all measured magnet multipoles in HERA [Zimmerman].

Stupakov introduces the concept of a quasi-closed orbit that closes on itself if the modulation period is an integer number of turns, in order to analytically investigate allowable levels of ripple in SSC dipole magnets. He concludes that "its envelope allows one to determine the range of particle deviations at any position of the ring for a given amplitude of the [modulation]."

The Henon map consists of a rotation in phase-space coordinates followed by a single sextupole kick. It is parameterized by a single quantity, the tune, since the sextupole strength can be normalized by rescaling phase space. The difficulty of determining the border of stability of even this simple system, using traditional analytic tools, is noteworthy. One reason for this is the need to go to high-order perturbation theory, making simple Hamiltonian approaches unsuitable. Giovannozzi describes one analytic method, and attempts to extend it to more general 1-D maps. The papers by Brüning and by Bazzani and Pusterla go on to add tune modulation to the Henon map. Brüning makes a connection with experiments at the SPS, in which extrinsic tune modulation was introduced at one or two frequencies, with variable parameters. Bazzani and Pusterla present analytical and numerical results, culminating in the dependence of a 1-D diffusion coefficient on the tune modulation amplitude.

Tune modulation due to finite chromaticity and synchrotron motion is an essential ingredient in the independent tracking models used by Dell, and by Parzen, to simulate RHIC. Parzen confirms the conventional wisdom that it makes a considerable difference to the dynamic aperture whether or not a particle launched with a momentum offset of $\Delta p/p = 0.005$ undergoes synchrotron oscillations. He goes on to make the phenomenological observation that "synchrobetatron coupling becomes important when the particle transverse displacement due to $\Delta p/p$ is about equal to the betatron oscillation amplitude." Dell compares the dynamic aperture results obtained by tracking 10^2 particles for 10^3 turns with those found by tracking 1 particle for 10^6 turns. He launches 100 particles on a surface of constant total initial action by randomly selecting the remaining free initial coordinates. Ten different magnetic multipole seeds are used in each method. He concludes that, while "apertures determined by the worst case values from 10^6-turn runs and multiparticle runs are essentially equal ... [the] use of 10^2 particles tracked for 10^3 turns requires approximately one tenth the computer time needed for 10^6-turn runs and thus enables more varied studies for a given computer budget."

In 1-D, the quantitative agreement between the analytic theory of tune modulation effects, simulation results, and accelerator experiments such as E778, is well established — when a single isolated resonance model is valid. The situation is conveniently encapsulated in a plot in the space of tune modulation depth and modulation tune, showing 4 distinct dynamical phases. Unfortunately the real world has 2 transverse degrees of freedom. Satogata and Peggs attempt to extend the agreement between analysis and simulation to 2-D, by investigating thick-layer diffusion. Here, the chaos caused by overlapping synchrobetatron sidebands of a horizontal primary resonance feeds into the vertical motion via a nearby secondary resonance. The conventional Hamiltonian model tortuously derives a diffusion coefficient for the vertical motion that exhibits steps when plotted versus the distance from the working point to the secondary resonance. In their paper, Satogata and Peggs observe that the vertical diffusion is best described by an exponential chaotic random divergence, and not by root time diffusion. The exponent that they observe also exhibits steps as a function of the distance to the secondary resonance, but as yet there is no quantitative agreement between simulation and analytical prediction.

Zimmerman presents an impressively detailed theoretical and numerical model of HERA, derived from his recently published thesis. Among other topics, he considers the use of a Lyapunov exponent method in forecasting long-time scale behavior, and derives a complementary version of the tune modulation dynamical phase diagram, emphasizing the width of the stochastic layer between sidebands rather than the behavior of particles near the center of the sideband islands. After predicting diffusion rates for HERA and comparing them with operational observations, he concludes "the observed dynamic aperture ... in the HERA proton ring at 40 GeV is in good agreement with the results of computer simulations and analytical predictions which include a modest tune modulation and the effect of the measured nonlinear field errors of all superconducting magnets."

Experiments both at the Tevatron and at the SPS have investigated diffusion effects. Quite different approaches were used, but Gerasimov nonetheless attempts to compare results from the superconducting storage ring with results from the normal conducting ring. "In view of the incompatibilities of the diffusion models with the CERN diffusion experiment data and tracking survival data one could naturally ask why the Fermilab diffusion experiment data were basically quite successfully fitted with diffusion models." Shi and Ohnuma develop a technique to study the evolution of particle distribution as a function of oscillation amplitude. "Using perturbation expansion with multiple scales, we have solved the Vlasov equation and the Fokker-Planck equation in the time domain." Unfortunately, "when the system is close to major resonances, the perturbation expansion ... may not converge ... large storage rings, however, are generally operated far from all major resonances."

In a short paper, Mane presents two simple examples of second-order symplectic integrators for spin motion. He claims that "the use of Lie group theory makes the generalization [from orbit symplectic generators] to include spin motion relatively straightforward." In a lengthy paper, Mane and Weng review normal form methods for solving nonlinear differential equations, and compare the relative merits of three ways to evaluate normal forms. They conclude that "...the superiority of the minimal normal form method ... for ordinary nonlinear autonomous differential equations ... has been demonstrated. The minimal normal form method has also been extended, to treat discrete maps. The application to the evaluation of one-turn maps for accelerators yields mixed results, hence the superiority of the minimal normal form is not as clearly visible."

4 Working Group Opinions

The last session of the working group was held with both subgroups, to see if we could form a group consensus on some issues. The 32 attendees were informally polled on two questions:

Is there a dynamic aperture? Survival plots, showing the logarithm of the number of turns before loss plotted versus the launch amplitude, have become commonplace. Since data on these plots typically extend only to about 10^6 turns, it is natural to speculate on how to extrapolate to the 10^9-turn time scale of interest. One school of thought claims that there is a minimum initial amplitude — the dynamic aperture — below which all particles are unconditionally stable (neglecting Arnol'd diffusion). The survival plot will be vertical at the dynamic aperture. On the other hand, some claim that all particles eventually make their way out to the vacuum chamber wall. When the participants were asked if they expect the survival plot to become vertical, demonstrating an unambiguous dynamic aperture, 24 said YES, 1 said NO, and 7 abstained.

What time scales do we understand? Although we routinely track for 10^6 turns, some argue that we don't use a complete enough model to trust the results. Others argue that all the important physics is in the model, and that we can already trust one-turn map techniques for very long times. Of the 18 people who responded to the question, 14 believed that we understand the 10^6- to 10^7-turn time scale, but 2 pessimists trust only in 10^3- to 10^4-turn results. The most optimistic opinion was that we can trust our present models even beyond 10^{10} turns.

Is there anything better, or faster, or more reliable than brute force single-particle tracking? We discussed three alternative approaches to tracking, without voting:

1) Lyapunov exponent methods. There was a loose consensus that the Lyapunov method, based on about 10^4 turns' worth of tracking data, is "a conservative stability test that gives a lower bound limit on the aperture for 10^5 to 10^6 turns."

2) One-turn truncated concatenated maps. After much discussion, we agreed to disagree. Notable comments from different ends — and dimensions — of the spectrum included:

"The reliability of one-turn map methods is case dependent, and so one must always test results with brute force codes anyway, removing the advantage of speed."

"There is no need to compare one-turn map and brute force tracking results. One should instead compare one-turn map tracking results with different truncation orders."

"There is no advantage in speed below about 20,000 turns."

"A larger source of uncertainty is the validity of the mathematical model, and the accuracy of the magnet data. How well do we model or measure colliders anyway?"

3) More particles for fewer turns. Discussion focused on the issue of whether accelerator motion is ergodic, so that a particle launched with equal horizontal and vertical displacements (for example) will adequately sample all parts of phase space. The consensus was that the motion is non-ergodic, so that many particles with different initial amplitude ratios should be launched to get truly reliable aperture results. However, there was no consensus on the optimum trade-off between number of particles and number of turns, for a fixed number of particle-turns.

Acknowledgement

I would like to thank Dusan Maletic for the unstinting and invaluable help he provided as secretary of working group B.