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II. Application to the trinucleon system ***

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**Relativistic three-particle dynamical equations: II. Application to
the trinucleon system**

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Abstract

We calculate the contribution of relativistic dynamics on the neutron-deuteron scattering length and triton binding energy employing five sets trinucleon potential models and four types of three-dimensional relativistic three-body equations suggested in the preceding paper. The relativistic correction to binding energy may vary a lot and even change sign depending on the relativistic formulation employed. The deviations of these observables from those obtained in nonrelativistic models follow the general universal trend of deviations introduced by off- and on-shell variations of two- and three-nucleon potentials in a nonrelativistic model calculation. Consequently, it will be difficult to separate unambiguously the effect of off- and on-shell variations of two- and three-nucleon potentials on low-energy three-nucleon observables from the effect of relativistic dynamics.

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1 INTRODUCTION

As the two-nucleon observables test the two-nucleon potential only on-shell one needs to consider the few-nucleon system to test the off-shell properties of this potential. Also, one needs to consider the few-nucleon system in order to study the effect of the three-nucleon interaction. There has been a great deal of experimental and theoretical activities in the three-nucleon system over the last three decades with the objective of extracting informations about the two- and three-nucleon interactions. In the recent past there has been many benchmark calculations involving realistic two- and three-nucleon potentials.[1, 2, 3] Though it has been possible to fit most of the low-energy three-nucleon observables using an appropriate *ad hoc* mixture of reasonable two- and three-nucleon potentials, not much physics was learn from these calculations. No reasonable criteria for preferring one nonrelativistic meson-theoretic[4] potential model over another for this system has been obtained from these calculations. Though these calculations have been successful in explaining a great deal of experimental data, they have revealed very little new information about the two- and three-nucleon interactions, once the potential models satisfy some reasonable criteria, such as possessing the correct tail.[3]

The most easily and commonly studied three-nucleon observables, which are very sensitive to variations of two- and three-nucleon interactions, are the triton binding energy, B_t , and the S -wave spin-doublet neutron-deuteron scattering length, a_{nd} . Quite sometime ago Phillips[3, 5] noted that, in nonrelativistic potential model calculations, these two observables are always correlated. Later many other correlations have been observed in the S -wave spin-doublet observables. Girard and Fuda[6] found that the S -wave asymptotic normalization parameter of triton is correlated with B_t or a_{nd} . A correlation has been observed between the r.m.s. radius of triton and B_t . [1, 2, 3] There has been correlations involving the D -state observables of the three-nucleon system.[1, 2, 3]

If two three-nucleon nonrelativistic dynamical models yield the same value for B_t or a_{nd} they should yield identical results for many other correlated three-nucleon observables.[1, 2, 3, 5] These observables of the three-nucleon system, which exhibit the correlated behavior, are usually most sensitive to the variations of the three-nucleon potential models. The low-energy correlations make it simple to classify the results of theoretical calculations, while at

the same time make the extraction of physically meaningful information that much harder.[3]

The importance of relativistic effects in the three-nucleon calculations has never been overemphasized. Both the bound-state and low-energy scattering calculations involve large momentum components which demand a relativistic dynamical treatment of the problem. Relativistic dynamical calculations in the three-nucleon problem have been mainly restricted to the study of the three-nucleon bound state problem[7, 8, 9, 10, 11] with one exception where relativistic effect on the neutron-deuteron scattering length has been studied[12]. However, the objective of all these studies has been the same. The authors have been mainly concerned in explaining the missing gap between the predictions of a nonrelativistic potential model for the three-nucleon system and experiment by incorporating some kind of relativistic dynamics. Both the four-dimensional Bethe-Salpeter-Faddeev equation[8, 9] in some approximate form and several types of three-dimensional reductions of this equation have been employed for this purpose.[7, 8, 9, 10, 11, 12]

Though the magnitude of relativistic corrections to B_t and a_{nd} , as emphasized in previous studies, is interesting, in our opinion it is most relevant to see if meaningful physics could be extracted from the relativistic treatment of the three-nucleon system. The nonrelativistic potential model calculations of the three-nucleon system involving meson-theoretic nucleon-nucleon potentials[4] did not allow us to extract meaningful informations about the two- and three-nucleon interactions because of the correlated behavior of the observables directly sensitive to these interactions.[3] The question to ask at this stage is whether the relativistic treatment of the three-nucleon problem is expected to change the scenario.

It is still unclear on how to progress from QCD to practical collision integral equations for hadronic and nuclear processes. Nevertheless, often for hadronic systems a Bethe-Salpeter (BS) type equation is postulated using some type of meson-baryon field theory with phenomenology, that presumably have a wider range of validity than nonrelativistic equations of the Lippmann-Schwinger (LS) type. Usually, the ladder approximation to the BS equation and its subsequent reduction to three-dimensional form [13, 14, 15, 16] have permitted numerical calculations. It is reasonable to require that all the approximate versions of the BS equation satisfy conditions of time-reversal symmetry, unitarity, and relativistic covariance. One of the approximate versions considered so far[15] and frequently used in numerical calculations[7, 8, 9, 11] in an approximate form do not even satisfy conditions of time-reversal

symmetry. However, at present time, in spite of these defects, one of the practical and feasible ways for performing a relativistically covariant three-nucleon calculation is through some of these approximate three-dimensional equations and we use them for studying the relativistic effect to the three-nucleon problem. At this point it should be noted that the solution of the approximate BS equation in ladder form is not necessarily a superior way of dealing with the relativistic effect.[11, 17]

In order to verify if new informations about two- and three-particle interactions could be obtained via a relativistic dynamical three-nucleon calculation, we have performed three-nucleon calculation for B_t and a_{nd} using several separable potential models and four approximate versions of three-dimensional relativistic three-particle equations suggested in the preceding paper.[16] The spin variables are treated nonrelativistically. We do not pretend to claim that the separable potential model presents a realistic description of the three-nucleon system. However, the numerical calculation is simplified by an order of magnitude in this model, and this model has been used successfully in understanding the essential features of the nonrelativistic three-nucleon problem. Here we employ the relativistic version of the three-particle separable potential model with a hope to see if new physics could be extracted from a study of the low-energy observables of the three-nucleon system.

We employ Yamaguchi and Tabakin-type[18] nucleon-nucleon 3S_1 and 1S_0 potentials in the present calculation. Tabakin-type nucleon-nucleon potentials yield nucleon-nucleon phase shifts in better agreement with experiment, which change sign at higher energies, compared to the Yamaguchi potential. If Tabakin-type potential is used in both 3S_1 and 1S_0 spin channels, it leads to an unrealistic triton ground state of several hundred MeV's.[19] The use of the Tabakin potential in one of the nucleon-nucleon spin channels and Yamaguchi in the other, as has been done in the present calculation, does not lead to a collapsed triton and lead to trinucleon observables in better agreement with experiment and realistic calculations.

We derive certain general theoretical inequalities among the different triton binding energies obtained using nonrelativistic and various relativistic dynamical formulations. These inequalities are verified in actual numerical calculations and are expected to be valid in general for other potential models. All the relativistic models satisfy conditions of relativistic covariance and unitarity. As there is no obvious theoretical reason for preferring one of the relativistic formulations over another, in view of these inequalities it is not to the point to

talk about the absolute value of the relativistic corrections to B_t or a_{nd} ; one could have corrections of different magnitudes and signs.

We present the nonrelativistic and relativistic three-nucleon models, which we use in numerical calculations, in Sec. II. Numerical results are presented in Sec. III and finally, a summary of our findings are given in Sec. IV.

2 DYNAMICAL MODELS

As we shall only be considering the three-nucleon system, it is convenient to consider three equal-mass particles of mass m , where m is the nucleon mass. In our calculation we use $\hbar c = 197.33$ MeV fm, and $m = 938.97$ MeV.

The nonrelativistic two-nucleon dynamics for a central S wave potential is governed by the following partial-wave Lippmann-Schwinger (LS) equation

$$t(q', q, k^2) = V(q', q) + 4\pi \int_0^\infty p^2 dp V(q', p) \frac{1}{k^2 - p^2 + i0} t(p, q, k^2), \quad (1)$$

where $V(q', q)$ is the usual momentum space potential. The relativistic two-nucleon dynamics for the same potential is taken to be governed by the following partial-wave Blankenbecler-Sugar (BIS) equation[13]

$$t(q', q, k^2) = V(q', q) + 4\pi \int_0^\infty p^2 dp \frac{m}{\omega_p} V(q', p) \frac{1}{k^2 - p^2 + i0} t(p, q, k^2), \quad (2)$$

where $\omega_p = (m^2 + p^2)^{1/2}$. Equation (2) satisfies the conditions of relativistic unitarity and covariance. However, these conditions are not enough to specify the relativistic dynamics properly. Actually, there are a host of such equations.[3, 16] In our study, however, at the two-nucleon level we shall only consider the dynamics given by BIS Eq. (2).

We shall consider only separable forms for two-nucleon potentials. There is a convenient way of defining phase equivalent nucleon-nucleon potentials using relativistic and nonrelativistic equations.

We take the relativistic nucleon-nucleon potential of the following form

$$[V_n(q', q)]_{rel} = -\lambda_n [v_n(q')]_{rel} [v_n(q)]_{rel}, \quad (3)$$

where $n = 0$ (1) represents the spin triplet (singlet) state, and the subscript rel (nr) denotes relativistic (nonrelativistic). Several analytic form factors have been used for the form factor $[v_n(q)]_{rel} [\equiv N_n g_n(q)]$, where N_0 (N_1) is the normalization for the momentum space spin triplet deuteron (singlet virtual-state) wave function $\phi(q) = N_0 g_0(q) (\alpha_0^2 + q^2)^{-1}$. Here, α_0^2 is the triplet deuteron binding energy in fm^{-2} ; similarly, α_1^2 is the singlet virtual state energy.

The relativistic t matrix in this case at the square of the center of mass (c.m.) energy $s = 4(m^2 + k^2)$ is given by

$$[t_n(q', q, k^2)]_{rel} = [v_n(q')]_{rel} [\tau_n^{-1}(k^2)]_{rel} [v_n(q)]_{rel}, \quad (4)$$

where

$$[\tau_n(k^2)]_{rel} = -\frac{1}{\lambda_n} - 4\pi \int_0^\infty q^2 dq \left(\frac{m}{\omega_q} \right) \frac{[v_n(q)]_{rel}^2}{k^2 - q^2 + i0}. \quad (5)$$

We generate a nonrelativistic two-nucleon t matrix, phase-equivalent to its relativistic version by the following transformation for the form-factors

$$[v_n(q)]_{nr} = (\sqrt{m/\omega_q}) [v_n(q)]_{rel}, \quad (6)$$

so that

$$[t_n(q', q, s)]_{nr} = [v_n(q')]_{nr} [\tau_n^{-1}(k^2)]_{rel} [v_n(q)]_{nr}, \quad (7)$$

The functional form of $[\tau]_{rel}$ of Eq. (7) is exactly identical to its relativistic counterpart (5).

The above recipe generates phase-equivalent two-nucleon potentials to be used in nonrelativistic and relativistic three-nucleon problem. The nonrelativistic and relativistic versions lead to the same deuteron binding α_0^2 in units of fm^{-2} . However, one uses a distinct relation for transforming this energy to MeV in relativistic and nonrelativistic versions. Consequently, the relativistic and nonrelativistic deuteron bindings are slightly different.

The nonrelativistic Faddeev equations for the three-nucleon system is given by[3]

$$\begin{aligned} \Xi_{n,n'}(p, p', E) &= Z_{n,n'}(p, p', E) + \sum_l \int_0^\infty q^2 dq Z_{n,l}(p, q, E) \left[-\frac{3}{2\pi} \tau_l^{-1}(mE - 3q^2/4) \right] \\ &\times \Xi_{l,n'}(q, p', E), \end{aligned} \quad (8)$$

with

$$Z_{n,n'}(p, q, E) = \frac{8\pi^2}{3} J_{n,n'} \int_{-1}^1 dx [v_n(\mathcal{P})]_{nr} G_{nr}(\vec{p}, \vec{q}, E) [v_{n'}(\mathcal{Q})]_{nr}, \quad (9)$$

where G_{nr} is the three-particle nonrelativistic propagator given by,

$$G_{nr}(\vec{p}, \vec{q}, E) = (p^2 + q^2 + pqx - mE - i0)^{-1}, \quad (10)$$

with

$$P^2 = p^2/4 + q^2 + pqx, \quad (11)$$

and

$$Q^2 = q^2/4 + p^2 + pqx. \quad (12)$$

Here J 's are the spin-isospin recoupling factors given by $J_{00} = J_{11} = 1/4$, and $J_{01} = J_{10} = -3/4$ for the spin doublet system. The scattering length in this case is given by $a_{nd} = -\Xi_{0,0}(0, 0, mE = -\alpha_0^2)$.

The three-dimensional relativistic generalization of these Faddeev equations has a form similar to Eq.(8) and is given by[13, 14, 16]

$$\begin{aligned} \Xi_{n,n'}(p, p', s) &= Z_{n,n'}(p, p', s) + \sum_l \int_0^\infty q^2 dq \frac{m}{\omega_q} Z_{n,l}(p, q, s) \left[-\frac{3}{2\pi} \tau_l^{-1} [(s - 3m^2 - 2\omega_q \sqrt{s})/4] \right] \\ &\times \Xi_{l,n'}(q, p', s), \end{aligned} \quad (13)$$

and Eq. (9) but with the relative momentum squares given by Eqs. (11) and (12) now changed to the following relativistic forms:

$$P^2 = (\omega_q + \omega_{pq})^2/4 - p^2/4 - m^2, \quad (14)$$

$$Q^2 = (\omega_p + \omega_{pq})^2/4 - q^2/4 - m^2. \quad (15)$$

Here we use notations $\omega_p = (m^2 + p^2)^{1/2}$, $\omega_{pq} = [m^2 + (\vec{p} + \vec{q})^2]^{1/2}$, etc. It should be noted that in the nonrelativistic limit Eq. (13) reduces to Eq. (8).

Finally, for the relativistic three-particle propagator G we use the following functions:[14, 16]

$$G_A(\vec{p}, \vec{q}, s) = \frac{2(\omega_p + \omega_q + \omega_{pq})}{\omega_{pq}[(\omega_p + \omega_q + \omega_{pq})^2 - s - i0]}; \quad (16)$$

$$G_B(\vec{p}, \vec{q}, s) = \frac{2(\omega_p + \omega_q)}{\omega_{pq}[(\omega_p + \omega_q)^2 - (\sqrt{s} - \omega_{pq})^2 - i0]}; \quad (17)$$

$$G_C(\vec{p}, \vec{q}, s) = \frac{1}{\omega_{pq}[\omega_p + \omega_q + \omega_{pq} - \sqrt{s} - i0]}; \quad (18)$$

$$G_D(\vec{p}, \vec{q}, s) = \frac{2(\omega_q + \omega_{pq})}{\omega_{pq}[(\omega_q + \omega_{pq})^2 - (\sqrt{s} - \omega_p)^2 - i0]} \quad (19)$$

In Eqs. (16) - (19) the parameter s is the square of the total c.m. energy of the three-particle system. All these propagators satisfy conditions of relativistic unitarity, governed by that part of the denominator in these propagators which corresponds to the pole for three-particle propagation in the intermediate state, e.g., at $\sqrt{s} = \omega_p + \omega_q + \omega_{pq}$. The condition of relativistic unitarity in these propagators is manifested in having the same residue at this pole.

All these equations satisfy two-particle unitarity via the use of the BIS equation. Equation (16) was implicit in the work of BIS but was explicitly advocated by Aaron, Amado, and Young[14] and obeys time-reversal symmetry, e.g. $G(\vec{p}, \vec{q}, s) = G(\vec{q}, \vec{p}, s)$, and both two- and three-particle unitarity. Equations (17) and (18) also have these virtues of Eq. (16). The propagators G_B and G_D were suggested recently in Ref. [16], G_C was suggested long ago.[15] It has been shown[20] that the propagator G_D follows from a suggestion by Ahmadzadeh and Tjon.[15] But in numerical applications of this propagator to the three-nucleon problem unnecessary nonrelativistic approximations have been used which violate conditions of unitarity.[20] The form (19) obeys three-particle unitarity, but violates time-reversal symmetry.

We have used all these forms, (16) - (19), in numerical calculation. As the propagators $G(\vec{p}, \vec{q}, s)$'s directly enter the Born term of the scattering equation, useful inequalities for the three-particle binding energies could be obtained with these propagators, which are later verified in numerical calculations. For example, from Eqs. (16) - (19) we have

$$G_A(\vec{p}, \vec{q}, s) = G_C(\vec{p}, \vec{q}, s) \frac{2(\omega_p + \omega_q + \omega_{pq})}{\omega_p + \omega_q + \omega_{pq} + \sqrt{s}}, \quad (20)$$

$$G_B(\vec{p}, \vec{q}, s) = G_C(\vec{p}, \vec{q}, s) \frac{2(\omega_p + \omega_q)}{\omega_p + \omega_q - \omega_{pq} + \sqrt{s}}, \quad (21)$$

$$G_D(\vec{p}, \vec{q}, s) = G_C(\vec{p}, \vec{q}, s) \frac{2(\omega_{pq} + \omega_q)}{\omega_{pq} + \omega_q - \omega_p + \sqrt{s}}, \quad (22)$$

The propagators G 's are directly proportional to the potentials in the three-nucleon system. It should be noted that both for three-nucleon bound-state and threshold scattering problems $s \simeq 3m$ and the variables p and q in Eqs. (20) - (22) run from 0 to ∞ . Consequently, ω_p , ω_q , and ω_{pq} are larger than m in this domain, and the factors multiplying $G_C(\vec{p}, \vec{q}, s)$ in

Eqs. (20) - (22) are larger than one. So the propagators and the potentials in models *A*, *B*, and *D* are stronger than that in the model *C*, implying $(B_t)_A > (B_t)_C$, $(B_t)_B > (B_t)_C$, and $(B_t)_D > (B_t)_C$. From Eqs. (20) and (21) one can see that the model potential *B* is stronger than the model potential *A*. Similarly, one can show that the model potential *D* is stronger than the model potential *A*. Consequently, $(B_t)_B > (B_t)_A > (B_t)_C$, and $(B_t)_D > (B_t)_A > (B_t)_C$. These are some useful inequalities. No such inequality could be established between models *B* and *D*.

Hence we have the following useful inequalities

$$(i)(B_t)_B > (B_t)_A > (B_t)_C, (ii)(B_t)_D > (B_t)_A > (B_t)_C, \quad (23)$$

which will be verified in the numerical calculation in the following section.

3 NUMERICAL RESULTS

For two-nucleon separable potentials in spin-triplet and spin-singlet channels we take the following Yamaguchi and Tabakin form-factors,[18] recently used by Rupp and Tjon:[9]

$$g_Y(q) = \frac{1}{q^2 + \beta^2}, \quad (24)$$

$$g_T(q) = \frac{q^2 + \nu^2}{q^2 + \gamma^2} \times \frac{q_c^2 - q^2}{(q^2 + \beta^2)^\kappa}, \kappa = 1.5, 2. \quad (25)$$

The Yamaguchi potential will be referred to as *Y*, and the Tabakin potential with $\kappa = 1.5, 2$ will be referred to as *T-1.5* and *T-2*, respectively. The constants of these potentials for the triplet and the singlet channels are slightly different from those of Rupp and Tjon and are given in Table I. These potentials fit the two-nucleon phase-shifts equally well as in the work of Rupp and Tjon. Potential (25) provides a change of sign of nucleon-nucleon phase shifts at higher energies in agreement with experiment.

We calculated the triton binding, B_t , and the neutron-deuteron scattering length, a_{nd} , in the nonrelativistic case as well as with each of the four versions of relativistic formulations *A - D*. Propagator *A* has been used before in numerical calculations of the three-nucleon problem. [7, 8, 9, 10, 12] Propagator *B* has been suggested only recently[16] and has never

been used before. Propagator C is the simplest and has been known for a long time,[15] but to the best of our knowledge has not been used in the three-nucleon problem.

Our results are exhibited in Table II. From Table II it is clear that the relativistic corrections to B_t and a_{nd} for various models may vary a lot, even the sign of the relativistic correction may change in agreement with inequality (23). All the relativistic models increase the triton binding energy, B_t , in relation to the nonrelativistic case, except model C which reduces the binding. The magnitude of the relativistic correction to B_t varies from 0.1 MeV to 0.7 MeV in different situations. The magnitude and even its sign changes when one changes the relativistic models. In view of this, and related flexibilities of the various relativistic models, it may not be quite meaningful to talk about the magnitude of relativistic effect with a view to reduce the discrepancy between experiment and nonrelativistic theoretical model calculation. The theoretical inequalities (23), however, hold true in all situations. In addition we observed in numerical calculations the following general inequality

$$(B_t)_D, (B_t)_B > (B_t)_A > (B_t)_{nr} > (B_t)_C. \quad (26)$$

Our principal finding is exhibited in Fig. 1 where we plot B_t versus a_{nd} for the present nonrelativistic and relativistic model calculations, as well as for many other nonrelativistic calculations taken from the literature. [21, 22, 23] The relativistic calculations differ in employing different relativistic dynamics and nucleon-nucleon potentials, the nonrelativistic calculations differ in variations of two-nucleon potential off-shell and/or three-nucleon potential. The trend of the relativistic calculations is identical to that of the nonrelativistic calculations. Hence, the effect of including relativistic dynamics in the three-nucleon problem can not be distinguished from the effect of varying the two- and three-nucleon potentials in nonrelativistic calculations. Consequently, a relativistic treatment of these low-energy observables may not enhance our knowledge of the underlying interactions, or dynamics. In other words, in the low-energy three-nucleon observables, the hope of separating the effect of on- and off-shell variations of the two- and three-nucleon potentials from the relativistic effect, in a model independent fashion, is remote.

Certain meson theoretic (two- and three-nucleon) potentials[4] when used in a certain relativistic formalism may reproduce low-energy three-nucleon observables. But this should not be considered as a mark of superiority of this model over others. An appropriate mixture

of on- and off-shell variations of the potentials and a relativistic formulation may reproduce certain experimental results, but not much physics could be learned from such studies. This happens because of the existence of a shape independent approximation to many of the low-energy observables, such as binding energy and scattering length, of the three-nucleon system, as in the two-nucleon system.[3, 21] These observables are insensitive not only to the variations of the shape of the potential, but also to inclusion of certain relativistic dynamics, provided that the triton binding is reproduced.

4 SUMMARY

We have calculated the contribution of relativistic effect on the neutron-deuteron scattering length and the triton binding energy employing several separable nucleon-nucleon potentials and three-particle relativistic equations. We have used combinations of Yamaguchi and Tabakin type potentials for the singlet and triplet nucleon-nucleon channels and four types of relativistic three-particle scattering equations. To the best of our knowledge, of these equations only those by Aaron, Amado, and Young (model *A*) has been used before in this context. Model *D* can be derived from a three-particle propagator derived by Ahmadzadeh and Tjon, but has not been used in numerical calculation before in this form. Previous numerical calculations with this propagator used unnecessary nonrelativistic approximations which violate conditions of relativistic covariance and unitarity, as has been pointed out recently.[20] Models *B* and *C* have not been used in numerical calculations before. All these models satisfy constraints of relativistic unitarity and covariance. However, these conditions are not enough to determine the dynamics. There still remains a lot of flexibility which results in very different relativistic corrections to the three-nucleon problem. The relativistic correction could be both positive and negative depending on the model chosen. The magnitude of relativistic correction to triton energy varies from 0.1 to 0.7 MeV (see, Table II), depending on the relativistic dynamics and nucleon-nucleon potential model employed. We have derived certain inequalities for the binding energies of different models which are verified in numerical calculations.

In addition to studying the relativistic corrections to B_t and a_{nd} we also studied the correlations among these two observables. They exhibit a correlated behavior in different

nonrelativistic model calculations employing different on-shell equivalent nucleon-nucleon potentials. The inclusion of a three-nucleon potential also does not change the situation. The present study of relativistic effect is also in accordance with this correlation. Hence the inclusion of relativistic dynamics and three-nucleon potential and off-shell variation of the nucleon-nucleon potential lead to similar correlated behavior of B_1 and a_{nd} . Consequently, it will be difficult to separate the effect of relativistic correction from the effect of a variation of the nucleon-nucleon potential off-shell from a study of these observables. This confirms the existence of a shape-independent approximation to these observables even after inclusion of the relativistic effect.[21]

Of course, there are other observables for the three-nucleon system, which should be directly sensitive to relativistic effect, such as the charge form factors. Because of the presence of the possible large effect of meson-exchange currents and of the non-nucleonic components in the nucleus, such observables are not easily tractable, and it has so far been difficult to draw model independent conclusion from studies of these observables.[1, 2]

We are aware that there is an inherent flexibility in deciding on the relativistic dynamics, in treating the spin variables relativistically, and in deciding the correct form of two- and three-nucleon potentials. We are far from exhausting all possibilities. But the tendency of existing the shape-independent approximation is so strong that we do not believe our conclusions to be so peculiar as to be of no general validity. Hence a relativistic framework may reduce the still existing discrepancy between theory and experiment, but this may not enhance our knowledge of the three-nucleon system.

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| | | <i>Yamaguchi</i> | <i>Tabakin - 1.5</i> | <i>Tabakin - 2</i> |
|---------|------------------------------|----------------------------|----------------------------|----------------------------|
| 3S_1 | λN^2 | 0.4012 (fm ⁻³) | 1.7316 (fm ⁻¹) | 256.20 (fm ⁻³) |
| | β (fm ⁻¹) | 1.4117 | 4.0335 | 5.1435 |
| | ν (fm ⁻¹) | | 0.8067 | 0.8400 |
| | γ (fm ⁻¹) | | 0.7324 | 0.7534 |
| | q_c (fm ⁻¹) | | 2.1205 | 2.1205 |
| 1S_0 | λN^2 | 0.1487 (fm ⁻³) | 0.9455 (fm ⁻¹) | 216.01 (fm ⁻³) |
| | β (fm ⁻¹) | 1.1560 | 4.057 | 5.074 |
| | ν (fm ⁻¹) | | 1.1643 | 1.1415 |
| | γ (fm ⁻¹) | | 0.9237 | 0.9065 |
| | q_c (fm ⁻¹) | | 1.6966 | 1.6966 |

Table 1. Yamaguchi and Tabakin potential parameters $\lambda N^2, \beta, \nu, \gamma, q_c$, etc. The quantity λN^2 is the usual strength of the separable potential, where N is the normalization of the two-nucleon state. These parameters are fitted for the 3S_1 state to $a = 5.424$ fm, $\alpha_0 = 0.23161$ fm⁻¹, and for the 1S_0 state to $a = -23.748$ fm, and $\alpha_1 = 0.03992$ fm⁻¹.

| Potential | | nr | A | B | C | D |
|-----------|----------|-------|-------|-------|-------|-------|
| YY | B_t | 10.65 | 10.73 | 10.92 | 10.39 | 10.91 |
| YY | a_{nd} | -0.77 | -0.83 | -0.94 | -0.61 | -0.94 |
| T-2Y | B_t | 8.06 | 8.14 | 8.34 | 7.91 | 8.30 |
| T-2Y | a_{nd} | 0.94 | 0.87 | 0.73 | 1.04 | 0.75 |
| YT-2 | B_t | 7.69 | 7.87 | 8.30 | 7.52 | 8.19 |
| YT-2 | a_{nd} | 1.30 | 1.15 | 0.82 | 1.42 | 0.90 |
| T-1.5Y | B_t | 7.99 | 8.09 | 8.35 | 7.85 | 8.27 |
| T-1.5Y | a_{nd} | 0.98 | 0.91 | 0.71 | 1.08 | 0.77 |
| YT-1.5 | B_t | 7.59 | 7.78 | 8.38 | 7.44 | 8.17 |
| YT-1.5 | a_{nd} | 1.39 | 1.22 | 0.75 | 1.49 | 0.93 |

Table 2. Triton binding energy B_t (MeV) and neutron-deuteron scattering length a_{nd} (fm) for different nucleon-nucleon potential models (Yamaguchi, Tabakin-1.5, and Tabakin-2) and relativistic (A, B, C, D) and nonrelativistic (nr) dynamics. The three-nucleon potential model XY has a triplet X and singlet Y nucleon-nucleon potential, where each of X and Y could be $Y, T-1.5,$ and $T-2$. For example, $YT-2$ denotes a triplet Yamaguchi and singlet Tabakin-2 potential.

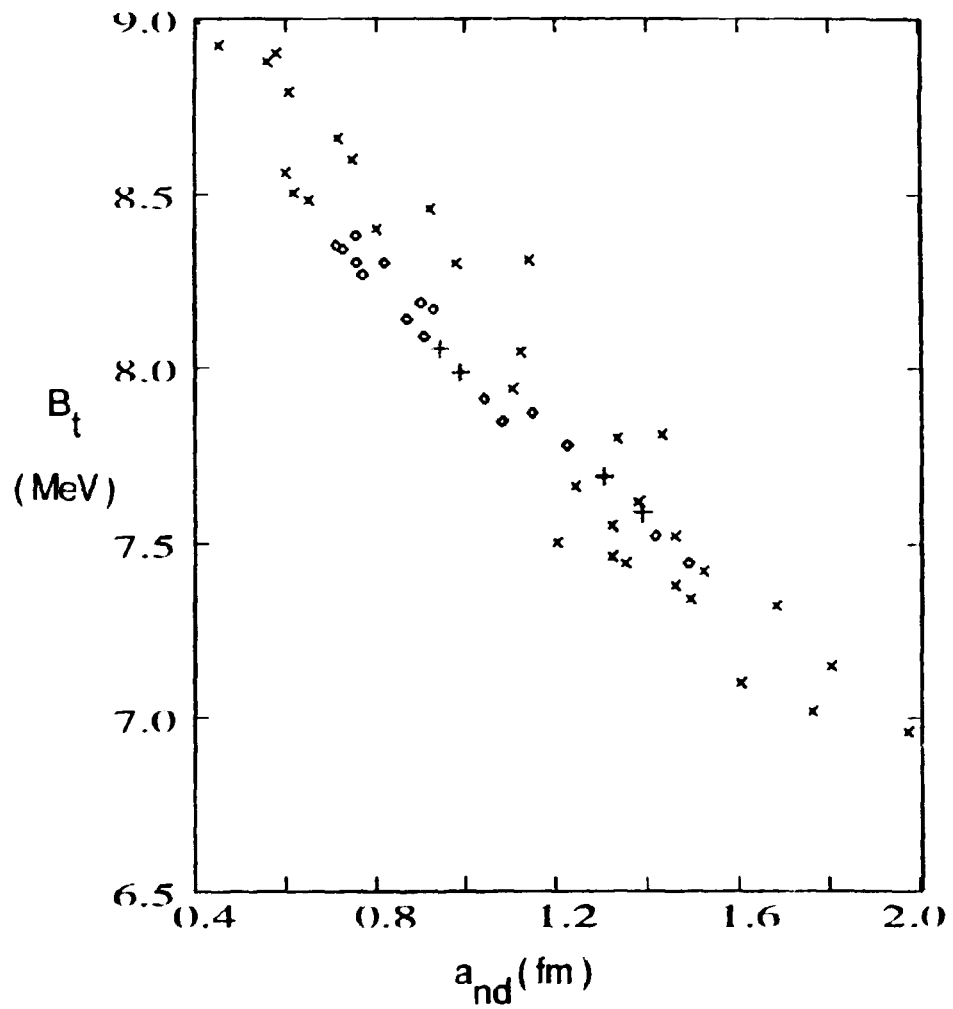


Figure 1: The B_t versus a_{nd} plot for various trinucleon models: the present relativistic models (\circ), the present nonrelativistic models ($+$), and other nonrelativistic models taken from the literature (\times).